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Amplitude Factorization Method in the Atmospheric Gravity-Wave Equation

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Abstract. A novel amplitude factorization method is applied to solve a discrete buoyancy wave equation with arbitrary wind and temperature height distribution. The solution is given in the form of a cumulative product of complex factors, which are computed by a nonlinear, inhomogeneous, two-member recurrence formula, initiated from a radiative condition on top. Singularities of the wave equation due to evanescent winds are eliminated by turbulent friction. The method provides an estimation of the minimal vertical resolution, required to attain a stable accurate solution. The areas of application of the developed numerical scheme are high resolution modelling of orographic waves for arbitrary orography in general atmospheric stratification conditions, and testing of adiabatic kernels of numerical weather prediction models.

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1 Introduction

Models used for buoyancy wave studies can be divided into linear analytical [1-16] and nonlinear numerical (finite difference) schemes [17-31].

Advantages of the linear analytical approach are the high spatial and temporal resolution, a possibility to study stationary and transient regimes in separation, and the existence of analytical means for stability etc. analysis. The main disadvantage is the restriction to the simplest ('analytical') flow regimes (like the linear shear or constant stability).

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Benefits of finite difference schemes lie in base-state generality, which supports realistic experimental conditions. Their main disadvantages are the moderate spatial resolution and inability to treat the steady and transient regimes separately.

In this paper, a novel method for the solution of the linear buoyancy wave equation is presented, combining the best of both approaches, being linear and (as much as possible) analytical, and at the same time numerical, accepting arbitrary vertical distributions of reference wind and temperature. The algorithm proves effective and fast, as it makes use of the direct computation of the attenuation factors of the wave amplitude via a straightforward recurrence formula with the initial value specification from a wave radiation condition on the top. The cumulative product of those factors from bottom to top yields a solution for the omega-velocity within a constant factor precision, the value of which is specified from bottom boundary condition. Thus, solution of the wave equation will take very little effort and the major computational time goes to preparation of the equation coefficients and summation of the obtained orthogonal modes over wave numbers to get the solution in ordinary physical coordinates. Also, the numerical solution has a clear physical meaning, as the modulus of the complex attenuation factor represents the actual decrease of the wave amplitude per single layer of the discrete model, whereas its argument is the phase angle increment per layer.

As the model deals with variable winds, a problem arises inevitably with critical wave-vectors and critical levels, corresponding to singularities of the wave equation. This problem is solved in the current numerical approach with inclusion of turbulent friction in forcing terms, yielding singularity removal. The use of turbulent viscosity for wave-equation regularization was proposed already in [5,32,33]. However, the theoretical estimates of the maximum stable vertical grid-step will show that the requirements for high vertical resolution remain, especially in the vicinity of an evanescent wind level. This is the point where the numerical efficiency of the method becomes crucial, enabling the application of sufficiently high spatial resolution where appropriate.

While there exist various 'ready' wave equations they do differ rather substantially in appearance, depending on the 'small' details of the dynamical model, its coordinate system etc. To avoid potential ambiguities in basic definitions, we start with a short introduction of the wave equation used in this investigation from the Miller-Pearce-White (MPW) non-hydrostatic, semi-elastic pressure-coordinate model [34,35].

2 Buoyancy-wave equation

2.1 Continuous model

When using the non-dimensional log-pressure coordinate $\zeta = \ln(p_0/p)$ instead of the common pressure-coordinate p (p_0 = 1000 hPa is the mean sea-level pressure), the lin-