English Series | NUMERICAL MATHEMATICS A Journal of Chinese Universities | Vol.14, No.1 Feb. 2005

MULTILEVEL AUGMENTATION METHODS FOR SOLVING OPERATOR EQUATIONS*

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Abstract *We introduce multilevel augmentation methods for solving operator equations based on direct sum decompositions of the range space of the operator and the solution space of the operator equation and a matrix splitting scheme. We establish a general setting for the analysis of these methods, showing that the methods yield approximate solutions of the same convergence order as the best approximation from the subspace. These augmentation methods allow us to develop fast, accurate and stable nonconventional numerical algorithms for solving operator equations. In particular, for second kind equations, special splitting techniques are proposed to develop such algorithms. These algorithms are then applied to solve the linear systems resulting from matrix compression schemes using wavelet-like functions for solving Fredholm integral equations of the second kind. For this special case, a complete analysis for computational complexity and convergence order is presented. Numerical examples are included to demonstrate the efficiency and accuracy of the methods. In these examples we use the proposed augmentation method to solve large scale linear systems resulting from the recently developed wavelet Galerkin methods and fast collocation methods applied to integral equations of the second kind. Our numerical results confirm that this augmentation method is particularly efficient for solving large scale linear systems induced from wavelet compression schemes.*

Key words *Multilevel augmentation methods, operator equations, Fredholm integral equations of the second kind.*

AMS(2000)subject classifications 65J10, 65R20

* Supported in part by the Natural Science Foundation of China under grants 10371137 and 10201034, the Foundation of Doctoral Program of National Higher Education of China under under grant 20030558008, Guangdong Provincial Natural Science Foundation of China under grant 1011170 and the Foundation of Zhongshan University Advanced Research Center. Supported in part by the US National Science Foundation under grants 9973427 and 0312113, by the Natural Science Foundation of China under grant 10371122 and by the Chinese Academy of Sciences under the program "One Hundred Distinguished Chinese Scientists".
Received: Jun. 15, 2004.

1 Introduction

Developing stable, efficient and fast numerical algorithms for solving operator equations including differential equations and integral equations is a main focus of research in numerical analysis and scientific computation. The development of such algorithms is particularly important for large scale computation. Solving an operator equation normally requires three steps of processing. The first step, at the level of approximation theory, is to choose appropriate subspaces and their suitable bases. The second step is to discretize the operator equations using these bases and to analyze convergence properties of the approximate solutions. This step of processing which results in a discrete linear system is a main task considered at the level of numerical solutions of operator equations. The third step, at the level of numerical linear algebra, is to design an efficient solver for the discrete linear system resulting from the second step. The ultimate goal is to efficiently solve the discrete linear system which gives an accurate approximate solution of the original operator equation. Theoretical consideration and practical implementation in the numerical solution of operator equations show that these three steps of processing are closely related. A good way of designing efficient algorithms for the discrete linear system should be taken into consideration choices of subspaces and their bases, methodologies of discretization of the operator equations and numerical solvers of the resulting discrete linear system. The well-known multigrid method (cf., [16, 17]) and the related two-grid method [3,4] are excellent examples of such algorithms.

A good algorithm for solving operator equations should be convenient for implementing adaptivity. When a computed approximate solution is confirmed to be not accurate enough, a local or global subdivision is often made aiming at a solution at a finer level. An efficient algorithm should be able to update the old computed approximate solution obtaining a new, more accurate approximate solution, at an additional expense proportional to the net gain in accuracy, avoiding solving the whole equation at the finer level. In other words, the additional computational costs to obtain the additional accuracy from a (local or global) subdivision should be proportional to the dimension of the difference space between the coarse level and the finer level, not at the expense in the order of the dimension of the finer level space. Presently, to our best knowledge, existing numerical methods in the literature are not yet able to meet this need. The research reported in this paper is an attempt to accomplish this goal.

We introduce a multilevel augmentation method for solving operator equations based on multilevel decompositions of the approximate subspaces, aiming at efficiently solving linear systems of large scale obtained from discretization of the operator equations. For this purpose, we require that both the range space of the operator and the solution space of the operator equation have direct sum decompositions. Our method consists of two steps. In the first step, we use the direct sum decompositions to discretize the operator equation and result in a linear system. Reflecting the direct sum decompositions of the subspaces, the coefficient matrix of the