English Series | NUMERICAL MATHEMATICS A Journal of Chinese Universities | Vol.14, No.2 May 2005

A STABILITY ANALYSIS OF THE (k) JACOBI MATRIX INVERSE EIGENVALUE PROBLEM*

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Abstract *In this paper we will analyze the perturbation quality for a new algorithm of the (k) Jacobi matrix inverse eigenvalue problem.*

Key words *eigenvalue, Jacobi matrix, (k) inverse problem.*

AMS(2000)subject classifications $65F18$

1 Introduction

Let

$$T_n = \begin{pmatrix} \alpha_1 & \beta_1 & & & 0 \\ \beta_1 & \alpha_2 & \beta_2 & & \\ & \beta_2 & \ddots & \ddots & \\ & & \ddots & \ddots & \beta_{n-1} \\ 0 & & & \beta_{n-1} & \alpha_n \end{pmatrix}$$

be an $n \times n$ unreduced symmetric tridiagonal matrix, and denote its submatrix $T_{p,q}, (p < q)$ as follows

$$T_{p,q} = \begin{pmatrix} \alpha_p & \beta_p & & & 0 \\ \beta_p & \alpha_{p+1} & \beta_{p+1} & & \\ & \beta_{p+1} & \ddots & \ddots & \\ & & \ddots & \ddots & \beta_{q-1} \\ 0 & & & \beta_{q-1} & \alpha_q \end{pmatrix} \quad p < q.$$

We call an unreduced symmetric tridiagonal matrix with $\beta_i > 0$ as a Jacobi matrix.
Consider $T_{1,n}$ and $T_{p,q}$ to be Jacobi matrices. The matrix

$$W_k = \begin{pmatrix} T_{1,k-1} & 0 \\ 0 & T_{k+1,n} \end{pmatrix}$$

is obtained by deleting the k^{th} row and the k^{th} column $(k = 1, 2, ..., n)$ from T_n.

*Received: Mar. 15, 2003.

Problem If we don't know the matrix $T_{1,n}$, but we know all eigenvalues of matrix $T_{1,k-1}$, all eigenvalues of matrix $T_{k+1,n}$, and all eigenvalues of matrix $T_{1,n}$,could we construct the matrix $T_{1,n}$.

Let

$$S1 = (\mu_1, \mu_2, \cdots, \mu_{k-1}),$$
$$S2 = (\mu_k, \mu_{k+1}, \cdots, \mu_{n-1}),$$
$$\lambda = (\lambda_1, \lambda_2, \cdots, \lambda_n)$$

are the eigenvalues of matrices $T_{1,k-1}, T_{k+1,n}$ and $T_{1,n}$ respectively. The problem is that from above 2n-1 data to find other 2n-1 data:

$$\alpha_1, \alpha_2, \cdots, \alpha_n, \quad \text{and} \quad \beta_1, \beta_2, \cdots, \beta_{n-1}.$$

Obviously, when $k = 1$ or $k = n$ this problem has been solved and there are many algorithms to construct $T_{1,n}$ [3][6], and its stability analysis can be founded in [2]. While $k = 2, 3 \ldots n-1$, a new algorithm has been put forward to construct $T_{1,n}$ [1]. In this paper we will give some stability properties of the new algorithm in case $k = 2, 3 \ldots n-1$.

2 Basic Theorem

Theorem 2.1[1] If there is no common number between $\mu_1, \mu_2, \cdots, \mu_{k-1}$ and $\mu_k, \mu_{k+1}, \cdots, \mu_{n-1}$, then the necessary and sufficient condition for the (k) problem having a solution is:

$$\lambda_1 < \mu_{j_1} < \lambda_2 < \mu_{j_2} < \cdots < \mu_{j_{n-1}} < \lambda_n, \tag{2.1}$$

where $\mu = (\mu_{j_1}, \mu_{j_2} \ldots \mu_{j_{n-1}})$, and $\mu_i, (i = 1, 2 \ldots n-1)$ are recorded as $\mu_{j_i}, (i = 1, 2 \ldots n-1)$ such that

$$\mu_{j_1} < \mu_{j_2} < \cdots < \mu_{j_{n-1}}. \tag{2.2}$$

Furthermore, if a given (k) problem has a solution, then the solution is unique.

Algorithm 2.2[1]

Given three vectors $\lambda = (\lambda_1, \lambda_2 \cdots \lambda_n)^T$,$S1 = (\mu_1, \mu_2 \cdots \mu_{k-1})^T$ and $S2 = (\mu_k, \mu_{k+1} \cdots \mu_{n-1})^T$ which are satisfied with (2.1), then we can solve (k) problem by following algorithm:

Step 1 Find α_k as

$$\alpha_k = \text{trace}(T_{1,n}) - \text{trace}(W_k) = \sum_{i=1}^{n} \lambda_i - \sum_{i=1}^{n-1} \mu_i. \tag{2.3}$$