Simulated Annealing for the 0/1 Multidimensional Knapsack Problem

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Abstract

In this paper a simulated annealing (SA) algorithm is presented for the 0/1 multidimensional knapsack problem. Problem-specific knowledge is incorporated in the algorithm description and evaluation of parameters in order to look into the performance of finite-time implementations of SA. Computational results show that SA performs much better than a genetic algorithm in terms of solution time, whilst having a modest loss of solution quality.

Keywords: Simulated annealing algorithm; multidimensional knapsack; static cooling schedule; finite-time implementation.
Mathematics subject classification: 65K05, 90C10, 90C59

1. Introduction

The NP-hard 0/1 multidimensional knapsack problem (MKP) represents an optimization model, which can formulate many practical problems such as capital budgeting problem, cargo loading [8] and cutting stock problems [2].

Several metaheuristics have been developed for MKP, including tabu search [3, 4] and genetic algorithm [1]. These metaheuristic approaches have yielded very good results so far.

In this paper, an SA is implemented to solve the 0/1 MKP with static cooling schedule. Here, a reasonable strategy is used to generate random solution in the neighborhood, the relevant parameters concerning cooling schedule are tested in order to obtain finite-time implementations of the algorithm.

Section 2 gives the 0/1 multidimensional knapsack problem formulation. Section 3 describes the approach and preliminary testing for identifying parameters. The computational results are presented in Section 4. The conclusions are summarized in Section 5.

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2. Problem formulation

The MKP is a classical 0-1 combinatorial optimization problem that can be applied to various fields such as economics. A set of n items and a set of m resources are given. Each item j ($j = 1, \cdots, n$) has assigned a profit p_j and a resource consumption value r_{ij} for each resource i ($i = 1, \cdots, m$). The problem is to identify a subset of all items that leads to the highest total profit and does not exceed the resource upper bound b_i. The MKP can be formulated as:

$$\text{maximize} \quad f = \sum_{j=1}^{n} p_j x_j \tag{2.1}$$

subject to

$$\sum_{j=1}^{n} r_{ij} x_j \le b_i, i = 1, \cdots, m, \quad x_j \in \{0, 1\}, \; j = 1, \cdots, n. \tag{2.2}$$

The variable x_j is an indicator of item j. If x_j is set to 1, it means item j is selected, or 0 means item j is not selected for $j = 1, \cdots, n$. Eq. (1) represents the total profit of selection items and Eq. (2) the m resource constraints. A well stated 0/1 MKP assumes that $p_j > 0$ and

$$r_{ij} \le b_i \le \sum_{j=1}^{n} r_{ij} \quad \text{for all} \; i = 1, \cdots, m, \; \text{and} \; j = 1, \cdots, n.$$

3. Simulated annealing

Simulated annealing is a local search algorithm capable of escaping from local optima. Its ease of implementation, convergence properties, and its capability of escaping from local optima has made it a popular algorithm over the past decades. Simulated annealing is so named because of its analogy to the process of physical annealing with solids, in which a crystalline solid is heated and then allowed to cool very slowly until it achieves its stable state, i.e. its minimum lattice energy state, and thus is free of crystal defects. Simulated annealing mimics this type of thermodynamic behavior in searching for global optima for discrete optimization problems (DOP).

At each iteration of a simulated annealing algorithm applied to a DOP, the objective function values for two solutions (the current solution and a newly generated neighboring solution) are compared. Better solutions are always accepted, while a fraction of inferior solutions are accepted in the hope of escaping local optima in search of global optima. The probability of accepting non-improving solutions depends on a temperature parameter, which is non-increasing with each iteration of the algorithm.

The key algorithmic feature of simulated annealing is that provides a means to escape local optima by allowing worse moves (i.e. moves to a solution that corresponds to a worse objective function value). As the temperature is decreased to zero, worse moves occur less frequently, and the solution distribution associated with the inhomogeneous Markov chain that models the behavior of the algorithm converges to a distribution in which all the