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Electrodynamics of relativistic particles through non-standard Lagrangian

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Abstract. The main aim of this paper is to discuss the electrodynamics of relativistic dynamics of particles bases on the notion of the non-standard Lagrangians which have gained increasing importance in the theory of nonlinear differential equations, dissipative dynamical systems and theoretical physics. The mathematical settings are constructed starting from the modified Euler-Lagrange equation and modified Hamiltons equations. Some illustrative examples are considered and discussed.

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Key words: non-standard Lagrangians, modified relativistic electrodynamics, modified Hamiltonian

1 Introduction

The notion of non-standard Lagrangians (NSL) is not new and it as in fact introduced by Arnold in 1978 [1]. In reality, NSL were not considered seriously in the past for two major reasons. First of all, the physical meaning of NSL is still obscure and besides their Hamiltonian formalism was problematic in particular when it is related to quantization process. However, in the progress of years, it was observed that NSL plays an important role in the theory of nonlinear differential equations [2-4], dissipative dynamical systems [5-15] and in many problems related to theoretical physics [16-18]. Their applications seem wide yet lots of works are required for a better understanding of their physical significances. Some advances to understand the root of NSL based on the theories of inverse variational problem were discussed in [15] yet the problem is still open. It should be emphasized that recent works prove that a number of dynamical systems may be described by two different Lagrangians: one is standard and another non-standard one.

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In other words, the resulting differential equations of motion may be obtained from two different Lagrangians [9-11]. Accordingly, one may argue that a complete description of physical systems require the knowledge of both types of Lagrangians. In this work, we would like to explore the implications of NSL in relativistic electrodynamics. It should be noted that NSL may came in different mathematical forms as discussed in [11], yet through this paper we pick the power-law form. It should be stressed that both the kinetic terms and the potential function in the NSL are non-standard yet physically interesting nonlinear dynamics were obtained. There is one more simple elucidation to add is that in our approach NSL refers to "standard Lagrangian function that modifies the Euler-Lagrange equations and accordingly the Hamilton's equations of motion".

The paper is organized as follows: in Sec. 2, we introduce basic settings mainly the power-law NSL and its corresponding Euler-Lagrange and Hamilton's equations of motions in the presence of electromagnetic forces (EM). In Sec. 3, we discuss the modified dynamics of relativistic particles in the absence and in the presence of the electromagnetic field for different values of ξ. The paper concludes in Sec. 4 with a brief summary of main results and perspectives.

2 Electrodynamics with non-standard Lagrangians and the modified Euler-Lagrange and Hamilton's equations

Through this work we define the power-law NSL by $L_{NSL}{=}L^{1+\xi}(\dot{q},q,t)$ where $L(\dot{q},q,t)\epsilon C^2$ $([a,b]\times\mathbb{R}^n\times\mathbb{R}^n;\mathbb{R})$ is the standard Lagrangian of the theory $(\dot{q},q,t)\to L(\dot{q},q,t)$ assumed to be a C^2 function with respect to all its arguments and ξ is a free parameter which is different from -1. Here $\dot{q}{=}dq/dt$ is the time-derivative of the generalized coordinate. The action functional of the theory is defined by:

$$S=\int_a^b \Lambda L^{1+\xi}(\dot{q}(t),q(t),t)dt, \tag{1}$$

and the basic problem is to define the extremum of the functional $S=D\to\mathbb{R}$ where D is the subset of D which is the set of all functions $q:[a+b]\to\mathbb{R}^n$ such that the temporal derivative of $\dot{q}$ exists and is continuous on $[a+b]$. In Eq. (1), the parameter Λ is a free parameter that is introduced for physical arguments (this parameter guarantees the correct physical dimensionalities for all terms). It is an easy exercise to prove that if $q(t)$ is a local minimizer to the action (1) then the following modified Euler-Lagrange equation (MELE) holds [11]:

$$\frac{\partial L}{\partial q}-\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right)=\xi\frac{1}{L}\frac{\partial L}{\partial \dot{q}}\left(\frac{\partial L}{\partial t}+\dot{q}\frac{\partial L}{\partial q}+\ddot{q}\frac{\partial L}{\partial \dot{q}}\right). \tag{2}$$

Two main features of Eq. (2) concern first its RHS which depends on the total derivative of $L(\dot{q},q,t)$ and the form of the momentum conjugate and its time derivative which take respectively the forms $p=\Lambda\partial L^{1+\xi}/\partial\dot{q}$ and $\dot{p}=\Lambda\partial L^{1+\xi}/\partial q$. We will prove that these