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NUMERICAL OPTIMIZATION OF RADIATED ENGINE NOISE WITH UNCERTAIN WAVENUMBERS

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Abstract. In this paper, we investigate an efficient numerical method to identify an optimal impedance factor for mitigating radiated engine noise. The engine tone-noise wavenumber is treated as a random variable. We prove the existence of the sensitivity derivative of the state variable (which is the acoustic pressure) with respect to the random wavenumber. The proposed numerical method is based on the stratified Monte Carlo algorithm whose convergence is accelerated by exploiting the sensitivity derivative information.

Key Words. optimal control, liner impedance factor, random wavenumber, finite element

1. Introduction

The purpose of this paper is to study the optimal design of the acoustic liner to minimize fan noise radiation from commercial aircraft engine nacelles. There have been numerous studies of this problem from an engineering perspective (e.g., [12, 26, 27] and references therein). In [8, 11], the search for the liner material minimizing engine noise radiation was treated as an optimal control problem and liner impedance factors that yielded significant noise reduction were found both theoretically and numerically.

Optimal control of systems governed by partial differential equations is a very active research subject [2, 14, 15, 16, 17, 20]. Most research in this area makes the natural (but rather unrealistic) assumption that the input data and model parameters of the control systems are precisely known. But it is well known that every physical system is subject to uncertainty due to varying operating conditions and imprecise measurements, and that when a mathematical model is formulated, further uncertainties may arise due to modeling and discretization errors [3, 18]. Practical experience suggests that uncertainties in the input data and the parameters of control systems can drastically reduce the reliability and accuracy of the deterministic optimal control approach.

Optimal control of stochastic partial differential equations clearly makes the control models more complex, but more flexible, realistic and hence of practical value. As the effect of parameter uncertainty is built into the model, one would expect that the optimal control will be less sensitive to changes in the model parameters, and hence more robust.

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The key to the numerical solution of the proposed problem is an efficient simulation method to evaluate the cost function. The Monte Carlo simulation is a generally applicable solution method for stochastic problems. However, it is a well-known that as the accuracy requirement increases, the number of realizations (deterministic problems to be solved) grows far too rapidly. Consequently, algorithms that employ the Monte Carlo method may be easy to program, but impossible to employ on problems of practical interest. Alternatives to the Monte Carlo method include moment methods and the polynomial chaos method [24, 23, 25]. However, moment methods are accurate only for small variance problems. The polynomial chaos method may prove to be a viable alternative.

In this paper, we apply a modified Monte Carlo algorithm using the sensitivity derivatives of the state variable with respect to the uncertain model parameter – the acoustic wavenumber in the present case. The method was first developed in [4] and tested extensively in [6, 7, 28]. We focus on the combination of this method and the stratified sampling method. We provide the existence result on sensitivity derivatives as well as the variance reduction analysis on the sensitivity-derivative enhanced stratified Monte Carlo method (SDSMCM). Numerical experiments demonstrate the efficiency of this numerical method.

2. Optimal control model for the optimal impedance factor

2.1. Model formulation. We assume the problem to be axisymmetric [5]. The nacelle geometry has the generic shape represented in Figure 1. The modal composition of the noise source is supposed to be known on the source plane Γ_1. The nacelle boundary is made up of two parts: the first part being the interior boundary Γ_2 to which some acoustic liner material is attached, and the second part being Γ_3 that constitutes the rest of boundary of the nacelle geometry. The boundary Γ_4 is assumed to be sufficiently far from the noise source so that the Sommerfeld radiation boundary condition is an adequate approximation. The nacelle symmetry axis is denoted by Γ_5.

If the meanflow is uniform with Mach number M_0, the governing equation for the acoustic pressure u [21] is

$$(1 - M_0^2)\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} - 2ikM_0\frac{\partial u}{\partial x} + k^2 u = 0. \tag{2.1}$$

For simplicity, we set the mean Mach number M_0 equal to zero. The acoustic pressure u then satisfies the Helmholtz equation,

$$\Delta u + k^2 u = 0 \quad \text{on } \Omega, \tag{2.2}$$

subject to the following boundary conditions on the boundary $\partial\Omega$ of Ω:

$$\begin{aligned} u\,\big|_{\Gamma_1} &= g, \\ \left(\tfrac{\partial u}{\partial n} + \tfrac{ik}{\xi}u\right)\Big|_{\Gamma_2} &= 0, \\ \tfrac{\partial u}{\partial n}\Big|_{\Gamma_3} &= 0, \\ \left(\tfrac{\partial u}{\partial n} + iku\right)\Big|_{\Gamma_4} &= 0, \\ \tfrac{\partial u}{\partial n}\Big|_{\Gamma_5} &= 0, \end{aligned} \tag{2.3}$$

where k is the wavenumber and ξ is the complex impedance factor whose real part represents resistance and the imaginary part reactance. Both the dependent and