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MONOTONIC ITERATIVE ALGORITHMS FOR A QUASICOMPLEMENTARITY PROBLEM*

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Abstract

We present two iterative algorithms, so called SCP and SA respectively, for solving quasicomplementarity problem (QCP). Algorithm SCP is to approximate QCP by a sequence of ordinary complementarity problems (CP). SA is a Schwarz algorithm which can be implemented parallelly. We prove the algorithms above are monotonically convergent.

Key words: Quasicomplementarity problem, Iterative algorithm, Monotonic convergence, Schwarz algorithm.

1. Introduction

Consider the following QCP: find $u \in R^n$ such that

$$\min\{Au - f, \quad u - Bu\} = 0, \tag{1}$$

where A, B: $R^n \to R^n$ are operators, $f \in R^n$. The quasivariational inequality which is equivalent to (1) sounds: to find $u \in R^n$ such that $u \geq Bu$ and

$$(Au, \ \ v - u) \geq (f, \ \ v - u), \quad \forall v \geq Bu. \tag{2}$$

If $Bu \equiv c \in R^n$ for any $v \in R^n$ then $QCP(1)$ reduces into CP. (1) appears in mathematical programming (see, for example, [2] and the references therein), also comes from the discretization of QCP in mathematical physics and control theory (see [1]). For various generalization, see [3] and the references therein.

We assume in this paper that A is a strictly T-monotonic operator, that is:

$$(Au - Av, \ \ (u - v)^+) \geq 0, \qquad \forall u, v \in R^n,$$

where the equality holds only if $(u - v)^+ = 0, \ \ v^+ = \max\{v, 0\}$. The examples in [6] show that strictly T-monotonic operator is widely applicable.

We propose two algorithms for solving (1). The first (SCP) is to solve iteratively a sequence of CP, which produces a sequence of approximate solutions convergent monotonically to a solution of (1). The second (SA) is a Schwarz algorithm, which is parallel algorithm and produce super(sub)solution sequence of (1), convergent monotornically to a solution of (1).

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Schwarz algorithms have been developed rapidly (for example, see [4], [6] and the references therein), But Algorithm SA is the first Schwarz algorithm for quasivariational inequality.

2. Sequential CP Algorithm

Algorithm SCP

1^0. Take $u^0 \in R^n$, $k := 0$;

2^0. Find $u^{k+1} \in R^n$ such that

$$\min\left\{Au^{k+1} - f,\ \ u^{k+1} - Bu^k\right\} = 0; \tag{3}$$

3^0. $k := k+1$, go to 2^0.

At first we study an operator F related to (3) and defined as follows: for any $x \in R^n$ define $y = Fx$ as the solution of the following CP:

$$\min\{Ay - f,\ \ y - Bx\} = 0. \tag{4}$$

It has a unique solution if operator A is continuous, strictly T-monotonic and coercive in some sense (see, for example, [5]). Then F is well-defined.

We call an operator B order-preserved if $v \le w$ implies $Bv \le Bw$.

Lemma 1. *Assume B is continuous, order-preserved and there exists $b \in R^n$ such that*

$$Bv \le b, \qquad \forall v \in R^n. \tag{5}$$

Assume A is continuous, strictly T-monotoinic and

$$\frac{(Av,\ \ v-b)}{\| v \|} \to +\infty \quad (\| v \| \to \infty). \tag{6}$$

Then the range of F, denoted by $R(F)$, is bounded. That is, there exist $p, q \in R^n$ such that

$$p \le Fx \le q, \qquad \forall x \in R^n.$$

Proof. Since (6) implies the coerciveness condition of A in [5], (4) has a unique solution for any $x \in R^n$ and F is well-defined. Hence we have

$$\min\{AFx - f, \quad Fx - Bx\} = 0, \qquad \forall x \in R^n.$$

Then for any $x \in R^n$ we have

$$(AFx,\ \ v - Fx) \ge (f,\ \ v - Fx), \qquad \forall v \ge Bx.$$

Letting $v = b$ in it we obtain

$$(AFx, Fx - b) \le (f, Fx - b), \quad \forall x \in R^n$$

which combining with (6) yields that F is bounded.

Remark 1. If there exists a constant α such that

$$(Av - Aw, v - w) \ge \alpha \| v - w \|^2, \quad \forall v, w \in R^n$$