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NEW ODE METHODS FOR EQUALITY CONSTRAINED OPTIMIZATION (2) — Algorithms*

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Abstract

As a continuation of [1], this paper considers implementation of ODE approaches. A modified Hamming's algorithm for integration of (ECP)-equation is suggested to obtain a local solution. In addition to the main algorithm, three supporting algorithms are also described: two are for evaluation of the right-hand side of (ECP)-equation, which may be especially suitable for certain kinds of (ECP)-equation when applied to large scale problems; the third one, with a convergence theorem, is for computing an initial feasible point. Our numerical results obtained by executing these algorithms on an example of (ECP)-equation given in [1] on five test problems indicate their remarkable superiority of performance to Tanabe's ODE version that is recently claimed to be much better than some well-known SQP techniques.

This work is a continuation of [1], so the same notation is used as before and the section numbers are continued. Implementation problems of ODE methods are considered in detail here. In Section 4, The main algorithm, a modified Hamming's algorithm for integration of (ECP)-equation, is described . Two supporting algorithms for evaluation of the right-hand side of (ECP)-equation are presented in Section 5, which may be especially suitable for certain kinds of (ECP)-equation when applied to large scale problems. Then, in Section 6, an algorithm for computing an initial feasible point and a related convergence theorem are given. Finally, Section 7 presents our numerical results obtained by executing these algorithms on an example of (ECP)-equation proposed in [1] on five test problems. These results show their remarkable superiority of performance to Tanabe's ODE version that is recently claimed to be much better than some well-known SQP techniques[2].

§4. Numerical Integration of (ECP)-Equation

Without loss of generality, from now on we will consider the implementation problems of ODE approaches only for minimization problem (ECP). Once an (ECP)-equation has been chosen and an initial point $x_0 \in \hat{X}$ has been obtained, what we

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need to do next is just to handle it numerically. Let the ECP-equation be of the form:

$$\begin{cases} \dfrac{dx}{dt} = -\varphi(x)P(x)A(x)\nabla f(x) \equiv p(x), & \text{(4.1a)} \\ x(0) = x_0. & \text{(4.1b)} \end{cases}$$

In the sequel, we will always assume that (ECP)-equation (4.1) has a complete-limit point x^*, which we refer to as limit point for short. Let us choose the (ECP)-rate factor (3.13), i.e., $\varphi(x) = 1/\|PA\nabla f\|_2$ so that the right-hand side is standardized:

$$\|p(x)\|_2 = 1. \tag{4.2}$$

As mentioned at the end of Section 3, this gives the easiest may to get to know the length of the trajectory from x_0 to any point $x(t)$ along the trajectory.

For (4.1) we can choose between a number of numerical integration schemes with differing techniques of step-size control. While it is impossible at this stage to specify a best integration scheme for our problem, it may be proper to use a higher order integration method here than in the unconstrained case, where approaches of Euler's type are often suggested, for now the search should stay closer to the trajectory (see Brown and Bartholomew [2]). Furthermore, since what we are really interested in is the limit point, rather than the trajectory itself, the integration method should possess a good round-off property, and its computation complexity should at the same time be as low as possible. In the light of these, Hamming's approach[4], a fourth order, multistep and predictor-corrector method, seems to be among the reasonable choices. We will incorporate it into a stepsize-variable integration algorithm.

Denote simply the k-th stepsize, $x(t_k)$ and $p(x(t_k))$ by α_k, x_k and p_k, respectively. Suppose that other three points x_1, x_2 and x_3 are ready to be used besides x_0. Then at the k-th step, $k = 3, 4, \cdots$, point x_{k+1} may be calculated by applying Hamming's formulas:

$$\left.\begin{aligned} &\text{(1) Prediction } & u_{k+1} &= x_{k-3} + \frac{8}{3}\alpha_k(p_k + p_{k-2} - \frac{1}{2}p_{k-1}) \\ & & v_{k+1} &= u_{k+1} - \frac{112}{121}d_k \ (d_3 = 0) \\ & & w_{k+1} &= p(v_{k+1}) \\ &\text{(2) Correction } & c_{k+1} &= \frac{1}{8}[9x_k - x_{k-2} + 3\alpha_k(w_{k+1} + 2p_k - p_{k-1})] \\ & & d_{k+1} &= u_{k+1} - c_{k+1} \\ & & x_{k+1} &= c_{k+1} + \frac{9}{121}d_{k+1} \\ & & p_{k+1} &= p(x_{k+1}) \end{aligned}\right\} \tag{4.3}$$