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MATRIX PADÉ APPROXIMATION: RECURSIVE COMPUTATIONS*[1)]

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Abstract

In this paper we consider computational aspect of the matrix Padé approximants whose definitions and properties were considered in an accompanying paper. A three-term recursive approach for the computation is established.

In [1] the authors have studied the general matrix Padé approximation problem. It turned out that it is necessary to consider not only left and right approximants, a duality imposed by the non-commutativity of the matrix multiplication, but also type I and type II approximants, depending on the normalization of the denominator. In this paper, we shall consider a recursive method for computing the approximants. We use the definitions and notations from [1].

We assume that $V, W \in Z_+^{p\times 1}$ and $U \in Z_+^{m\times 1}$, so that we do not have to mention this condition every time. On the other hand, we mainly consider the computation of type I right MPAs. The computation of the second type will be shown to be equivalent to the computation of the first type under some conditions. So the right subscript I or II in the notations will be deleted if there is no confusion. Consider the set

$$[V,U,W] = \{(N,M) : NM^{-1} \in [V,U,W]^f\}.$$

The problem we want to solve here is to compute $[V,U,W]$ from two "previous" ones. We shall need some normality condition for f which is different from the one we have defined earlier. We call the new concept I-normality, defined as follows:

Definition 1 (I-normality). *If for any V, U and W which satisfy condition*

$$\sum_{k=1}^{p} w_{kj} = \sum_{k=1}^{p} v_{kj} + \sum_{k=1}^{m} u_{kj}, \quad j = 1,2,\cdots,m, \tag{1}$$

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the matrix $H(V,U,W)$ is nonsingular, then we say that f is I*-normal.*

For $(N,M) \in [V,U,W]$, we introduce the following notations for the elements of the numerator, denominator and residual:

$$N_{ij}(z) = \sum_{k=0}^{v_i} n_{ij}^{(k)} z^k, \quad M_{ij}(z) = \sum_{k=0}^{u_i} m_{ij}^{(k)} z^k, \quad \text{and} \quad (fM-N)_{ij}(z) = \sum_{k=0}^{\infty} e_{ij}^{(k)} z^k.$$

We introduce the following matrices:

$$N(V,U,W,V')^T = [N_1^{(v_1'+1)}(V,U,W)^T \cdots N_1^{(v_1)}(V,U,W)^T \cdots$$
$$N_p^{(v_p'+1)}(V,U,W)^T \cdots N_p^{(v_p)}(V,U,W)^T],$$
$$M(V,U,W)^T = [M_1^{(0)}(V,U,W)^T \cdots M_1^{(u_1)}(V,U,W)^T$$
$$\cdots M_m^{(0)}(V,U,W)^T \cdots M_m^{(u_m)}(V,U,W)^T],$$

and

$$E(V,U,W,W')^T = [E_1^{(w_1+1)}(V,U,W)^T \cdots E_1^{(w_1')}(V,U,W)^T \cdots E_p^{(w_p+1)}(V,U,W)^T$$
$$\cdots E_p^{(w_p')}(V,U,W)^T],$$

where

$$N_i^{(k)}(V,U,W) = [n_{i1}^{(k)}\ n_{i2}^{(k)}\ \cdots\ n_{im}^{(k)}], \quad M_i^{(k)}(V,U,W) = [m_{i1}^{(k)}\ m_{i2}^{(k)}\ \cdots\ m_{im}^{(k)}],$$
$$E_i^{(k)}(V,U,W) = [e_{i1}^{(k)}\ e_{i2}^{(k)}\ \cdots\ e_{im}^{(k)}].$$

The degrees $V'+1$ and orders W' satisfy $V'+1 \in Z_+^{p\times 1}$ and $V' \le V, W' \ge W$. It can be seen that $N(V,U,W,-1)$ is the coefficient matrix of N, $M(V,U,W)$ is the coefficient matrix of M, and $E(V,U,W,\infty)$ is the coefficient matrix of $R = fM - N$. Hence, from the definition of MPA, we have

$$H(V'+1,U+1,V+1)M(V,U,W) = N(V,U,W,V'), \tag{2}$$
$$H(V+1,U+1,W+1)M(V,U,W) = 0, \tag{3}$$
$$H(W+1,U+1,W'+1)M(V,U,W) = E(V,U,W,W').$$

Lemma 1. *Let f be* I*-normal. Then for any V,U,W satisfying* (1)*, we have*

(i) *The matrix*

$$\begin{bmatrix} N(V,U,W,V') \\ E(V,U,W,W') \end{bmatrix} \tag{5}$$

is nonsingular, where

$$-1 \le V' \le V, \quad W' \ge W \text{ and } |V-V'| + |W'-W| = m. \tag{6}$$

(ii) *The leading coefficient matrix of the denominator M*

$$M^{hc}(V,U,W)^T = \left[M_1^{(u_1)}(V,U,W)^T \cdots M_m^{(u_m)}(V,U,W)^T\right]$$

is nonsingular.

(iii) *For V', W' satisfying* (6)*, and $W'' \ge W'$ and $(V,\ U+1,\ W'')$ satisfying* (1)*, the matrix*

$$\begin{bmatrix} N(V,U,W,V') \\ E(V,U,W,W') \end{bmatrix} - \begin{bmatrix} N(V,U+1,W'',V') \\ 0 \end{bmatrix} \tag{7}$$