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Composite Implicit Iteration Process for Asymptotically Hemi-Pseudocontractive Mappings

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Abstract. In Banach space, the composite implicit iterative process for uniformly L-Lipschitzian asymptotically hemi-pesudocontractive mappings are studied, and the sufficient and necessary conditions of strong convergence for the composite implicit iterative process are obtained.

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1 Introduction and preliminaries

Throughout this work, we assume that E is a real Banach space. E^* is the dual space of E and $J:E\to 2^{E^*}$ is the normalized duality mapping defined by

$$J(x)=\{f\in E^*:<x,f>=\|x\|\|f\|,\|f\|=\|x\|\},\quad \forall x\in E,$$

where $\langle\cdot,\cdot\rangle$ denotes duality pairing between E and E^*. A single-valued normalized duality mapping is denoted by j.

Let C be a nonempty subset of E and $T:C\to C$ a mapping, we denote the set of fixed points of T by $F(T)=\{x\in C;Tx=x\}$.

Definition 1.1. ([1]) T is said to be asymptotically nonexpansive, if there exists a sequence $\{k_n\}\subset[1,\infty)$ with $\lim_{n\to\infty}k_n=1$ such that

$$\|T^nx-T^ny\|\le k_n\|x-y\|,\quad \forall x,y\in C \text{ and } n\ge 1.$$

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(2) ([2]) T is said to be uniformly L-Lipschitzian, if there exists $L>0$ such that

$$\|T^n x-T^n y\|\le L\|x-y\|,\quad \forall x,y\in C \text{ and } n\ge 1.$$

(3) ([3]) T is said to be asymptotically pseudocontractive, if there exists a sequence $\{k_n\}\subset[1,\infty)$ with $\lim_{n\to\infty}k_n=1$, for any $x,y\in C$, there exists $j(x-y)\in J(x-y)$ such that

$$\langle T^n x-T^n y,\, j(x-y)\rangle\le k_n\|x-y\|^2,\ \ n\ge 1.$$

(4) ([4]) T is said to be asymptotically hemi-pseudocontractive, if $F(T)\neq\emptyset$ and there exists a sequence $\{k_n\}\subset[1,\infty)$ with $\lim\limits_{n\to\infty}k_n=1$ such that, for any $x\in C$ and $p\in F(T)$, there exists $j(x-p)\in J(x-p)$ such that

$$\langle T^n x-p,\, j(x-p)\rangle\le k_n\|x-p\|^2,\ \ n\ge 1.$$

Remark 1.1. It is easy to see that if T is an asymptotically nonexpansive mapping, then T is a uniformly L-Lipschitzian and asymptotically pseudocontractive mapping, where $L=sup_{n\ge1}\{k_n\}$; if T is an asymptotically pseudocontractive mapping with $F(T)\neq\emptyset$, then T is an asymptotically hemi-pseudocontractive mapping.

Let C be a nonempty closed convex subset of E and $T:C\to C$ be a uniformly L-Lipschitzian asymptotically hemi-pseudocontractive mapping, for any given $x_1\in C$, we introduce a composite implicit iteration process $\{x_n\}$ as follows:

$$\begin{cases} x_{n+1}=(1-\alpha_n)x_n+\alpha_n T^n y_n, \\ y_n=(1-\beta_n)x_n+\beta_n T^n x_{n+1}, \quad \forall n\ge 1, \end{cases} \tag{1.1}$$

where $\{\alpha_n\},\{\beta_n\}$ are two real sequences in $[0,1]$.

As $\beta_n=0$ for all $n\ge1$, then (1.1) reduces to

$$x_{n+1}=(1-\alpha_n)x_n+\alpha_n T^n x_n. \tag{1.2}$$

Remark 1.2. For any given $x_n\in C$, define the mapping $A_n:C\to C$, such as:

$$A_n x=(1-\alpha_n)x_n+\alpha_n T^n[(1-\beta_n)x_n+\beta_n T^n x],\quad \forall x\in C,$$

where C is a nonempty closed convex subset of E and $T{:}C\to C$ is a uniformly L-Lipschitzian. Then

$$\begin{aligned}\|A_n x-A_n y\| &=\|\alpha_n(T^n[(1-\beta_n)x_n+\beta_n T^n x]-T^n[(1-\beta_n)x_n+\beta_n T^n y])\|\\ &\le\alpha_n\beta_n L\|T^n x-T^n y\|\\ &\le\alpha_n\beta_n L^2\|x-y\|\end{aligned}$$

for all $x,y\in C$. Thus A_n is a contraction mapping if $\alpha_n\beta_n L^2<1$ for all $n\ge1$, and so there exists a unique fixed point $x_{n+1}\in C$ of A_n, such that $x_{n+1}=(1-\alpha_n)x_n+\alpha_n T^n[(1-\beta_n)x_n+\beta_n T^n x_{n+1}]$. This shows that the composite implicit iteration process (1.1) is well defined.