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Multiplicative Noise Removal Based on Unbiased Box-Cox Transformation

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Abstract. Multiplicative noise removal is a challenging problem in image restoration. In this paper, by applying Box-Cox transformation, we convert the multiplicative noise removal problem into the additive noise removal problem and the block matching three dimensional (BM3D) method is applied to get the final recovered image. Indeed, BM3D is an effective method to remove additive Gaussian white noise in images. A maximum likelihood method is designed to determine the parameter in the Box-Cox transformation. We also present the unbiased inverse transform for the Box-Cox transformation which is important. Both theoretical analysis and experimental results illustrate clearly that the proposed method can remove multiplicative noise very well especially when multiplicative noise is heavy. The proposed method is superior to the existing methods for multiplicative noise removal in the literature.

AMS subject classifications: 62E17, 62F10, 65H05, 68U10

Key words: Multiplicative noise removal, Box-Cox transformation, maximum likelihood estimation, block-matching three dimensional method (BM3D).

1 Introduction

In real image applications, most images are generated through image recording systems. During the digital image acquisition, images are commonly affected by noise. For instance, additive Gaussian white noise is the most frequent noise in image systems. In the past decades, various methods have been considered to reduce additive Gaussian white noise in images. For instance, besides the total variation-based methods [13, 15, 41, 46],

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there are wavelet-based methods [23], fourth-order method [33], nonlocal-based methods [9, 10], sparse representation methods such as dictionary-based methods [25], [31] and the block matching three dimensional (BM3D) method [19], of which BM3D method is very efficient to remove additive Gaussian white noise. Another classical degradation to real images is impulse noise which includes salt-and-pepper noise and random-valued impulse noise. For salt-and-pepper noise, pixel of the damaged image takes the smallest or biggest pixel value (e.g. 0 or 255 for an 8-bit image), and other pixel value keeps; for random-valued impulse noise, pixel of the damaged image takes random value, and other pixel remains. In literature, many approaches have been also proposed to impulse noise removal, see, for instance, [5,11,12,28,35,47]. Moreover, Poisson noise often arises in radiographic imaging applications, and many efficient methods are devoted to removing this kind of noise, such as the methods proposed in [8,17,30,34,48].

In recent years, multiplicative noise removal problem has attracted much attention since it arises in many practical applications such as laser, microscope, and synthetic aperture radar (SAR) images. When monochromatic radiation is scattered from a surface, whose roughness is of the order of a wavelength, it causes wave interference which results in speckle or multiplicative noise in an image. Different from additive noises, a recorded image g contaminated by multiplicative noise is the multiplication of the original image u and a noise v pixel by pixel

$$g = uv. \tag{1.1}$$

Here u, g, and v are mn-by-1 vectors corresponding to the original clean m-by-n image, observed m-by-n image and m-by-n noise, respectively. Because of the special degraded mechanism, almost all information of the original image may disappear when it is distorted by multiplicative noise and the degraded image is very difficult to be recovered. Many methods have been proposed for multiplicative noise removal in the literature. For example, in [40], Rudin, Lions and Osher first considered the multiplicative noise image restoration and constructed constrained minimization methods for Gaussian multiplicative noise removal. In 2008, by utilizing Maximum A Posterior analysis, Aubert and Aujol proposed a variational method for multiplicative noise removal where the noise follows Gamma distribution [4]. In the same year, Shi and Osher applied the inverse scale space method to multiplicative noise degraded models by converting multiplicative noise into additive noise removal problem, see, for instance, [42]. Huang, Ng and Wen proposed a global convex model to remove multiplicative noise in logarithm domain in [27]. In recent years, some other efficient methods also have been proposed. Indeed, Chesneau, Fadili and Starck proposed a stein block thresholding method in [18]. An efficient hybrid approach was proposed by Durand, Fadili and Nikolova in [24]. Steidl and Teuber considered using the I-divergence as a data fitting term to reduce multiplicative Gamma noise in [43]. In [6], Bioucas-Dias and Figueiredothe handled the multiplicative noise removal problem by using variable splitting and constrained optimization. In addition, Teuber and Lang proposed efficient nonlocal filters for multiplicative noise removal in [44] and [45]. Huang et al. proposed a dictionary method for multiplicative noise