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A New Hybrid Trigonometric WENO Scheme for Hamilton-Jacobi Equations

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Abstract. In this paper, we introduce a new hybrid weighted essentially non-oscillatory (WENO) scheme grounded in trigonometric polynomials for the solution of Hamilton-Jacobi equations. This innovative approach utilizes trigonometric polynomial reconstruction as an alternative to the conventional algebraic polynomial reconstruction. Notably, the proposed scheme demonstrates reduced truncation errors in smooth regions and enhanced resolution in nonsmooth regions, outperforming the classical trigonometric WENO scheme — cf. [J. Zhu and J.X. Qiu, *Commun. Comput. Phys.* **8** (2010)]. In particular, we have designed a novel streamlined hybrid strategy aimed at improving the computational efficiency of the scheme, which surpasses the classical WENO scheme in this regard. A comprehensive suite of numerical experiments has been undertaken to demonstrate the merits of the proposed scheme.

AMS subject classifications: 65M06, 35L99

Key words: WENO scheme, Hamilton-Jacobi equation, finite difference scheme, hybrid scheme.

1. Introduction

In this paper, our primary focus is on developing high-order numerical approximations for the Hamilton-Jacobi (HJ) equations

$$\psi_t + H(x_1,\dots,x_n,t,\psi,\nabla\psi) = 0, \quad (x_1,\dots,x_n)\in\Omega, \tag{1.1}$$

where H is the Hamiltonian function. This equation holds a paramount significance and occupies a pivotal position in numerous fields, including seismic wave propagation, geometric

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optics, and optimal control. Despite the smoothness of the initial conditions and the Hamiltonian, Eqs. (1.1) often have no classical solution or the solution is not unique. To tackle the issue of well-posedness, Crandall *et al.* [11–13] laid the groundwork for the entropy conditions and viscous solutions, clarifying their properties and constructing a first-order monotone scheme [14]. Building on this, further research emerged. Recognizing issues with accuracy and dissipation in the monotone approach, Osher and Sethian [31] introduced advanced, higher-order upwind schemes. This advancement paved the way for the modification of a multitude of numerical schemes originally designed for solving hyperbolic conservation laws, enabling them to address HJ equations as well.

The ENO methods [35,36] effectively addressed numerical oscillations in problems with discontinuities. Harten *et al.* [17, 18] and Shu [34] tailored these high-order methods to HJ equations, resulting in the production of monotone fluxes [32]. Subsequently, Lafon and Osher [23] extended the methods to unstructured grids. In addition, weighted ENO schemes have been also used in [21,29]. Later on, Jiang and Peng [20] introduced a WENO-JP scheme, followed by central [5], Hermite [33], mapped [6], and unequal-sized WENO schemes [41] for structured meshes. At the same time, alternative numerical methods [8, 15, 16, 28, 30], including finite element methods [1, 4, 19, 24], have been developed.

In the real world, high-frequency oscillating phenomena are widespread and pose significant challenges for accurate simulation. It is worth mentioning that numerical methods based on trigonometric polynomial functions offer a superior approach to capturing these complexities. Trigonometric interpolation, first explored by Baron [3], laid the groundwork for the development of more advanced numerical schemes. Building on this foundation, trigonometric polynomials have been successfully incorporated into both ENO [9] and WENO [40] frameworks. These methods turned out to be very efficient in simulating high-frequency phenomena. Wang and Zhang [37–39] further refined the trigonometric weighted essentially non-oscillatory (TWENO) framework by introducing concept of non-uniform stencils. This led to the creation of the US-TWENO scheme [37], MR-TWENO scheme [38], and MUS-TWENO scheme [39], each offering unique advantages in capturing high-frequency oscillations. While these advanced schemes provide remarkable accuracy, their inherent nonlinearity can diminish computational efficiency. Therefore, ongoing research continues to explore ways to optimize these methods, striking a balance between accuracy and performance.

Zhu and Qiu [40] applied a TWENO scheme to Hamilton-Jacobi equation, devising a comprehensive framework tailored specifically for this purpose. They adopted the same stencil as in the WENO-JP scheme [20] and reconstructed the numerical flux using trigonometric polynomials. However, numerical results show that the TWENO scheme does not exhibit exceptional performance in solving the Hamilton-Jacobi equations. To highlight its potential advantages, we develop a hybrid strategy, blending it with high-order trigonometric polynomials. A wide array of hybrid schemes exist for WENO schemes, encompassing approaches like TVB [10], KXRCF [22], etc. Li and Qiu [25–27] have thoroughly examined and studied various hybrid WENO schemes, totaling in dozens. Among the myriad hybrid schemes for the WENO scheme, the one proposed by Zhu [42] stands out due to its straightforward construction and practical ease of application, capturing widespread attention and