East Asian Journal on Applied Mathematics
doi: 10.4208/eajam.2025-099.281025

Vol. **16**, No. 4, pp. 788-814
November 2026

A Reduced Order Modeling Method with Variable-Separation-Based Domain Decomposition for Parametric Dynamical Systems

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Received 3 June 2025; Accepted (in revised version) 28 October 2025.

Abstract. This paper proposes a model order reduction method for a class of parametric dynamical systems. Using a temporal Fourier transform, we reformulate these systems into complex-valued elliptic equations in the frequency domain, containing frequency variables and parameters inherited from the original model. To reduce the computational cost of the frequency-variable elliptic equations, we extend the variable-separation-based domain decomposition method to the complex-valued context, resulting in an offline-online procedure for solving the parametric dynamical systems. At the offline stage, separate representations of the solutions for the interface problem and the subproblems are constructed. At the online stage, the solutions of the parametric dynamical systems for new parameter values can be directly derived by utilizing the separate representations and implementing the inverse Fourier transform. The proposed approach is capable of being highly efficient because the online stage is independent of the spatial discretization. Finally, we present three specific instances of parametric dynamical systems to demonstrate the effectiveness of the proposed method.

AMS subject classifications: 35R60, 37M99, 65P99

Key words: Parametric dynamical system, model order reduction, domain decomposition, variable-separation method, Fourier transformation.

1. Introduction

Parametric dynamical systems have been widely adopted in science and engineering to model complex real-world problems characterized by various uncertainties. These uncertainties may stem from multiple sources, including physical properties, geometric configurations, initial conditions, and boundary conditions. The numerical simulation of such

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parametric systems poses substantial computational challenges, especially in the case of repeated parameter evaluation for design optimization, control analysis, and uncertainty quantification. Although traditional high-fidelity numerical approaches such as finite element method (FEM), finite difference method, and finite volume method can provide accurate solutions, they often entail extreme-scale computations with prohibitive computational costs.

To overcome arising computational bottlenecks in the simulation of parametric dynamical systems, a variety of innovative numerical approaches have been developed over past few decades. In particular, the model order reduction (MOR) technique has emerged as a powerful tool — cf. Refs. [1, 2, 16, 18, 33, 42, 46]. It is used to construct reduced-order models by approximating solutions in a low-dimensional subspace, which significantly reduces the computational cost. A prominent MOR approach is the reduced basis (RB) method — c.f. [12, 17, 19, 20, 24, 25], which follows an offline-online computational strategy. In the offline phase, high-fidelity solutions for a carefully selected parameter set are precomputed and stored. The online phase then efficiently approximates solutions for new parameters via a linear combination of these precomputed solutions. In this method, the key challenge lies in constructing an optimal set of basis functions that accurately capture the essential features of the full order model. Having the reduced basis functions obtained, one can use intrusive or non-intrusive approach to construct a reduced order model. Non-intrusive MOR shows the flexibility and efficiency because the full order numerical systems are not required. To enhance the applicability of MOR to nonlinear systems and describe relationship between the inputs and outputs, MOR methods based on various machine or deep learning techniques have been proposed [7, 27, 49, 50]. These methods roughly contain data-driven machine learning [32, 38, 53], physical-informed machine learning [7, 27, 49], and physics-data combined machine learning [15, 41]. Among these methods, the most effective and easiest approach is the data-driven machine learning, which usually converges rapidly because the loss functions are often simple. The physically-informed machine learning can demonstrate strong theoretical constraints and inductive biases by integrating governing physical rules and domain knowledge into the learning process. The physics-data combined machine learning has been used to study nonlinear dynamical systems in small-data regimes [15]. It has adopted a step-by-step training scheme, which seamlessly integrates the governing physical laws and the limited labeled data into feedforward neural networks.

An alternative approach for effective reducing the computational cost of dynamical systems is the frequency-domain method, which relies on two key transformations — viz. the Fourier transform [10, 11, 14, 28, 29] and the Laplace transform [30, 31] in the time dimension. The Fourier transform stands out as a significant discovery in mathematical sciences, playing a crucial role in modern scientific and technological advancements, which has been widely applied in the analysis of continuous-time systems, including signal and image processing, analog circuit analysis, and communication systems. The fundamental principle behind this approach is to convert time-dependent PDEs into frequency-dependent elliptic problems, which are stationary in time but parameterized by frequency. The time-domain solution is then reconstructed by applying an inverse Fourier transform to the frequency-