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Piecewise Linear Maximum Entropy Method for Fredholm Integral Equations with Weakly Singular Kernel

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Abstract. Approximate solution of Fredholm integral equations with certain types of weakly singular algebraic kernels is obtained by the piecewise linear maximum entropy method. During implementation of the method, two-dimension numerical integration is reduced to one-dimension numerical integration, which allows to lower the computational cost. The results of numerical experiments are consistent with theoretical findings and show the effectiveness of the method.

AMS subject classifications: 65R20, 94A17

Key words: Fredholm integral equation, piecewise linear maximum entropy method, weakly singular kernel.

1. Introduction

Consider the integral equation

$$f(x)-\int_0^1 k(x,t)f(t)dt = g(x), \quad x\in[0,1], \tag{1.1}$$

where $k(x,t)$ is algebraic or logarithmic weakly singular kernel and $g(x)$ and $f(x)$ are given and unknown functions, respectively. Such second kind Fredholm integral equations (SKFIEs) often appear in science and engineering. In this work, we assume that the right-hand side $g(x)$ is a continuous function and consider the Eqs. (1.1) with two specific kernels

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— viz. the equations

$$f(x)-\int_0^1 |x-t|^{-\alpha} f(t)dt = g(x), \quad 0<\alpha<1, \tag{1.2}$$

$$f(x)-\int_0^1 \ln|x-t| f(t)dt = g(x). \tag{1.3}$$

Due to the weak singularity of the kernel function, it is difficult to solve the integral equation. Numerical methods for second kind Fredholm integral equations with weakly singular kernel include spectral Jacobi-collocation method [19], Lagrange interpolation method [17], Galerkin method [14], wavelet method [15], etc. The collocation method for Eq. (1.2) studied in [19] is based Gauss-Jacobi quadrature formula. The barycentric Lagrange interpolation formula [17] was developed for Eq. (1.2), and transformed the integral equation into a matrix equation. The Galerkin method [14] using Legendre polynomials as the basis functions was applied to Eqs. (1.2), (1.3), and the convergence rate of the numerical solutions was given. In the reproducing kernel Hilbert space method [2] adopted for solving Eq. (1.3), the approximate solution was represented by finite number of terms of a series.

The concept of entropy first appeared in thermodynamics, representing the degree of chaos in a thermodynamic system. The maximum entropy principle was proposed by Jaynes [7, 8], and was widely used in the function recovery [1], data modeling [16], invariant measures [6, 10], solving integral equations [9, 13, 18] and other fields. The maximum entropy principle with polynomials was proposed to solve SKFIE [13]. The piecewise linear maximum entropy method (PLMEM) has been also used to find approximate solution of SKFIE and second kind Vollterа integral equations — cf. [9, 18]. However, the maximum entropy method in [9, 18] employs time expensive double numerical integration. In this work, we use a piecewise linear maximum entropy method and obtain an approximate form without singularity of the Eqs. (1.2) and (1.3). This method substantially reduces the computational cost of numerical integration.

The paper is arranged as follows. The maximum entropy method is introduced in Section 2. In Section 3, we apply the PLMEM to Fredholm integral equations (1.2) and (1.3) with weakly singular algebraic and logarithmic kernels. The results of numerical experiments presented in Section 4 show the efficiency of the method. The conclusion is given in Section 5.

2. Maximum Entropy Method

Let Σ be the Lebesgue σ algebra on $[0,1]$, μ the Lebesgue measure, and $([0,1],\Sigma,\mu)$ the corresponding measure space. The generalized Boltzmann entropy of f is defined by

$$H(f) = -\int_0^1 f(x)\ln f(x) d\mu(x) + \int_0^1 f(x) d\mu(x) \tag{2.1}$$

with $f(x)\ln f(x) := 1$ if $f(x)=0$.