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Solving Schrödinger Bridge Problem via Continuous Normalizing Flow

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Abstract. The Schrödinger bridge problem (SBP), which can be understood as an entropy-regularized optimal transport, seeks to compute stochastic dynamic mappings connecting two given distributions. SBP has shown significant theoretical importance and broad practical potential, with applications spanning a wide range of interdisciplinary fields. While theoretical aspects of the SBP are well-understood, practical computational solutions for general cases have remained challenging. This work introduces a computational framework that leverages continuous normalizing flows and score matching methods to approximate the drift in the dynamic formulation of the SBP. The learned drift term can be used for building generative models, opening new possibilities for applications in probability flow-based methods. We also provide a rigorous Γ-convergence analysis for our algorithm, demonstrating that the neuron network solutions converge to the theoretical ones as the regularization parameter tends to infinity. Lastly, we validate our algorithm through numerical experiments on fundamental cases.

AMS subject classifications: 49Q22, 68T07
Key words: Schrödinger bridge, continuous normalizing flow, Γ-convergence.

1. Introduction

Finding mappings between two probability distributions is the focus of many modern machine learning applications, from generative modeling to understanding physical systems [34]. While deterministic methods such as optimal transport (OT) have offered robust frameworks for addressing this task, many real-world phenomena are inherently governed by stochastic processes, necessitating more sophisticated approaches. The Schrödinger bridge problem, originally proposed by Schrödinger [44], addresses this challenge by seeking the most likely stochastic process connecting two probability distributions. SBP is of

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great theoretical importance in mathematical physics, as they provide deeper insights into the evolution of physical systems and the study of quantum information. It also demonstrates broad practical potential across various applications in interdisciplinary fields such as modeling natural stochastic dynamical systems [16, 43], image processing [31] and shape correspondence [9].

Let $\Omega \subset \mathbb{R}^d$ be a convex domain in $\mathbb{R}^d$ with a smooth boundary, which could possibly be unbounded in general. The Schrödinger problem can be stated as follows [26]: For two probability distributions ρ_0 and ρ_1 on Ω, find a path measure $\mathbb{P}^* \in \mathscr{P}(C([0,T];\Omega))$ such that

$$\mathbb{P}^* = \underset{\mathbb{P}\in\mathscr{D}(\rho_0,\rho_1)}{\operatorname{argmin}} \ \mathbb{KL}(\mathbb{P}\|\mathbb{Q}), \tag{1.1}$$

where $\mathbb{Q} \in \mathscr{P}(C([0,T];\Omega))$ is a reference measure. Here, $\mathscr{P}(C([0,T];\Omega))$ indicates the set of measures on the path space $C([0,T];\Omega)$ (continuous curves taking values in Ω), and $\mathscr{D}(\rho_0,\rho_1) \subset \mathscr{P}(C([0,T];\Omega))$ is the set with marginal measures ρ_0 at time $t=0$ and ρ_1 at time $t=T$. The KL divergence between two probability measure μ and ν on Ω is defined by

$$\mathbb{KL}[\mu\|\nu] = \begin{cases} \displaystyle\int_\Omega \log\left(\frac{d\mu}{d\nu}\right) d\mu, & \text{if} \quad \mu \ll \nu, \\ \infty, & \text{otherwise,} \end{cases}$$

where $d\mu/d\nu$ denotes the Radon-Nikodym derivative of μ with respect to ν. Note that the KL divergence is non-negative by Jensen's inequality and achieves zero only if $\mu=\nu$. Moreover, it is a convex functional with respect to both arguments. In practice, $\mathbb{Q}$ is often taken to be the path measure induced by Brownian motion or some relatively simple diffusion process.

Previous research has demonstrated a close relationship between SBP and the optimal stochastic control problem [40], thereby offering an alternative approach to solving the SBP. Given that $\mathbb{Q} = \sqrt{2}\sigma\mathbb{W}$, where $\mathbb{W}$ is the path measure induced by the standard Brown motion, then solving SBP (1.1) is equivalent to finding the optimal stochastic control of the optimization problem — cf. [40],

$$\begin{aligned} \min_u \quad & \mathbb{E}\left[\frac{1}{2}\int_0^T |u_t(X_t,t)|^2 dt\right], \\ \text{s.t.} \quad & dX_t = u_t(X_t,t)dt + \sqrt{2}\sigma\, dW_t, \\ & X_0 \sim \rho_0, \quad X_T \sim \rho_1, \end{aligned} \tag{1.2}$$

or the problem with the Fokker-Planck equation constraint

$$\begin{aligned} \min_{\rho,u} \quad & \frac{1}{2}\int_0^T \int_\Omega \rho(x,t)|u(x,t)|^2\, dx\, dt, \\ \text{s.t.} \quad & \partial_t\rho + \nabla\cdot(\rho u) = \sigma^2\Delta\rho, \\ & \rho(x,0)=\rho_0(x), \quad \rho(x,T)=\rho_1(x). \end{aligned} \tag{1.3}$$