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Integrable mKdV Models as Reductions of AKNS Integrable Systems via Dual Similarity Transformations

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Abstract. This paper explores integrable modified Korteweg-de Vries (mKdV) models using dual similarity transformations. Two representative examples involving distinct similarity transformations are presented, along with their corresponding reduced Ablowitz-Kaup-Newell–Segur matrix spectral problems. These examples illustrate how reduced matrix mKdV integrable models can be systematically generated through such transformations.

AMS subject classifications: 37K10, 35Q53, 37K40

Key words: Lax pair, AKNS matrix spectral problem, zero-curvature equation, integrable hierarchy, similarity transformation.

1. Introduction

The Lax pair formulation in soliton theory is a fundamental tool for constructing integrable models [18]. To gain deeper insights into the structure of integrable systems, especially multi-component ones, it is essential to explore a wide range of illustrative examples. These examples help classify integrable models within the Lax pair framework. A natural starting point is to consider matrix spectral problems built from matrix loop algebras [48]. Lax pairs often lead to bi-Hamiltonian structures [41], which ensure the existence of infinitely many commuting symmetries and conservation laws, key features of integrability. Furthermore, the Lax pair formulation provides a basis for solving Cauchy problems via the inverse scattering transform [4, 8].

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Similarity transformations have proven effective in deriving reduced matrix spectral problems and their associated integrable models [13, 23, 25, 42], with notable examples including nonlocal integrable models involving reflection-type symmetries [3]. Starting from the matrix Ablowitz-Kaup-Newell-Segur (AKNS) spectral problems, classical equations such as the nonlinear Schrödinger (NLS) and modified Korteweg-de Vries equations can be obtained as reduced models. More recently, dual similarity transformations have led to novel reduced integrable models [27]. A key challenge in this framework is to carefully handle the reductions imposed on the potential matrices by both transformations, while maintaining the invariance of the zero-curvature condition [29]. A recent comprehensive classification of lower-order nonlocal integrable models associated with the matrix AKNS framework has identified three types of nonlocal NLS models and two types of nonlocal mKdV models [28].

The inverse scattering transform remains a powerful and effective tool for solving Cauchy problems, including those arising in nonlocal integrable models [2, 40]. In addition to this method, various other analytical techniques have proven valuable for studying reduced integrable models, particularly in the construction of soliton solutions. Classical approaches such as the Hirota bilinear method, Bäcklund transformations, Darboux transformations, and the Riemann-Hilbert method continue to play important roles. Furthermore, several mathematical frameworks have been developed to address nonlocal reductions of integrable models (see, e.g. [14–16, 28, 44, 45, 50]).

The aim of this study is to construct dual similarity transformations of distinct forms and to derive the corresponding reduced integrable mKdV models based on the matrix AKNS spectral problems. In Section 2, we revisit the matrix AKNS spectral problems and the associated integrable mKdV hierarchies. A general framework is presented for implementing dual similarity transformations and obtaining reduced integrable models. Section 3 explores two representative cases that yield consistent similarity transformations, one involving off-diagonal matrices and the other involving diagonal matrices. These transformations are then applied to the matrix AKNS spectral problems to generate novel reduced integrable mKdV models. The examples underscore the structural diversity and richness of the reduced matrix AKNS integrable systems. The closing section synthesizes the main contributions and offers final remarks.

2. Dual Similarity Reductions of AKNS Integrable Models

2.1. Revisiting matrix AKNS hierarchies: Toward reduced integrable models

Within the AKNS framework for integrable models, the dependent variable $u = u(p, q)$ is a column vector composed of two matrix-valued potentials, defined as follows:

$$p = p(x, t) = (p_{jk})_{m\times n}, \quad q = q(x, t) = (q_{kj})_{n\times m},$$

where m and n are two natural numbers. For each $r \geq 0$, a pair of matrix AKNS spectral problems is given by

$$-i\phi_x = U\phi, \quad -i\phi_t = V^{[r]}\phi. \tag{2.1}$$