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# Inverse Obstacle Scattering with a Single Moving Emitter

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**Abstract.** This paper is concerned with time domain forward scattering and inverse scattering problems with a single moving point source as the emitter. Approximate solutions are provided for the forward scattering problem with a moving emitter. Regarding the inverse problem, in addition to a basic indicator function based on the approximate solutions, a novel indicator function is developed to construct the direct sampling method to recover both point-like and extended scatterers. Numerical experiments demonstrate that the proposed algorithms are effective in reconstructing both two-dimensional and three-dimensional scatterers with a single moving emitter.

**AMS subject classifications**: 65M32, 35L05

**Key words**: Time domain, inverse scattering problem, direct sampling method, moving emitter.

## 1. Introduction

Scattering and inverse scattering problems have achieved considerable developments over the past decades. In general, the scattering problems can be divided into two categories, frequency domain problems and time domain problems. For frequency domain problems, substantial theoretical achievements have been published [4,11,12], and diverse numerical algorithms have been developed, such as factorization methods [20], iterative methods [3], sampling methods [10, 18, 22], and other imaging methods [2, 19, 21]. Additionally, various related problems are studied, such as the co-inversion of sources and scatterers [5, 32].

Compared with frequency domain problems, the study of time domain problems is more complex. However, time domain problems have received more and more attentions since time-dependent data is generally more accessible. Owing to advancements in various technologies such as the time domain boundary integral equation method [7,25] and the rapid solution of wave equation [1], the time domain direct scattering problem has been well analyzed and numerically solved. For time domain inverse scattering problems, sampling

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methods, including the linear sampling method and direct sampling methods, are important among various numerical methods. The time domain linear sampling method is proved to be effective for Dirichlet obstacles [9], Neumann and Robin obstacles [17], penetrable medium [16] and cracks [31]. Meanwhile, the research on time domain direct sampling methods has also made significant progress. The advantage of this type of method lies in the fact that it does not require solving the time-dependent wave equation. Effective and efficient sampling schemes based on the migration method are proposed in [14], and the numerical algorithms based on the Green's function and time convolution are proposed in [8, 30]. Based on the Fourier-Laplace transform, an inherent connection between the time domain and frequency domain direct sampling methods is established in [15]. The asymptotic properties of the indicator function for the point-like scatterer are proved according to the Fourier-Laplace transform in [13].

In this paper, we consider the forward and inverse obstacle scattering problems with a single moving point source as the emitter. Inverse scattering problems with moving point sources have engineering applications including geoacoustic inversion [26] and Doppler tomography imaging [27]. For typical forward and inverse scattering problems, the incident wave is usually chosen to be a spherical wave or a plane wave. Few studies concern with incident waves from moving point sources. Nevertheless, the research of the inverse source problem with moving point sources is helpful for the analysis in this paper. For instance, Nakaguchi *et al.* [24] provides an algebraic expression for the wave field generated by a moving point source, and Wang *et al.* [29] investigates the mathematical design of moving point sources as acoustic emitters. Additionally, numerical schemes to reconstruct three-dimensional time-dependent point sources are proposed in [6], and a deterministic-statistical approach for reconstructing the moving path of a moving source is proposed in [23].

For the forward scattering problem with a moving point source, we derive approximate solutions to the scattering problem under the assumption that the speed of the moving source is below the sound speed. For the inverse problem, we propose an indicator function that is directly dependent on the approximate solution, as well as a novel indicator function based on the time convolution. Numerical experiments with a single moving emitter validate the efficiency of the direct sampling methods based on the proposed indicator functions, and both point-like and extended scatterers can be accurately reconstructed.

The rest of the paper is organized as follows. In Section 2, we present the analysis of the forward scattering problem, and the corresponding approximate solutions are exhibited. Section 3 is devoted to the construction and analysis of time domain direct sampling methods. Numerical experiments are provided in Section 4 to verify the effectiveness of our methods. The last section is the conclusion.

## 2. Forward Scattering Problem and Approximate Solutions

Considering the forward scattering problem with a moving emitter, let $D \subset \mathbb{R}^3$ be the region where the sound-soft scatterer is located. Assume that $D$ is bounded with piecewise Lipschitz boundary $\partial D$ and connected exterior $\mathbb{R}^3 \setminus \overline{D}$. Denote by $s(t)$ the trajectory of