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High Order Well-Balanced and Positivity-Preserving AWENO Scheme for Rotating Shallow Water Equations with Coriolis Force

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Abstract. The rotating shallow water equations (RSWEs), incorporating the Coriolis force, induce the geostrophic equilibrium with a nonzero velocity field, which poses more challenges in designing well-balanced numerical schemes than solving the classical SWEs. To tackle this issue, we leverage the Coriolis force's primitive function, specifically, the approach of apparent topography. Additionally, we introduce a positivity-preserving and well-balanced finite difference AWENO scheme for solving the RSWEs. Our approach applies characteristic-wise WENO interpolation directly to conservative variables, eliminating the need for the commonly used WENO linearization technique. To realize the purpose of a well-balanced property, the Lax-Friedrichs numerical flux is modified by appropriately adjusting the viscosity coefficients to avoid the dissipation from non-zero momentum in equilibrium-state simulations. The corresponding theoretical proof is provided. Furthermore, a positivity-preserving limiter is introduced for numerical simulations involving dry topography to avoid the negative water depth. Numerical results demonstrate that the proposed scheme preserves the equilibria exactly with the optimal convergence order and outperforms non-well-balanced methods in resolving sharp gradients, and accurately simulates the evolution of geophysical flows with the impact of the Coriolis force.

AMS subject classifications: 35L65, 35L67, 65M06, 76M20

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1. Introduction

The rotating shallow water equations under Coriolis force are commonly employed to

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model atmospheric dynamics and mid-latitude oceanic flows over relatively large spatial and temporal scales [12, 22, 30, 35, 38, 39, 43]. The RSWEs can be represented as balance laws

$$\begin{aligned} &h_t+(hu)_x+(hv)_y=0,\\ &(hu)_t+\left(hu^2+\frac{1}{2}gh^2\right)_x+(huv)_y=-ghb_x+fhv,\\ &(hv)_t+(huv)_x+\left(hv^2+\frac{1}{2}gh^2\right)_y=-ghb_y-fhu, \end{aligned} \tag{1.1}$$

where h denotes the fluid height, and b represents the bottom. g represents the gravitational acceleration, and f represents the Coriolis force parameter. u and v are the velocities along the x- and y-directions, respectively, while t represents time.

In numerous instances, solutions with steady-state are of great significance, as many cases of important practical waves can be regarded as minor perturbations from those equilibrium states. When the influence of Coriolis forces is omitted ($f = 0$), such as in the application of SWEs to model river and coastal flows, a fundamental steady-state solution arises

$$u\equiv 0,\quad v\equiv 0,\quad h+b\equiv \text{Const}.$$

A considerable amount of effort has been devoted to developing numerical formats that maintain this equilibrium state [1, 16, 21, 24–26, 29, 33, 34, 37, 49], and significant achievements have been made.

However, incorporating the Coriolis force significantly enriches the structure of stationary solutions, such as the geostrophic equilibrium states of the system (1.1), which are both stationary and constant along streamlines. Oceanic and atmospheric circulations often represent perturbations of such equilibria. The geostrophic equilibrium states admit the subsequent conditions [29, 51]

$$uh_x+vh_y=uu_x+vu_y=uv_x+vv_y=0.$$

Since the velocity vector is invariant along streamlines, the latter reduce to straight lines. Thus, it is logical to orient the coordinate system along the streamlines. Then there exist two specific types of geostrophic equilibria, when the divergence condition $u_x + v_y = 0$ is met, commonly known as jets in the rotational frame [9, 29, 51]

$$\begin{aligned} &u=0,\quad v_y=0,\quad vh_y=0,\quad g(h+b)_y=0,\quad g(h+b)_x=fv,\\ &v=0,\quad u_x=0,\quad uh_x=0,\quad g(h+b)_x=0,\quad g(h+b)_y=-fu. \end{aligned}$$

With the idea of apparent topography proposed by Bouchut [5], the above equations can be rewritten as

$$u=0,\quad v_y=0,\quad h_y=0,\quad g(h+b)_y=0,\quad K:=h+b-V\equiv \text{Const}, \tag{1.2}$$

$$v=0,\quad u_x=0,\quad h_x=0,\quad g(h+b)_x=0,\quad L:=h+b+U\equiv \text{Const}, \tag{1.3}$$

where $(U,V)^T$ are the primitives of the Coriolis force and

$$V_x:=fv/g,\quad U_y:=fu/g.$$