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A COARSENING ALGORITHM ON ADAPTIVE GRIDS BY NEWEST VERTEX BISECTION AND ITS APPLICATIONS*

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Abstract

In this paper, an efficient and easy-to-implement coarsening algorithm is proposed for adaptive grids obtained using the newest vertex bisection method in two dimemsions. The new coarsening algorithm does not require storing the binary refinement tree explicitly. Instead, the structure is implicitly contained in a special ordering of triangular elements. Numerical experiments demonstrate that the proposed coarsening algorithm is efficient when applied for multilevel preconditioners and mesh adaptivity for time-dependent problems.

Mathematics subject classification: 65M55, 65N55, 65N22, 65F10.
Key words: Adaptive finite element method, Coarsening, Newest vertex bisection, Multilevel preconditioning.

1. Introduction

Adaptive methods are now widely employed in the scientific computation to achieve better accuracy with minimum degree of freedom. A typical adaptive finite element method through local refinement can be written as the following loop:

$$\textbf{SOLVE} \rightarrow \textbf{ESTIMATE} \rightarrow \textbf{MARK} \rightarrow \textbf{REFINE/COARSEN}. \tag{1.1}$$

In this paper, we shall consider the modules **COARSEN** and **SOLVE**. More precisely, we propose a new efficient and easy-to-implement coarsening algorithm and apply to multilevel preconditioners for adaptive grids obtained by the newest vertex bisection in two spatial dimensions.

Classical recursive bisection and coarsening algorithms [21] are widely used in adaptive algorithms (see, for example, ALBERTA [31] and deal.II [3]). These algorithms make use of binary-tree related data structures and subroutines to store and access the bisection history.

We propose a new node-wise coarsening algorithm which does not require storing the bisection tree explicitly. We only store coordinates of vertices and connectivity of triangles which is the minimal information to represent a mesh for standard finite element computation. We can built a kind of tree structure into a special ordering of the triangles. By doing this way, we simplify the implementation of adaptive mesh refinement and coarsening and thus provide an easy-access interface for the usage of mesh adaptation without too much sacrifice in time.

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The coarsening algorithm can be applied to construct efficient multilevel solvers for elliptic problems. Based on the special geometric relation between nested triangulations obtained by our coarsening algorithm, we develop a new multilevel preconditioner which numerically outperforms several classical multilevel preconditioners.

The proposed coarsening algorithm can also be employed for mesh adaptation, especially for time-dependent problems. For steady-state problems, quasi-optimal meshes can be obtained in practice using (1.1) without the **COARSEN** step [11]. However, it is not the case for time-dependent problems as the local features might change dramatically in time. We provide a numerical example for the application of our coarsening algorithm to time-dependent problems.

The rest of the paper is organized as follows. We review the classical coarsening algorithm and introduce our new algorithm in Section 2. We explain the data structures and implementation of our coarsening algorithm in Section 3. In order to demonstrate the performance of the proposed coarsening algorithm, then we show two applications of our coarsening algorithm: one is multilevel preconditioners for stationary problems in Section 4 and the other is mesh adaptation for time-dependent problems in Section 5.

2. Coarsening Algorithms

In this section, we present a new coarsening algorithm for triangular meshes obtained by the newest vertex bisection method. Unlike the classical recursive coarsening algorithm, the proposed algorithm is non-recursive and requires neither storing nor maintaining the bisection tree information such as the *parents*, *brothers*, *generation*, etc.

2.1. Conformity and shape-regularity of triangulations

Let $\Omega \subset \mathbb{R}^2$ be a polygonal domain. A triangulation $\mathcal{T}$ (also known as mesh or grid) of Ω is a set of triangles (also indicated by elements) which is a partition of Ω. The set of nodes (also indicated by vertices or points) of the triangulation $\mathcal{T}$ is denoted by $\mathcal{N}(\mathcal{T})$ and the set of all edges by $\mathcal{E}(\mathcal{T})$. As a convention, all triangles $t \in \mathcal{T}$ and edges $e \in \mathcal{E}(\mathcal{T})$ are closed sets.

We define the *first ring* of a point $p \in \Omega$ or an edge $e \in \mathcal{E}(\mathcal{T})$ as

$$\mathcal{R}_p := \{t \in \mathcal{T} \,|\, p \in t\} \qquad \text{and} \qquad \mathcal{R}_e := \{t \in \mathcal{T} \,|\, e \subset t\},$$

respectively; and define the *local patch* of p or e as

$$\omega_p := \bigcup_{t \in \mathcal{R}_p} t \qquad \text{and} \qquad \omega_e := \bigcup_{t \in \mathcal{R}_e} t,$$

respectively. Note that ω_p and ω_e are subdomains of $\Omega \subset \mathbb{R}^2$, while $\mathcal{R}_p$ and $\mathcal{R}_e$ are sets of triangles which can be viewed as triangulations of ω_p and ω_e, respectively. The cardinality of a set S is denoted by $\#S$. For each vertex $p \in \mathcal{N}(\mathcal{T})$, the *valence* of p is defined as the number of triangles in $\mathcal{R}_p$, i.e., $\#\mathcal{R}_p$.

For finite element discretizations, there are two standard conditions imposed on triangulations. The first condition is the conformity. A triangulation $\mathcal{T}$ is *conforming* if the intersection of any two triangles t_1 and t_2 in $\mathcal{T}$ either consists of a common vertex, a common edge, or empty. The second condition is the shape-regularity. A set of triangulations $\mathscr{F}$ is called *shape-regular* if there exists a constant σ such that

$$\max_{t \in \mathcal{T}} \frac{\operatorname{diam}(t)}{|t|^{\frac{1}{2}}} \leq \sigma, \quad \text{for all } \mathcal{T} \in \mathscr{F}, \tag{2.1}$$