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A NEW COUPLED SUBDIFFUSION MODEL AND ITS PARTITIONED TIME-STEPPING ALGORITHM*

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Abstract

This paper investigates an interface-coupled fractional subdiffusion model, featuring two subdiffusion equations in adjacent domains connected by an interface allowing bidirectional energy transfer. The fractional derivative, accounting for long-term medium effects, introduces challenges in theoretical analysis and computational efficiency. We propose a partitioned time-stepping algorithm using higher-order extrapolations on the interface term to decouple the system with improved temporal accuracy, combined with finite element spatial approximations. Rigorous theoretical analysis demonstrates unconditional stability and optimal L^2 norm error estimates, supported by several numerical experiments.

Mathematics subject classification: 35R11, 65M60, 65M12.
Key words: Time-fractional derivative, Interface-coupled problem, Partitioned time-stepping method, Stability, Error estimate.

1. Introduction

There are many problems in which different physical models, different parameter regimes, or different solution behaviors are coupled across interfaces, such as atmosphere-ocean coupling and fluid-solid interaction problems [1, 2, 4, 25, 32]. In recent years, significant attention has been devoted to multi-domain, multi-physics coupled problems [3, 5, 12, 13, 19]. Particularly,

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many innovative numerical simulation methods have been developed for fluid-fluid interactions [6, 10, 15, 17, 29]. Connors *et al.* [8, 9] introduced a partitioned time-stepping method to naturally decouple multidomain multiphysics systems, using a simplified atmosphere-ocean interaction model with deterministic friction parameter κ as a linear heat-heat coupled system. They proposed fully implicit, implicit-explicit partitioned, and data-passing partitioned schemes, proving first-order time accuracy for both partitioned schemes and extending the method to nonlinear models. Given that coupled fluid-fluid and fluid-structure interaction dilemmas result in large and ill-conditioned systems of algebraic equations, partitioned techniques have frequently been employed to break down the interconnected problem into smaller, better-conditioned subproblems [9, 20, 33].

Ongoing research demands increasingly precise models to study sophisticated materials and anomalous diffusion phenomena in physics. Fractional differential operators effectively describe phenomena involving long-term or nonlocal interactions [11, 14, 23]. Consequently, there has been significant interest in developing its theory and numerical analyses for fractional differential equations [18, 30, 37, 38]. Many numerical methods have been studied for the time-fractional PDEs. Liu *et al.* [26] presented a finite difference method in both space and time for the time-fractional diffusion equation. Sun and Wu [35] considered and analyzed a finite difference scheme for the fractional diffusion wave equation. Lin and Xu [24] presented a finite difference scheme in time and Legendre spectral method in space for the time-fractional diffusion equation. Li and Yi [21] investigate an implicit-explicit scheme for 2D nonlinear time-fractional subdiffusion equation with proving the stability and convergence. Lv and Xu [28] discussed a high order finite difference method to approximate the fractional derivative in time, resulting in a time stepping scheme for the underlying equation, and analyzed the stability and convergence of the time stepping scheme. Li *et al.* [22] established a fractional Gronwall inequality for the $L1$ approximation to the Caputo fractional derivative and provided optimal error estimates for several fully discrete linearized Galerkin finite element methods for nonlinear problems. Wang and Zheng [36, 40] explore finite element and finite difference approximations of time-fractional diffusion equations, focusing on regularity properties and conducting detailed error analysis. Zhang *et al.* [39] study a weighted and shifted Grünwald-Letnikov difference Legendre spectral method for 2D nonlinear time fractional mobile/immobile advection-dispersion equation and prove its stability and convergence. Luo *et al.* [27] present a numerical scheme based on compact finite difference for solving time fractional equation with the accuracy is not dependent on the fractional order. Qi and Xu [31] examine the stochastic time-fractional heat diffusion equation with a random diffusion coefficient field and fractionally multiplicative noise, providing the problem's well-posedness and the numerical scheme's convergence.

However, multidomain coupled subdiffusion problems have received less attention. Drawing inspiration from decoupling methods for parabolic problems with dual subdomains [33, 34], we introduce a partitioned time-stepping algorithm for a linear interface-coupled subdiffusion model. We employ higher-order extrapolations on the interface term to decouple the system, requiring only information exchange across the interface at each time step. This is followed by the independent resolution of each subproblem, with both subdomain solvers operating as black boxes.

The structure of this paper is as follows. In Section 2, we present some notations and collect preliminary lemmas. Section 3 is dedicated to proving the stability of the algorithm. In Section 4, we provide an error analysis of the algorithm and Section 5 present some numerical experiments that illustrate the theoretical results.