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SPECTRAL SOLUTIONS OF A FRACTIONAL-ORDER MATHEMATICAL MODEL FOR LUNG CANCER, SENSITIVITY ANALYSIS, AND FEEDBACK CONTROL*

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Abstract

A fractional-order mathematical model of lung cancer is used to describe the dynamics of tumor growth and the interactions between cancer cells and immune cells. To obtain approximate solutions and better understand the behavior of the state functions, a pseudo-operational collocation scheme employing shifted Jacobi polynomials as basis functions is introduced. Initially, the existence and uniqueness of solutions to the model are established using the Leray-Schauder fixed-point theorem. Error bounds for the residual functions are estimated within a Jacobi-weighted L^2-space. To enhance the accuracy and reliability of the results, two distinct strategies are implemented: sensitivity analysis and feedback control. The feedback control of the proposed pseudo-operational spectral method is performed using the method of Lagrange multipliers, marking its first application in this context. Spectral solutions are derived by applying the pseudo-operational scheme to both the original model and the model with control functions. Improved performance and outputs are anticipated following the application of the feedback control strategy. Finally, comprehensive biological interpretations of the results are provided, offering insights into the practical implications of the model.

Mathematics subject classification: 65K05, 93A30, 92B05, 37M05.
Key words: Fractional-order model of lung cancer, Fractional operators, Existence and uniqueness, Jacobi collocation method, Feedback control strategy.

1. Introduction

Fractional calculus is the generalization of the classical integer-order calculus. Fractional operators possess memory effects that integer-order integral and derivative operators are not able to represent them. These operators are especially applicable in modeling systems that emerged in physics, chemistry, biology, environmental data, and so on. Since fractional operators are non-local, they take notice of the entire history of a phenomenon rather than its current behavior. Indeed, fractional calculus provides deeper intuitions and more precise models [3, 15, 30, 39].

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Mathematical modeling of diverse diseases helps researchers to understand how diseases can spread through the body or population. They furnish insights into figuring out the mechanisms, dynamics, transmission ways, and factors influencing the propagation of diseases. Mathematical models can simulate the effect of diverse interferences, for instance, vaccination, quarantine, and social distances on disease transmission. Therefore, researchers and healthcare professionals can predict the future treatment of the disease and evaluate various strategies to control it [4, 5, 7, 10, 11, 18, 36]. Among different types of cancers, the study of causes, mechanisms, and therapy strategies of lung cancer is vital for its high mortality rate. That is why, researchers have simulated various models to describe the growth of lung cancer in lung tissues [1,6,13,31,42,46,49]. From the point of view of the numerical solution, Amilo *et al.* [4] solved numerically their suggested model of progression of the lung cancer using an Adams-Bashforth predictor-corrector method. Authors in [38] presented a fractional model of tumor-immune system interaction related to lung cancer and applied an Adams-type predictor-corrector method to estimate numerical solutions to the proposed model. Hassani *et al.* [21], used the generalized Laguerre polynomial method to obtain optimal solutions of a model of lung cancer.

Spectral methods, including the Galerkin, tau, and collocation, are a class of techniques utilized to solve different functional equations where the solution to the given problem is represented as a linear combination of basis functions [14, 26, 27, 29, 35, 50]. Basis functions can be eigenvalues of some second-order differential equations such as Jacobi, Laguerre, and Hermite polynomials [2, 12, 16, 23, 24, 43, 45, 47, 48, 51]. Orthogonal Jacobi polynomials play a significant role in different areas of mathematics and its applications. These polynomials are employed in various numerical techniques to approximate and interpolate unknown functions in given functional equations. Jacobi polynomials include two parameters $\theta, \vartheta > -1$ which by varying values of θ and ϑ, different special cases of these polynomials appear (for instance, Legendre polynomials for $\theta = \vartheta = 0$, Chebyshev polynomials of the first and second kinds for $\theta = \vartheta = -0.5$ and $\theta = \vartheta = 0.5$, respectively). In this way, the effect of varying two parameters on approximate solutions can be investigated. The trace of Jacobi polynomials can be seen in many research works as basis functions. For instance, to solve fractional integro-differential equations [12, 27, 35, 45, 47, 50] and time- or time-space fractional partial differential equations (PDEs) including the pantograph PDEs, Fisher-Kolmogorov equation, diffusion-wave equations, hyperbolic PDEs, telegraph equations, distributed-order fractional Schrodinger equation and so on [17, 22–24, 33, 48].

As we know, Jacobi polynomials have not been used to numerically solve systems of fractional differential equations obtained from mathematical modeling of diseases. For this reason, this research deals with applications of these polynomials for such models. A fractional-order system of differential equations, presented in [4], reveals the interactions between cancer cells and immune cells in lung tissues and cancer cells that have spread to other parts of the body as follows:

$$\begin{cases} {}_0^C\mathcal{D}_\tau^\zeta \mathcal{N}(\tau) = \lambda \mathcal{N}(\tau)\left(1 - \dfrac{\mathcal{N}(\tau)}{\kappa}\right) - \mu \mathcal{N}(\tau)\mathcal{P}(\tau) - \beta_1 \mathcal{N}(\tau)\mathcal{I}(\tau), \\ {}_0^C\mathcal{D}_\tau^\zeta \mathcal{I}(\tau) = \varphi_1 \mathcal{I}_0 + \varphi_2 \mathcal{N}^2(\tau) - \varphi_3\, \mathcal{I}(\tau) - \beta_2 \mathcal{I}(\tau)\mathcal{P}(\tau), \\ {}_0^C\mathcal{D}_\tau^\zeta \mathcal{P}(\tau) = \gamma \mathcal{N}(\tau)\, \mathcal{P}(\tau) - \delta \mathcal{P}(\tau) - \beta_3\, \mathcal{I}(\tau)\mathcal{P}(\tau), \end{cases} \tag{1.1}$$

where $\mathcal{N}(\tau)$ represents the number of cancer cells in lung tissues at time $\tau, \mathcal{I}(\tau)$ represents the number of immune cells in lung tissues at time τ, and $\mathcal{P}(\tau)$ represents the number of cancer cells that have spread to other parts of the body at time τ, $\tau \in [0, T]$. ${}_0^C\mathcal{D}_\tau^\zeta(.)$ is the