NEWTON ITERATIVE INVERSION METHOD FOR INVERSE OBSTACLE SCATTERING IN A LAYERED MEDIUM*

Hao Wu and Jiaqing Yang[1]

School of Mathematics and Statistics, Xi'an Jiaotong University, Xi'an 710049, China
Emails: wuhao2022@stu.xjtu.edu.cn, jiaq.yang@mail.xjtu.edu.cn

Abstract

Consider the inverse acoustic scattering of time-harmonic point sources by a locally perturbed interface with bounded obstacles embedded in the lower half-space. A Newton-type iterative method is proposed to simultaneously reconstruct the locally rough interface and embedded obstacles by taking partial near-field measurements in the upper half-space. The method relies on a differentiability analysis of the scattering problem with respect to the locally rough interface and the embedded obstacle, which is established by introducing a kind of new shape derivatives and reducing the original model to an equivalent system of integral equations defined in a bounded domain. With a slight modification, the inversion method can be easily generalized to reconstruct local perturbations of a global rough interface. Finally, numerical results are presented to illustrate the effectiveness of the inversion method with the multi-frequency data.

Mathematics subject classification: 65N21, 78A46, 35J05.
Key words: Newton iterative algorithm, Shape derivative, Inverse scattering, Locally rough interface, Embedded obstacle, Multi-frequency data.

1. Introduction

This paper is concerned with the two-dimensional inverse scattering of time-harmonic acoustic point sources by a locally rough interface with bounded obstacles embedded in the lower half-space. Such problems can find important applications in diverse scientific fields such as geophysics, underwater exploration and nondestructive testing, where the unbounded interface and embedded obstacles need to be reconstructed simultaneously from scattered field measurements over certain subdomains of the upper half-space.

In this paper, we consider the interface to be different from the planar interface over a finite interval, namely, locally rough surfaces. In this case, the whole space is divided into the upper and lower half-space, where an impenetrable obstacle is assumed to be embedded in the lower half-space. Considering a point source as an incident wave, then the wave motion is governed by the two-dimensional Helmholtz equation with a piecewise constant wavenumber. Since the interface is locally perturbed, the classical Sommerfeld radiation condition can be used to guarantee the uniqueness of the solution to the scattering problem. In the above setting, it is well-known that if there are no embedded obstacles, the well-posedness has been proved in associated function spaces by either the variational method, e.g. [2, 7, 43], or the integral equation method, e.g. [9, 10, 25, 38]. For the case of the presence of embedded obstacles, it was shown in [45] that a new technique was proposed to prove the well-posedness by reducing

the original problem into an integral equation defined on a bounded region. Motivated by this technique, we study in this paper the inverse problem for simultaneously recovering the infinite interface and embedded obstacles from the near-field measurements. It is noticed in [45] that a global uniqueness theorem has been proved that the interface and embedded obstacles can be uniquely determined from near-field measurements on a bounded line segment in the upper half-space. With this result, we aim to develop an efficient algorithm to solve the inverse problem numerically. Notice that if there are no embedded obstacles, many inversion algorithms are developed for solving inverse rough interface scattering problems in the literature such as the Kirsch-Kress method [26], Newton-type algorithms [11, 42], algorithms based on the transformed-field expansions [4], and non-iterative qualitative methods [27, 30, 31, 36, 46]. For the case of a planar interface with buried obstacles, we refer to the MUSIC-type scheme [1], the asymptotic factorization method [15], the sampling method [14], and the direct imaging algorithm [24, 29]. More generally, for the case of a rough interface with buried obstacles, the inversion algorithm was recently proposed in [28], which proposes a new version of the linear sampling method (LSM) for the numerical recovery of rough interfaces and embedded obstacles by constructing a modified near-field equation. Compared to the non-iterative LSM method, which can provide a good initial value for the iterative method, we aim to develop a Newton-type iterative method to further accurately recover the shape of the interface and embedded obstacles. As related works, we also refer to [3, 5, 6, 8, 13, 22, 23, 33, 34, 41, 44, 47] for the case of impenetrable rough surfaces.

The iterative method is a popular approach to solve the inverse scattering problem, where the key point is to analyze the Fréchet differentiability of the far-field or near-field with respect to the scatterer or physical parameters. A lot of literature can be found on Fréchet differentiability analysis, but they are mostly restricted to the case of bounded obstacles, e.g. [17–21, 37, 39, 40]. It is more challenging for infinite surface scattering problems, especially for the penetrable interfaces case. So far, the relevant results can only be provided when the surface is impenetrable and locally rough. It was shown by Bao and Lin [5] that the Fréchet differentiability analysis of the inverse locally rough scattering problem with a Dirichlet boundary condition can be proved based on the Dirichlet reflection principle of the Laplace operator. This approach was also extended by Qu *et al.* [41] into the inverse locally rough scattering problems with a Neumann boundary condition. However, the reflection principle fails in the impedance and penetrable interface scattering problems. Recently, Liu and Yang [35] established the Fréchet differentiability analysis of locally rough surfaces with Dirichlet and impedance boundary by transforming the original problem into a bounded boundary integral equation and repeatedly using Green's theorem. Nevertheless, this approach also fails in the case of penetrable interfaces, because in this case the scattering problem cannot be transformed into a bounded boundary integral equation, and to the authors' knowledge, there is no related works on Fréchet differentiability analysis for infinite rough interface scattering problems.

In this paper, we will propose a Newton-type iterative method to simultaneously reconstruct the locally rough interface and embedded obstacles, in which the main difficulty is how to establish the corresponding Fréchet differentiability analysis. In order to overcome this difficulty, we will first introduce the concept of shape derivatives, where the shape transformation function are defined, which allows us to use the one-dimensional variables t and s to characterize the shape variation of rough interfaces and embedded obstacles. Further, we introduce the perturbation problem with respect to t and s. With the help of a special hemispherical interface problem, the perturbation problem can also be reduced to a system of integral equations de-