Mathematical Model of Electric Field Distribution at a Critical State in Bubble Electrospinning

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Abstract: Bubble electrospinning is a novel method to produce ultrafine nanofibers. In this work, a mathematical model of the electric field distribution in a critical state was established for bubble electrospinning process and was numerically simulated using finite element method. The results indicated that the maximum electric force occurred at the tip of the bubble, it is probably for this reason that the first jet occurs at the tip of the bubble, when the electrical force exceeds the surface tension, which also possibly induces bubble bursting.

Keywords: Bubble electrospinning, electric field, mathematical model, numerical simulation.

1. Introduction

Electrospinning is an economical and straightforward method to produce ultrafine nanofibers from polymer solutions [1,2]. In this process, the electrostatic force, which is generated in an electrical field from a variety of polymer solutions, enables the production of ultrafine nanofibers. The traditional electrospinning setup uses a hollow needle as the spinning nozzle. Each nozzle only generates one polymer jet that results in a very limited fiber productivity, about 0.1-1.0g/h [3].

Bubble electrospinning, as a novel electrospinning technique, was designed to increase the number of jets which finally formed nanofibers. In bubble electrospinning, multiple jets can be easily obtained by the bursting of the polymer bubble which is produced by releasing the compressed gas via a gas tube (the reader can refer to [4,5] for a detailed description of the bubble electrospinning process and its principles). The electrostatic force on bubble surface plays a key role in bursting of the bubble. Although various experimental studies have been conducted, a mathematical model has not yet been obtained. In this work, a 2D mathematical model of the electric field distribution in a critical state was established for bubble electrospinning process. For simplicity, some assumptions were considered such as the bubble was exposed to a uniform electrical field and the polymer solution was perfectly dielectric. In the critical state, we assumed that the bubble was elliptical and it was symmetrical around the central axis in the rectangular coordinate in a two-dimensional condition.

2. Mathematical model of electrical field distribution

According to our previous studies [4-6], the bubble was subjected to electric field and stretched from a spherical shape to a cone shape. In order to simplify the calculation, we assumed that the bubble is elliptical in shape at the critical state in this process. Joukowski Transformation, which is a conformal map historically used to understand some principles of airfoil design, was used to describe the deformation of the bubble. The transform is

$$z = \frac{c}{2}\left(\zeta + \frac{1}{\zeta}\right), \tag{1}$$

where $z = y + xi$ and $\zeta = \eta + \xi i$ are complex variable, $c = \sqrt{b^2 - a^2}$, b and a presents major semi-axis and minor semi-axis of the ellipse, respectively. According to Eq. (1), the equations can be obtained as follows:

$$y = \frac{c}{2}\left(\eta + \frac{\eta}{\xi^2 + \eta^2}\right) = \frac{c}{2}\left(\rho + \frac{1}{\rho}\right)\cos\psi, \tag{2}$$

$$x = \frac{c}{2}\left(\xi - \frac{\xi}{\xi^2 + \eta^2}\right) = \frac{c}{2}\left(\rho - \frac{1}{\rho}\right)\sin\psi. \tag{3}$$

When $$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, \tag{4}$$

$$\rho_0 < 1,\ \rho_0 = \frac{c}{a+b}, \tag{5}$$

$$\rho_0 > 1,\ \rho_0 = \frac{a+b}{c}, \tag{6}$$

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Where ρ_o is the radius of circle in the ζ plane. And ψ is the polar angle and ρ is the radial distance from the origin in the cylindrical coordinates. So the elliptical bubble is generated in the z plane by employing the Joukowsky transform to a circular bubble in the ζ plane, as shown in Figure 1.

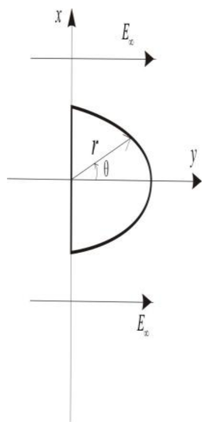

Figure 1 (a) the *z* plane.

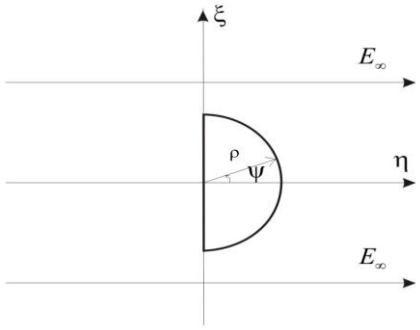

Figure 1 (b) the ζ plane.

The behavior of a fluid cylinder suspended in another fluid and subjected to an electrical field has been discussed by MN Reddy and A Esmaeeli [7]. In their paper, a mathematical model of the electrical field distribution for a perfect dielectric fluid immersed in another a perfect dielectric fluid was established. Based on their study, the electric potential outside the polymer circular bubble, which is surrounded by air in the ζ plane, can be expressed as

$$\begin{aligned}\phi_\zeta &= E_{\zeta\infty}\left[\rho - \frac{S-1}{S+1}\frac{\rho_o^2}{\rho}\right]\cos\psi \\ &= E_{\zeta\infty}\left[\eta + \frac{\eta}{\xi^2+\eta^2} - \left(1+\frac{S-1}{S+1}\rho_o^2\right)\frac{\eta}{\xi^2+\eta^2}\right],\end{aligned} \tag{7}$$

$$S = \frac{\varepsilon_i}{\varepsilon_o}. \tag{8}$$

where $E_{\zeta\infty}$ presents uniform Electrical field strength in the ζ plane, ε_i and ε_o are the relative permittivities of the bubble and the air, respectively.

According to Eqs. (2) and (3), we can get

$$\eta + \frac{\eta}{\xi^2+\eta^2} = \frac{2}{c}y, \tag{9}$$

$$\frac{\eta}{\xi^2+\eta^2} = \frac{2cy}{c^2 + f_1 + f_2 + \sqrt{2[c^2 f_1 + f_1(f_1+f_2)]}}, \tag{10}$$

$$f_1(x,y) = x^2 + y^2, \tag{11}$$

$$f_2(x,y) = \sqrt{c^4 + 2c^2(x^2-y^2) + (x^2+y^2)^2}, \tag{12}$$

The electric potential outside the polymer's elliptical bubble in the z plane can be expressed as

$$\phi_z = E_{\zeta\infty}\left[\frac{2}{c}y - \left(1+\frac{S-1}{S+1}\rho_o^2\right)\frac{2cy}{c^2 + f_1 + f_2 + \sqrt{2[c^2 f_1 + f_1(f_1+f_2)]}}\right], \tag{13}$$

Since the electric field is irrotational, it is possible to define the electric field strength E_y and E_x, which are the Y-axis direction and the X-axis direction components of the electrical field strength, respectively.

$$E_y = \frac{\partial\phi_z}{\partial y} = E_{\zeta\infty}\left[\frac{2}{c} - \left(1+\frac{S-1}{S+1}\rho_o^2\right)\left(\frac{1}{c} - \frac{\sqrt{c^2 f_3 + f_1(f_1+f_2)}}{\sqrt{2}cf_2}\right)\right], \tag{14}$$

$$E_x = \frac{\partial\phi_z}{\partial x} = E_{\zeta\infty}\left(1+\frac{S-1}{S+1}\rho_o^2\right)\frac{\sqrt{2}cxy}{f_2\sqrt{c^2 f_3 + f_1(f_1+f_2)}}, \tag{15}$$

$$f_3(x,y) = x^2 - y^2. \tag{16}$$

Where E_{sy} and E_{sx} are the electrical field strength on the elliptical bubble surface respectively. According to Eqs. (4) and (5), we obtained

$$E_{sy} = E_{\zeta\infty}\left[\frac{2}{c} - \left(1+\frac{S-1}{S+1}\frac{c^2}{(a+b)^2}\right)\left(\frac{1}{c} - \frac{\sqrt{c^2 f_{3y} + f_{1y}(f_{1y}+f_{2y})}}{\sqrt{2}cf_{2y}}\right)\right] \tag{17}$$

$$E_{sx} = E_{\zeta\infty}\left(1+\frac{S-1}{S+1}\frac{c^2}{(a+b)^2}\right)\frac{\sqrt{2}cxy}{f_{2x}\sqrt{c^2 f_{3x} + f_{1x}(f_{1x}+f_{2x})}} \tag{18}$$

$$f_{\mu y} = f_\mu\left(\frac{a^2(b^2-y^2)}{b^2}, y\right), \tag{19}$$

$$f_{\mu x} = f_\mu\left(x, \frac{b^2(a^2-x^2)}{a^2}\right), \tag{20}$$

$$(\mu = 1,2,3).$$