Ann. Appl. Math.
doi: 10.4208/aam.OA-2024-0015

Vol. **40**, No. 4, pp. 347-393
November 2024

An Enriched Petrov-Galerkin Method for Darcy Flow in Fractured Porous Media

Huangxin Chen* and Zixuan Wang

School of Mathematical Sciences and Fujian Provincial Key Laboratory on
Mathematical Modeling and High Performance Scientific Computing,
Xiamen University, Xiamen, Fujian 361005, China

Received 27 May 2024; Accepted (in revised version) 3 July 2024

Abstract. We develop a locally mass-conservative enriched Petrov-Galerkin (EPG) method without any penalty term for the simulation of Darcy flow in fractured porous media. The discrete fracture model is applied to model the fractures as the lower dimensional fracture interfaces. The new method enriches the approximation trial space of the conforming continuous Galerkin (CG) method with bubble functions and enriches the approximation test space of the CG method with piecewise constant functions in the fractures and the surrounding porous media. We propose a framework for constructing the bubble functions and consider a decoupled algorithm for the EPG method. The solution of the pressure can be decoupled into two steps with a standard CG method and a post-processing correction. The post-processing correction based on the bubble functions in the matrix and the fractures can be solved separately, which is useful for parallel computing. We derive a priori and a posteriori error estimates for the problem. Numerical examples are presented to illustrate the performance of the proposed method.

AMS subject classifications: 65M60, 65N30, 76S05

Key words: Discrete fracture model, enriched Petrov-Galerkin method, local mass conservation, post-processing, error analysis.

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*Corresponding author.

Emails: `chx@xmu.edu.cn` (H. Chen), `wangzixuan@stu.xmu.edu.cn` (Z. Wang)

1 Introduction

Modeling and simulation of flows in fractured porous media have wide applications in the fields such as petroleum reservoir engineering and geotechnical engineering [21, 23, 38, 44, 55]. We distinguish two types of the fractures: fractures usually called conductors which have the permeability higher than that in the surrounding porous media and those called barriers in which the permeability is lower than that in the matrix. In the porous media with the former type of fracture, the fluid has a tendency to flow into the fracture. However, when the fracture has a lower permeability, the fluid spontaneously tends to avoid flowing into the fracture, which is treated as a geological barrier.

Various types of models have been well established to describe flows in fractured porous media, which include the dual-porosity model [9, 51], the discrete fracture model (DFM) [8, 24, 25, 36, 39, 56], the embedded DFM [37, 48] and the discrete fracture network model [11, 22]. Among the models mentioned above, the DFM is a prominent and popular model. In [25], the authors proposed to model the matrix and the fracture as the same dimensional domains, which requires very fine grids to simulate the flow in the fracture and results in significant degrees of freedom and computational costs. In [36], the authors demonstrated that the fracture can be treated as a lower dimensional interface, which can greatly reduce the computational costs and handle the problem with high or low permeability in the fracture. This reduced model takes into account of the interactions between the fracture and the matrix. The DFM discussed in this paper refers to this approach [8, 36, 39, 56]. Besides, lots of achievements [35, 53, 54] based on the DFM have been presented in the literature recently.

In addition, we focus on the numerical methods preserving local mass conservation, which is significant in numerous applications, especially when the coupling of flow and transport is considered [21, 55]. The discontinuous Galerkin (DG) method [3, 6, 12, 19, 28, 31, 40, 45] can tackle rough coefficient cases and capture the nonsmooth features of the solution very well since the function space itself is naturally discontinuous. However, the main disadvantage of the primal DG method is that it requires a very huge number of degrees of freedom, which results in expensive computational costs. Quite a few works have also constructed alternative post-processing strategies to preserve local mass conservation [16, 18, 27, 30, 46]. Among these methods, the enriched Galerkin (EG) method [32–34, 43] precisely enriches the continuous Galerkin (CG) finite element space with piecewise constant functions, which has fewer degrees of freedom than the DG method while maintaining the local mass conservation. Besides, in order to improve the quality of the numerical approximation, particularly when the singularity of the solution occurs around