A Three-Dimensional Mass-Conserved Multiphase Lattice Boltzmann Flux Solver for Incompressible Flows with Large Density and Viscosity Ratios

Ehsan Azadi and Mohammad Taeibi Rahni*

Department of Aerospace Engineering, Sharif University of Technology, Tehran, Iran

Abstract. Multiphase lattice Boltzmann flux solver (MLBFS) uses the finite volume method to solve Navier-Stokes (NS) and Cahn-Hilliard (CH) equations. However, instead of macroscopic fluxes, the corresponding mesoscopic fluxes, defined using a local lattice Boltzmann method, are evaluated at cell faces. Since the development of the original three-dimensional (3D) MLBFS [16], several improvements have been made in it, including modification of CH equation to conserve the mass, determination of CH mesoscopic fluxes to eliminate the weighted essentially non-oscillatory scheme, and simplification of mesoscopic fluxes to improve the computational efficiency. However, these improvements have been implemented and studied in two-dimensional or axisymmetric MLBFS. In this study, MLBFS (based on the mentioned improvements) has been extended to 3D flows and applied to simulate three incompressible multiphase benchmark cases with large density and viscosity ratios up to 1000 and 100, respectively. The results of benchmark cases (Laplace law, bubble rising, and drop impact on a dry surface) agree well with previous credential data. Our simulations show the original CH equation considerably reduces the bubble/drop mass, while the modified CH equation conserves it completely. According to results, using the simplified mesoscopic fluxes (instead of the original ones) saves about 13% of computational time.

AMS subject classifications: 76P05, 76T10, 76D05, 76M12

Key words: 3D multiphase flows, lattice Boltzmann flux solver, large density and viscosity ratios, mass-conserved solver.

1 Introduction

Multiphase flows are widely observed in nature and in industries dealing with thermo-fluid mechanics. Both experimental methods [1, 2] and numerical simulations [3–5] are used to understand the behaviors of multiphase flows and how to control them. Due

---

*Corresponding author.
Emails: azadi_ehsan@ae.sharif.edu (E. Azadi), taeibi@sharif.edu (M. Rahni)

to complex topological changes of the interfaces between different phases and different fluid properties of phases, numerical simulations of such flows are often very challenging [3,4]. Usually, multiphase flow simulations are based on 1) continuum-based Navier-Stokes (NS) solvers or 2) kinetic-based lattice Boltzmann method (LBM), each of these approaches has its own advantages and disadvantages [3–5]. In the first approach, in addition to the NS equations used to balance the mass and momentum in the flow field, an interface (sharp-interface or diffuse-interface) related formulation is used. In this approach, finite volume method (FVM) can be used to discretize the governing equations, leading to the simplicity of defining and implementing macroscopic variables, boundary conditions, and source terms, as well as the possibility of using both uniform and non-uniform grids [3,5]. However, NS solvers require complex methods to determine fluxes, and handle the difficulty of pressure-velocity coupling [3,5]. In the second approach, the distribution functions (DF) (developed based on the mesoscopic properties of the flow) are used. The Chapman-Enskog (CE) expansion analysis shows that the NS and diffuse-interface tracking equations can be recovered by LBM [4,6]. The simple kinetic streaming-collision procedure in LBM leads to efficient numerical algorithms, especially for parallel computing. Since the interfacial phenomenon is mainly due to intermolecular interactions between phases, the LBM has an appropriate physical base for simulation of multiphase flows. However, LBM has some drawbacks, including coupled time step and mesh spacing, limitation in the use of uniform grids, and complex implementation of physical boundary conditions and source terms. Moreover, numerous lattice directions in LBM require a large capacity of computational memory [4,7].

In 2014, an interesting lattice Boltzmann flux solver (LBFS) was developed for single-phase flows to overcome most of the mentioned drawbacks of NS equations solvers and LBM [8,9]. In LBFS, first, NS equations were discretized by FVM, where macroscopic variables and source terms were defined at cell centers and macroscopic fluxes were defined at cell faces. Then, CE analysis was used to reconstruct NS equations from the standard LBM. As a result, the equivalents of NS macroscopic fluxes are obtained by the moments of DFs of LBM, which here are called mesoscopic fluxes. Finally, the macroscopic fluxes were replaced by their corresponding mesoscopic ones. These mesoscopic fluxes are obtained by applying a local LBM on cell faces. Using the local LBM eliminates the need to store numerous DFs in so many lattice cells, and consequently reduces the required virtual memory. On the other hand, using mesoscopic fluxes overcome the difficulties related to determinization of macroscopic fluxes and pressure-velocity coupling in NS solvers. Note that, unlike LBM, in LBFS, boundary conditions and source terms are easily applied by macroscopic variables. In addition, both uniform and non-uniform grids can be used [10–14]. Thus, LBFS uses the suitable features of both FVM-based NS solvers and LBM at the same time.

In 2015, a two-dimensional (2D) multiphase lattice Boltzmann flux solver (MLBFS) was developed by Wang et al. [15], where NS and Cahn-Hilliard (CH) equations were used as macroscopic governing equations. Note that, CH equation is a diffuse-interface model (based on the phase-field method) to determine the interface behavior in multi-