Advances in Applied Mathematics and Mechanics
Adv. Appl. Math. Mech., Vol. **18**, No. 5, pp. 1554-1575

DOI: 10.4208/aamm.OA-2022-0336
October 2026

An Efficient Numerical Scheme Based on Sinc Functions for the Analysis of 1D Peridynamic Models

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Received 15 December 2022; Accepted (in revised version) 31 October 2025

Abstract. The present work is concerned with developing a numerical scheme (viz., sinc-Galerkin method) based on sinc functions for the investigation of various 1D peridynamic models. Four different peridynamic models corresponding to four micro-modulus functions have been investigated here. The *a posteriori* errors in the approximation has been estimated and compared with the absolute errors whenever exact solutions are available. The results of the investigation ensure that the proposed scheme may be quite effective and useful for the analysis of 1D peridynamic models.

AMS subject classifications: 74A70, 74H15, 45K05, 65T60

Key words: Peridynamic models, Sinc function, Cardinal representation of functions, integro-partial differential equations.

1 Introduction

The peridynamic (PD) theory is a nonlocal continuum theory introduced by Silling [1] in an attempt to deal with the discontinuities in continuum mechanics. This theory is a reformulation of the equation of motion in solid mechanics, better suited for modeling bodies with discontinuities, such as cracks and failures. Unlike the classical continuum theory, the PD theory is formulated using an integro-partial differential equation (IPDE), which avoids the evaluation of the spatial partial differential equations. This attribute provides the PD theory a unique capability to handle discontinuities in materials, whereas the classical theory fails due to the nonexistence of spatial derivatives in the region of discontinuities. The PD theory has been successfully applied to damage analysis of metallic and nano-scale structures [2–4]. Later, its application was also extended to damage and failure analysis of composite materials [5, 6] and polycrystals [7, 8].

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The IPDE appearing as the governing equation of peridynamics is difficult to solve analytically. However, a few analytical solutions exist in the literature [9–13]. One of the main difficulties in the analytical treatment of PD models is that the formal solution obtained by the Fourier transform is often divergent or contains singular-type integrals. As a result, it needs a higher computational effort to solve peridynamic equations. Consequently, an efficient numerical scheme to deal with this type of models will be of great interest.

Various approximation schemes/numerical methods have been implemented so far for approximating PD-based nonlocal models. Some of them include finite difference, finite element, quadrature, particle-based methods as well as recently developed peridynamic differential operator [14–19] etc. In this article, the authors have developed a numerical scheme based on sinc functions (viz. sinc-Galerkin method) to obtain an approximate solution for various peridynamic models in one dimension.

Numerical schemes based on sinc functions have drawn wide attention in recent years. It has been used with great success to solve one-dimensional problems, e.g., interpolation, numerical integration, convolutions, approximation of derivatives, regularization of singularities etc. [20].

In the present work, the authors have investigated four 1D peridynamic models corresponding to four different micromodulus functions (MFs), one being discontinuous and the rest being smooth. In one of the examples, the body force density (two-point concentrated loads) is taken non-zero, which is relatively difficult to exercise, as pointed out by Mikata [13]. The proposed scheme effectively provides highly accurate approximate solutions to various peridynamic models in one-dimension.

The paper is organized as follows. One-dimensional PD models with four different MFs are discussed in Section 2. In Section 3, an appropriate basis comprising translates of sinc function has been constructed for the cardinal representation of the solution of PD models. The representation of the integral operators in the basis is obtained in Section 4. Section 5 describes the sinc-Galerkin method for obtaining an approximate solution of an integro-partial differential equation that appears in the mathematical modeling of 1D peridynamic problems. The scheme for obtaining the approximate solution has been summarized in Section 6 so that it can be coded in any computer language efficiently. Some examples have been exercised in Section 7 to test the efficiency of the scheme developed here. The usefulness and limitations of the present scheme have been discussed in the concluding Section 8.

2 One-dimensional peridynamic models

The equation of motion for one-dimensional linear bond-based peridynamics is given by [9,13]

$$\int_{-\infty}^{\infty} C(\xi)\{u(x-\xi,t)-u(x,t)\}\, d\xi + b(x,t) = \rho \frac{\partial^2 u}{\partial t^2}(x,t), \quad x\in\mathbb{R}, \quad t\in\mathbb{R}^+, \tag{2.1}$$