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The Spread Model of SHIR Rumors under Impulse Intervention

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Abstract: In the rumor propagation model, different people have different attitudes towards a rumor. This article considers the four attitudes of different people to rumors: unknown (S), hesitation (H), spread (I), and resistance (R)'s rumor spreading model, on this basis, considers impulsive supervision by the government. Under impulsive intervention, that is, the government's role in refuting rumors, both hesitating and disseminating will tend to zero, and the number of people resisting rumors will increase and tend to increase rapidly. It is stable, and the rate of increase of resisters is proportional to the correlation coefficient p of impulse supervision. The numerical simulation results have taken different p values, and the conclusions drawn also well confirm that the larger the p value, the speed of the increase of resisters is also Faster.

Keywords: Rumor spreading; Pulse Interference; Double Delay; Comparison principle

1. Introduction

For a company's reputation and interests are closely related, the spread of rumors may lead to unprecedented impact on earnings or even bankruptcy. This is especially true in the country. The spread of some bad rumors can easily cause social panic and social unrest, which is not conducive to social progress and the country's prosperity. Therefore, studying the dynamics of the spread of rumors helps the government to control the spread of rumors in time.

Based on the initial spread of SIR rumors, the former considers different attitudes towards disease transmission, ignorant people (I), hesitant people (H), people who spread it (S), and people who resist the spread (R).

2. Literature Review

Previously, Li and Ma [1] discussed the influence of government punishment and individual sensitivity on rumor propagation and took into account some rumors related to hot events. The results showed that increasing the severity of government punishment and individual sensitivity could effectively control rumor propagation. [2] believes that compared with educated people, uneducated people have a great chance to receive the information. He investigated the influence of education level on the final scale of the rumor, and concluded that the more educated people inside, the smaller the final scale of the rumor. [3] considered the global asymptotic stability of neutral inertial BAM neural network with time-varying delay. Using the theory of variation and homomorphism, a suitable lyapunov functional is proposed. Based on the matrix equation, the delay correlation sufficient conditions for the existence and global asymptotic stability of a class of neutral inertial BAM neural networks are established. [4] established a rumor propagation model considering the proportion of the wise in the crowd, believing that the velocity of rumor propagation is a variable that changes with time in our model.

[5] extended the SIR information propagation model to analyze the influence of network structure on rumor propagation when considering group propagation, and studied the basic reproduction times of rumor propagation in the model, indicating that rumor propagation with larger groups is more effective than with more groups. In the small world network, [6] studied a rumor propagation model that considered the change of forgetting rate over time. The larger the initial forgetting rate or the faster the forgetting rate, the smaller the final size of rumor propagation. The numerical solution also shows that the final scale of rumor propagation is much larger under the variable forgetting rate than under the constant forgetting rate. [7] studied the influence of rumor control measures on rumor propagation through numerical simulation, and proposed the method of crisis management. [8] proposed a dynamic model of rumor propagation I2SR,

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considering that each communicator in the network rotates between high active state and low active state according to a certain probability. Considering that exposed nodes can become removed nodes at a rate, [9] presented a new SEIR model of rumor propagation on heterogeneous networks. [10] proposed I2S2R rumor propagation model with general correlation function in homogeneous network and I2S2R rumor propagation model in heterogeneous network. The dynamic model of rumor propagation I2S2R is established in homogeneous networks, and the free equilibrium problem is discussed by taking two general correlation functions into consideration.

[11] considered the different attitudes of individuals to rumor propagation, analyzed the local and global stability, balance and rumor existence balance of rumors, and found that those who hesitated to spread rumors had a positive impact on rumor propagation. [12] proposed a time-delayed SEIRS epidemic model with changes in pulse inoculation and total population size. The results show that short - time or large - pulse vaccination rate is a sufficient condition for disease eradication. [13] proposed a non-markov model to describe the complex contagion adopted by a sensitive node that must take into account social reinforcement from different levels and neighbors. [14] study the kinetics of double the spread of rumors and at the same time, this paper introduces the two double rumor spreading model: DSIR model and C - DSIR model. Provided by the state vector expressions and double rumors spread mechanism, introduced a select parameters theta to express differences attractive, results show that the new rumors the start time of the closer it gets to the best of time, so the more strongly they depend on each other. By analyzing the characteristics and modes of rumor propagation with forgetting effect, [15] established a class of SIRS rumor propagation model with time delay in scale-free network environment, and calculated the basic regeneration number in the propagation process. [16] studied the stability and Hopf branch of a SEIR pollution-infectious disease model with time delay, saturated infection rate and saturated treatment function, and analyzed the local stability of disease-free equilibrium and endemic disease equilibrium by means of eigenvalue theory and routh-hurwitz criterion. Meanwhile, the delay is taken as the branch parameter to obtain the conditions for the existence of Hopf branch. [17] systematically sorted out several classical models of infectious diseases and derived several methods for the basic regeneration number of infectious diseases models.

[18] studied the threshold dynamics of a random time-delay SIR epidemic model with immunization, and obtained sufficient conditions for the extinction and persistence of the pandemic. [19] expressed concern about the asymptotic nature of transient immunity in the stochastic delayed SIR epidemic model, and obtained the threshold between the average persistence of epidemic and extinction. [20] studied the effects of vaccine immunization and out-group migration on the transmission behavior of SIR infectious diseases. [21] proposed a novel SIR model, in which both delayed infection and non-uniform transmission are considered as two factors influencing the disease transmission behavior.

However, considering the pulsed interference of external information on investor sentiment, as well as the double delay of disseminators and resisters, no comprehensive research has been conducted. Here, we will solve these problems to find out more influential factors affecting the spread of investor sentiment.

3.Model Description

This article has added the role of government supervision to the previous model. Because the government may not find out in time and take effective control measures in the early stage of the spread of rumors in the society, but from a certain moment, measures to stop the rumors are taken, so consider, The government’s role at time t is a pulse,Based on the above,the rumour propagation process is shown below:

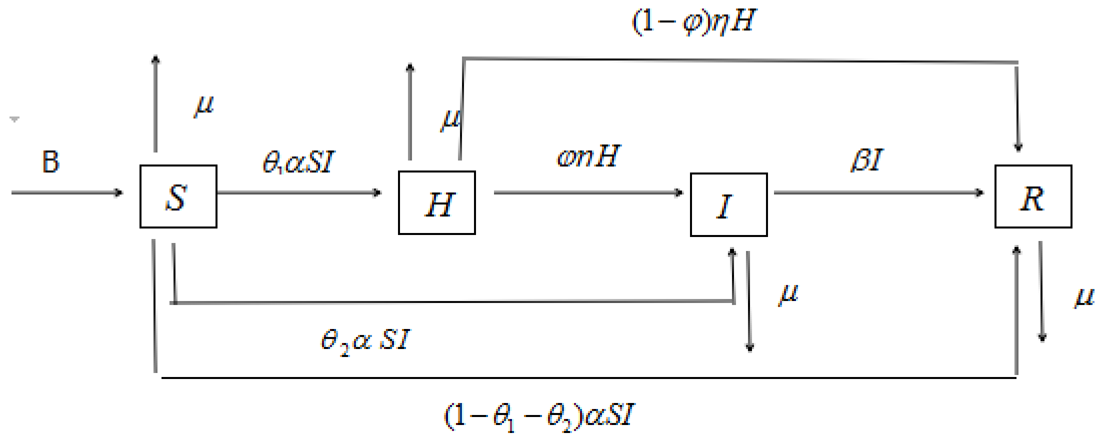

Chart 1:the rumour propagation process

Thus, the dynamic mean-field reaction rate equations can be written as:

$$\begin{cases} \frac{dS}{dt} = B - \alpha SI - \mu S \\ \frac{dH}{dt} = \theta_1 \alpha SI - \eta H - \mu H \\ \frac{dI}{dt} = \theta_2 \alpha SI + \varphi \eta H - \beta I - \mu I \\ \frac{dR}{dt} = (1-\theta_1-\theta_2)\alpha SI + (1-\varphi)\eta H + \beta I - \mu R \end{cases}$$

$$\begin{cases} S(k^+) = (1-p)S(k) \\ H(k^+) = H(k) \\ I(k^+) = I(k) \\ R(k^+) = R(k) + pS(k) \end{cases} \quad (1)$$

Let's say $N(t) = S(t) + I(t) + R(t)$, it is easy to get to $N'(t) = B - \mu N(t)$ which is $\lim_{t\to\infty} N(t) \leq \frac{B}{\mu}$ so $S(t), I(t), R(t)$ is the final bounded function.

3.1 Lemmas about the existence and stability of periodic solutions

3.1.1 Related Lemmas

Lemma 1. System (1) There is a rumor-free periodic solution $E* = (S*, 0, 0, R*)$ where $S* = \frac{B(1-pe^{-\mu t})}{\mu[1-(1-p)e^{-\mu}]}$, $R* = \frac{pBe^{-\mu t}}{\mu[1-(1-p)e^{-\mu}]}$.

Proof: The steady state of the model is when the hesitating and spreading people are zero, so the rumor-free periodic solution $E* = (S*, 0, 0, R*)$ of the impulse model can be solved. At this time, the model is equivalent to

$$\begin{cases} \frac{dS}{dt} = B - \mu S \\ \frac{dR}{dt} = -\mu R \\ S(k^+) = (1-p)S(k) \\ R(k^+) = R(k) + pS(k) \end{cases} \quad (2)$$

For the first formula of (2), we can obtain:

$$S(t) = \frac{B}{\mu} - \left(\frac{B}{\mu} - S(0)\right)e^{-\mu t} \quad (3)$$

When t=1, $S(1)=\frac{B}{\mu}-(\frac{B}{\mu}-S(0))e^{-\mu}$
According to the boundary value conditions:

$$S(0)=(1-p)S(1) \tag{4}$$

Get the Solution $S(0)=\frac{(1-p)B(1-e^{-\mu})}{\mu[1-(1-p)e^{-\mu}]}$ (5)

From (4) (5)we can get

$$S(1)=\frac{B(1-e^{-\mu})}{\mu[1-(1-p)e^{-\mu}]} \tag{6}$$

Substituting (4) into (3) can be obtained

$$S*(t)=\frac{B}{\mu}-\left[\frac{B}{\mu}-\frac{(1-p)B(1-e^{-\mu})}{\mu[1-(1-p)e^{-\mu}]}\right]e^{-\mu t}=\frac{B(1-pe^{-\mu t})}{\mu[1-(1-p)e^{-\mu}]} \tag{7}$$

Next, we can solve the second formula of (2)

$$R(t)=R(0)e^{-\mu t} \tag{8}$$

Because the second formula of boundary value condition knows:

$$R(0)=R(1)+pS(1)=R(0)e^{-\mu t}+\frac{pB(1-e^{-\mu})}{\mu[1-(1-p)e^{-\mu}]} \tag{9}$$

Because the second formula of boundary value condition knows:
Finished up:$R(0)=\frac{pB}{\mu[1-(1-p)e^{-\mu}]}$Substituting (8) to get:$R*(t)=\frac{pBe^{-\mu t}}{\mu[1-(1-p)e^{-\mu}]}$
For this we have found a balance point without rumors $E*$.

To prove the stability of the rumorless equilibrium point, we introduce the theorem

Lemma 2. There is a periodic system

$$\begin{cases}\frac{dy}{dt}=f(t,y), f\in[R\times R^n,R^n]\\ y(k^+)=\phi_k(y(k)),\phi_k\in[R^n,R^n]\end{cases} \tag{10}$$

Among them $f(t+1,y)=f(t,y),\forall t\in R,\phi_{k+1}=\phi_k$, the linear approximate solution of (10) about its periodic solution $y(t)$ is:

$$\begin{cases}\frac{dx}{dt}=A(t)x\\ x(k+)=B_kX_k\end{cases} \tag{11}$$

Suppose $\varphi(t)$ is a basic solution matrix of (10) that satisfies:$\frac{d\varphi(t)}{dt}=A(t)\varphi(t),\varphi(0)=E$

If the absolute value of $M=B_1\varphi(1)$ all the characteristic roots of is less than 1, the zero solution of system (11), that is, the periodic solution of (10) is locally asymptotically stable.

Lemma 3. consider the following impulsive differential inequality:

$$\begin{cases}m'(t)\le p(t)m(t)+q(t),t\ne t_k\\ m(t_k^+)\le d_km(t_k)+b_k,t=t_k,k=1,2,\cdots,\end{cases} \tag{12}$$

Where $p(t),q(t)\in C[R_+,R],d_k\ge 0$ and b_k is a constant, if

(i) Sequence $\{t_k\}$ satisfies $0\le t_0<t_1<t_2<\cdots$, and $\lim_{k\to\infty}t_k=\infty$

(ii) $m(t)\in PC'[R_+,R]$ And if $m(t)$ is continuous to the left of point $t_k(k=1,2,\cdots)$, then

$m(t)\le m(t_0)(\prod_{t_0<t_k<t}d_k)\,exp\left\{\int_{t_0}^t p(s)ds\right\}+\sum_{t_0<t_k<t}(\prod_{t_0<t_k<t}d_k\,exp\left\{\int_{t_0}^t p(s)ds\right\})\,b_k+\int_{t_0}^{t\int\left\{\int_s^t p(v)dv\right\}_0}\prod_{t_0<t_k<t}d_k\,exp$ is true.

3.1.2 Proof of local stability

Theorem 1. Let $\sigma=\frac{Bp(\alpha\theta_2\eta+\alpha\theta_2\mu+\theta_1\phi\eta)e^{-\mu}(1-e^{-\mu})}{\mu(\beta+\mu)(\eta+\mu)(e^{-\mu}-1+p)}$, if $\sigma<1$, then the rumor-free periodic solution $E*=(\frac{B(1-pe^{-\mu t})}{\mu[1-(1-p)e^{-\mu}]},0,0,\frac{pBe^{-\mu t}}{\mu[1-(1-p)e^{-\mu}]})$ of the model is locally asymptotically stable.

Proof: do transformation $x(t)=S(t)-S*(t),y(t)=H(t)-0,z(t)=I(t)-0,w(t)=R(t)-R*(t)$

$$\begin{pmatrix}\frac{dx}{dt}\\ \frac{dy}{dt}\\ \frac{dz}{dt}\\ \frac{dw}{dt}\end{pmatrix}=\begin{pmatrix}-\mu & 0 & -\alpha S* & 0\\ 0 & -\eta-\mu & \theta_1\alpha S* & 0\\ 0 & \phi\eta & \theta_2\alpha S*-\beta-\mu & 0\\ 0 & (1-\phi)\eta & (1-\theta_1-\theta_2)\alpha S*+\beta & -\mu\end{pmatrix}\begin{pmatrix}x\\ y\\ z\\ w\end{pmatrix} \tag{13}$$

It is not difficult to obtain the basic solution matrix as:

$$\Phi(1)=\begin{pmatrix} e^{-\mu t} & 0 & \phi_{13} & 0 \\ 0 & \phi_{22} & \phi_{23} & 0 \\ 0 & 0 & \phi_{33} & 0 \\ 0 & \phi_{42} & \phi_{43} & e^{-\mu t} \end{pmatrix} \tag{14}$$

among them

$\phi_{33}(t) = exp\left[\int_0^t(\theta_2\alpha+\frac{\phi\eta\theta_1\alpha}{\eta+\mu})S*-\beta-\mu\right] = exp\left[-(\beta+\mu)t+(\theta_2\alpha+\frac{\theta_1\phi\eta\alpha}{\eta+\mu})\frac{Bpe^{-\mu}(1-e^{-\mu t})}{\mu(e^{-\mu}-1+p)}\right]\phi_{13}(t) = -\alpha e^{-\mu t}\int_0^t S*(t)\phi_{33}(\tau)\,e^{\mu\tau}d\tau$

$$\phi_{23}(t)=\theta_1\alpha e^{-\mu t}\int_0^t S*(\tau)\phi_{33}(\tau)\,e^{\mu\tau}d\tau$$

$$\phi_{22}(t)=exp[-(\eta+\mu)t]$$

$$\phi_{42}(t)=(1-\phi)\eta e^{-(\eta+\mu)t}\int_0^t\phi_{22}(\tau)\,e^{(\eta+\mu)\tau}d\tau$$

$$\phi_{43}(t)=(1-\theta_1-\theta_2)\alpha e^{-\mu t}\int_0^t S*(\tau)\,\phi_{33}(\tau)e^{\mu\tau}d\tau+\beta te^{-\mu t}$$

Boundary value condition:

$$\begin{pmatrix} x(k^+) \\ y(k^+) \\ z(k^+) \\ w(k^+) \end{pmatrix}=\begin{pmatrix} 1-p & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ p & 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} x(k) \\ y(k) \\ z(k) \\ w(k) \end{pmatrix} \tag{15}$$

$$M=\begin{pmatrix} 1-p & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ p & 0 & 0 & 1 \end{pmatrix}\Phi(1)=\begin{pmatrix} (1-p)e^{-\mu} & 0 & 0 & 0 \\ 0 & e^{-(\eta+\mu)} & 0 & 0 \\ 0 & 0 & \phi_{33}(1) & 0 \\ pe^{-\mu} & 0 & 0 & e^{-\mu} \end{pmatrix} \tag{16}$$

$\lambda_1=(1-p)e^{-\mu}$, $0<p<1, \mu>0$, so $\lambda_1<1$

$\lambda_2=e^{-(\eta+\mu)}$ where $0<\eta<1$, so $\lambda_2<1$

$\lambda_3=(\varepsilon+\mu)\left[\frac{Bpe^{-\mu}(1-e^{-\mu})(\alpha\theta_2\eta+\alpha\theta_2\mu+\theta_1\phi\eta)}{\mu(\beta+\mu)(\eta+\mu)(e^{-\mu}-1+p)}-1\right]$, $\sigma<1$, so $\lambda_3<1$

$\lambda_4=e^{-\mu}$ there $0<\mu<1$ so $\lambda_4<1$

According to Theorem 1, the rumor-free periodic solution is locally asymptotically stable.

3.2.Global stability

Theorem 2. If $R_0=\frac{\alpha B[\theta_2(\mu+\eta)+\theta_1\phi\eta]}{\mu(\mu+\beta)(\mu+\eta)}<1$, then the rumor-free periodic solution $E*$ of the system is globally stable.

Proof: when $R_0<1$ you can choose a sufficiently small $\varepsilon>0$ to $r_0=\int_0^\tau[\theta_1\alpha(S*+\varepsilon)-(\eta+\mu)]<0$

From the first equation of the system $S'(t)\le B-\mu S(t)$, we can make the following impulse comparison differential equation:

$$\begin{cases} u(t)\le B-\mu u(t) \\ u(t^+)=(1-p)u(t), t=n\tau \\ u(0)=S(0) \end{cases} \tag{17}$$

It is not difficult to solve $u(t)\le\frac{B(1-pe^{-\mu t})}{\mu[1-(1-p)e^{-\mu}]}=S*(t)$ from the impulse comparison theorem, we know that for arbitrarily small $\varepsilon>0$,there exist a positive integers T_1, when $t>T_1$, there is always

$$S'(t)\le u(t)<S*(t)+\varepsilon \tag{18}$$

From (18) and the second equation of system (1), there are

$$\begin{cases} H'(t) \le [\theta_1 \alpha(S* + \varepsilon) - (\eta+\mu)]H(t) \\ H(t^+) = H(t) \end{cases}, t \ne n\tau \tag{19}$$

Therefore $H((n+1)\tau) \le H(n\tau)\, exp\left\{\int_{n\tau}^{(n+1)\tau} [\theta_1 \alpha(S* + \varepsilon) - (\eta+\mu)]dt\right\}$, because $r_0 < 0$ it has to recursive formula $H(n\tau) \le H(0^+)\, exp\{nr_0\}$, $\lim_{n\to\infty} H(n\tau) = 0$ and for $\forall n\tau < t < (n+1)\tau$, $S*(t) + \varepsilon < \frac{B}{\mu} + 1$ Therefore $0 < H(t) \le H(n\tau)\, exp\left\{\int_{n\tau}^{(n+1)\tau} [\theta_1 \alpha(S* + \varepsilon) - (\eta+\mu)]dt\right\} \le H(n\tau)\, exp\left\{\theta_1 \alpha(\frac{B}{\mu} + 1)\right\}$

So you can get $\lim_{t\to\infty} H(t) = 0$ and $\exists T_2 > T$ when $t > T_2$ there is always $H(t) < \varepsilon$ .From the third equation that is easy to get the inequality: $-(\beta+\mu)I \le I'(t) \le \mu\varepsilon - (\beta+\mu)I$ and because the arbitrariness of ε,$\lim_{t\to\infty} I(t) = \lim_{t\to\infty} I(n\tau)\, exp\{-(\beta+\mu)(t-n\tau)\} = 0$there is a positive integer $T_3 > T_2$,such that when $t > T_3$, there is always $I(t) < \varepsilon$

Similar to the fourth equation:$-\mu R(t) \le R'(t) \le (1-\phi)\eta\varepsilon + \beta\varepsilon - \mu R$ is corrent, because the arbitrariness of ε, there exist $T_4 > T_3$,when$t > T_4$,$\lim_{t\to\infty} R(t) = \frac{pBe^{-\mu t}}{\mu[1-(1-p)e^{-\mu}]} = R*(t)$

From the first equation of the system: $S'(t) \ge B - (\alpha+\varepsilon)SI - (\mu+\varepsilon)S$

Do the pulse comparison equation：

$$\begin{cases} v'(t) = B - (\alpha+\varepsilon)SI - (\mu+\varepsilon)S \\ v(t) = (1-p)v(t) \\ v(0) = S(0) \end{cases} \tag{20}$$

The solution is $v(t) = \frac{B(1-pe^{-\mu t})}{(\mu+\varepsilon)[1-(1-p)e^{-\mu}]}$

So there is $S(t) \ge v(t) \ge v(t) - \varepsilon \ge S*(t) - \varepsilon$,combined with $S'(t) \le u(t) < S*(t) + \varepsilon$and the arbitrariness of ε we can get $\lim_{t\to\infty} S(t) = S*(t)$.

In summary, the balance point of no rumors is globally stable.

3.3. Numerical simulation

The image of the influence of p value on the propagation is shown below. The image on the left is the one without pulse intervention, and the image on the right is the one with pulse intervention

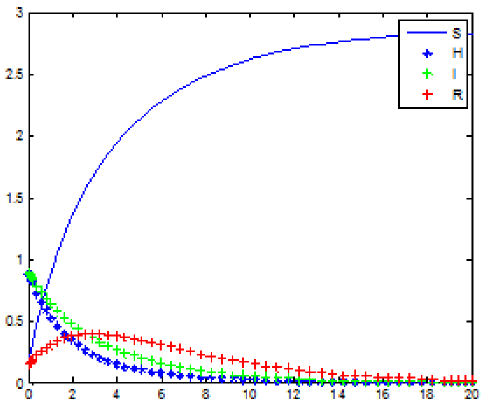

Figure 1:When p=0, there is no pulse intervention

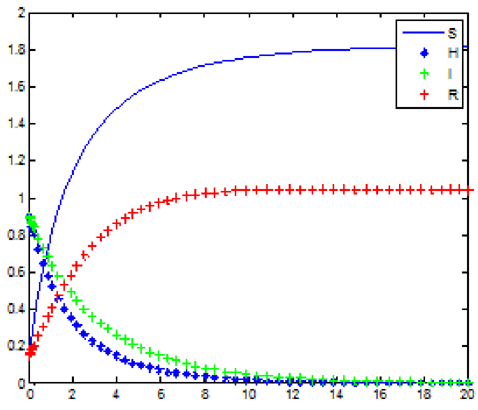

Figure 2:When p=0.2, there is small pulse intervention

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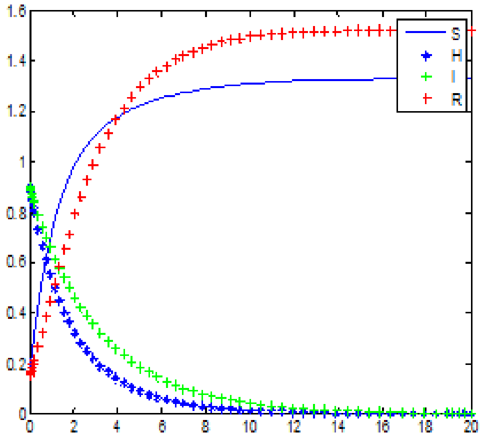

Figure 3:When p=0.4, there is a small pulse intervention

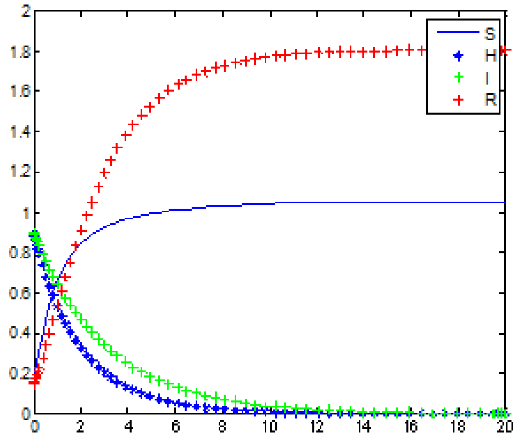

Figure 4:When p=0.6, there is a pulse intervention

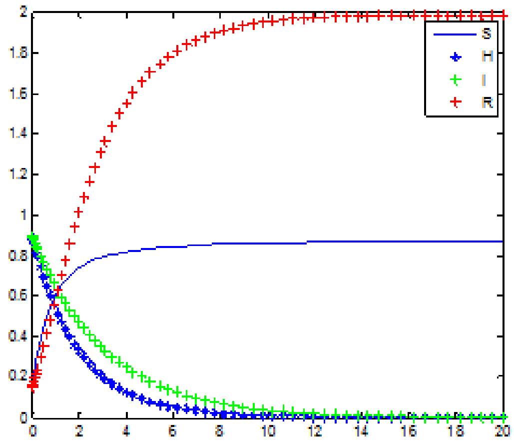

Figure5:When p=0.8, there is a big pulse intervention

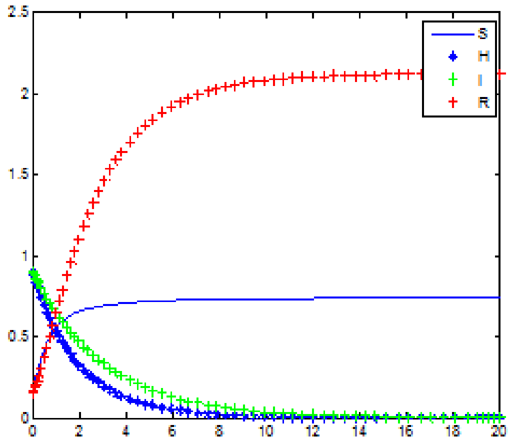

Figure 6:When p=1, there is a bigger pulse intervention

4. Discussion

(1) By comparing FIG. 1 with FIG. 2, it can be found that in the case of any pulse intervention, the resisters first increased and then disappeared, and although the hesitant and propagator both disappeared at last, the hesitant and propagator with pulse intervention disappeared more quickly.

(2) FIG. 3-6 shows the rumor propagation process under different pulse intervention values. By comparison, it is not difficult to see that pulse intervention accelerates the rumor disappearance process by affecting the growth of the boycotts.

5. Conclusion

(1) Compared with the rumor propagation process under the natural state, pulse interference is applied to accelerate the rumor propagation process. The value represents the role of the government in clarifying rumors at a certain moment. The greater the value is, the faster the rumor disappears and the faster the rate of the boycotts grows.

(2) The pulse interference can control the rumor propagation process by increasing the number of boycotts. The larger the value is within a certain threshold range, the more favorable it is for rumor control.

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