ISSN 1746-7659, England, UK
Journal of Information and Computing Science
Vol. 15, No. 2, 2020, *pp*.124-133

The Propagation Process of Financial Investment Emotion with Double Time Delay under Impulse Interference

Hongxing Yao[1], Yuan Zhao[2]
School of science, Jiangsu University, Zhenjiang 212013, China
E-mail: hxyao@ujs.edu.cn; E-mail: 2669063842@qq.com
(Received May 28, 2020, accepted June 29, 2020)

Abstract: This article puts forward the double pulse delay investor sentiment spread model, using the stroboscopic map of discrete system, prove the existence of periodic solution, by impulsive differential inequalities proved the global asymptotic stability of the emotional balance, use comparison principle prove persistent rumors of impulsive differential equation, analyses the threshold delay affect mood, finally, the numerical simulation results, the result confirms my analysis very well.

Keywords: Financial Investment; Emotional Communication; Pulse Interference; Double Delay; Persistence

1. Introduction

Investor sentiment is one of the research emphases in behavioral finance. It is generally believed that the stock market is the barometer of China's macro economy. Similarly, investor sentiment can be compared to the weathervane of the stock market. In real trading situations, investors make decisions not only based on some simple established facts and obvious information, but also rely on their own intuition, others' comments, other investors' opinions and psychological activities to make the final decision. Therefore in the behavioral finance theory of investor sentiment by educational world attention, more and more investor sentiment volatility can affect the stock market share price volatility and trading main body of the trading strategy, under this background, combining the actual operation condition of our country stock market, investor sentiment was discussed thoroughly in our country stock market and the impact of income whether for investors or the government regulators have far-reaching significance. This paper considers the emotional transmission process based on the models of infectious disease and rumor transmission.

2. Literature Review

Previously, Li and Ma [1] discussed the influence of government punishment and individual sensitivity on rumor propagation and took into account some rumors related to hot events. The results showed that increasing the severity of government punishment and individual sensitivity could effectively control rumor propagation. [2] believes that compared with educated people, uneducated people have a great chance to receive the information. He investigated the influence of education level on the final scale of the rumor, and concluded that the more educated people inside, the smaller the final scale of the rumor. [3] considered the global asymptotic stability of neutral inertial BAM neural network with time-varying delay. Using the theory of variation and homomorphism, a suitable lyapunov functional is proposed. Based on the matrix equation, the delay correlation sufficient conditions for the existence and global asymptotic stability of a class of neutral inertial BAM neural networks are established. [4] established a rumor propagation model considering the proportion of the wise in the crowd, believing that the velocity of rumor propagation is a variable that changes with time in our model.

[5] extended the SIR information propagation model to analyze the influence of network structure on rumor propagation when considering group propagation, and studied the basic reproduction times of rumor propagation in the model, indicating that rumor propagation with larger groups is more effective than with more groups. In the small world network, [6] studied a rumor propagation model that considered the change of forgetting rate over time. The larger the initial forgetting rate or the faster the forgetting rate, the smaller

+ Corresponding author. Tel.: +86-13912802960;
E-mail address: hxyao@ujs.edu.cn

the final size of rumor propagation. The numerical solution also shows that the final scale of rumor propagation is much larger under the variable forgetting rate than under the constant forgetting rate. [7] studied the influence of rumor control measures on rumor propagation through numerical simulation, and proposed the method of crisis management. [8] proposed a dynamic model of rumor propagation I2SR, considering that each communicator in the network rotates between high active state and low active state according to a certain probability. Considering that exposed nodes can become removed nodes at a rate, [9] presented a new SEIR model of rumor propagation on heterogeneous networks. [10] proposed I2S2R rumor propagation model with general correlation function in homogeneous network and I2S2R rumor propagation model in heterogeneous network. The dynamic model of rumor propagation I2S2R is established in homogeneous networks, and the free equilibrium problem is discussed by taking two general correlation functions into consideration.

[11] considered the different attitudes of individuals to rumor propagation, analyzed the local and global stability, balance and rumor existence balance of rumors, and found that those who hesitated to spread rumors had a positive impact on rumor propagation. [12] proposed a time-delayed SEIRS epidemic model with changes in pulse inoculation and total population size. The results show that short - time or large - pulse vaccination rate is a sufficient condition for disease eradication. [13] proposed a non-markov model to describe the complex contagion adopted by a sensitive node that must take into account social reinforcement from different levels and neighbors. [14] study the kinetics of double the spread of rumors and at the same time, this paper introduces the two double rumor spreading model: DSIR model and C - DSIR model. Provided by the state vector expressions and double rumors spread mechanism, introduced a select parameters theta to express differences attractive, results show that the new rumors the start time of the closer it gets to the best of time, so the more strongly they depend on each other. By analyzing the characteristics and modes of rumor propagation with forgetting effect, [15] established a class of SIRS rumor propagation model with time delay in scale-free network environment, and calculated the basic regeneration number in the propagation process. [16] studied the stability and Hopf branch of a SEIR pollution-infectious disease model with time delay, saturated infection rate and saturated treatment function, and analyzed the local stability of disease-free equilibrium and endemic disease equilibrium by means of eigenvalue theory and routh-hurwitz criterion. Meanwhile, the delay is taken as the branch parameter to obtain the conditions for the existence of Hopf branch. [17] systematically sorted out several classical models of infectious diseases and derived several methods for the basic regeneration number of infectious diseases models.

[18] studied the threshold dynamics of a random time-delay SIR epidemic model with immunization, and obtained sufficient conditions for the extinction and persistence of the pandemic. [19] expressed concern about the asymptotic nature of transient immunity in the stochastic delayed SIR epidemic model, and obtained the threshold between the average persistence of epidemic and extinction. [20] studied the effects of vaccine immunization and out-group migration on the transmission behavior of SIR infectious diseases. [21] proposed a novel SIR model, in which both delayed infection and non-uniform transmission are considered as two factors influencing the disease transmission behavior.

However, considering the pulsed interference of external information on investor sentiment, as well as the double delay of disseminators and resisters, no comprehensive research has been conducted. Here, we will solve these problems to find out more influential factors affecting the spread of investor sentiment.

3. Model Description

Based on the original SIR rumor spread, the former takes into account the different attitudes to the spread of the disease, which will be divided into groups (E) the ignorant people who don't understand (for), (I) communicators (accept rumors spread information and to the people around), and the resistance (R) (know rumours but refused to surrounding people spreading rumours) here for emotional communication in the process of financial investment investors can be divided into three categories of people: did not receive some kind of mood of the person (S) that is susceptible people; the person who receives an emotion and transmits it (I) is the transmitter; the person who refuses to spread the emotion (R) is the person who resists it. In the traditional rumor propagation model, time delay is added:

(1) In the process of communication, communicators(I) have propagation time delay δ due to delayed communication or information transmission speed.

(2) Groups that are already boycotts may receive too many messages and forget them, so that they stop boycotting after a period of time τ.

(3) For the transmission of a certain emotion, there will be some messages that affect the change of emotion and thus affect the decision within a period of time. It is considered as a periodic pulse, and the probability of the occurrence of the pulse causing the change of emotion is p.

(4) Population input is a constant B.

The transmission rate between individuals is α, and the probability of any group exiting the investment process in the process of financial communication is μ. When people (S) without emotional infection contact with the transmitter, the probability of them becoming resisters (R) is θ. Then there is no resistance, and the probability of actively transmitting emotions finally becomes a communicator is $1-\theta$; the probability that the disseminator will reject the spreading due to the delay of the spreading speed and thus become the resister is $(1-\theta)\alpha e^{-\mu\delta}S(t-\delta)I(t-\delta)$,the probability that the resister will become the susceptible because of receiving all kinds of information for a long time is $(1-\theta)\alpha S(t-\delta-\tau)I(t-\delta-\tau)e^{-\mu(\delta+\tau)}$.

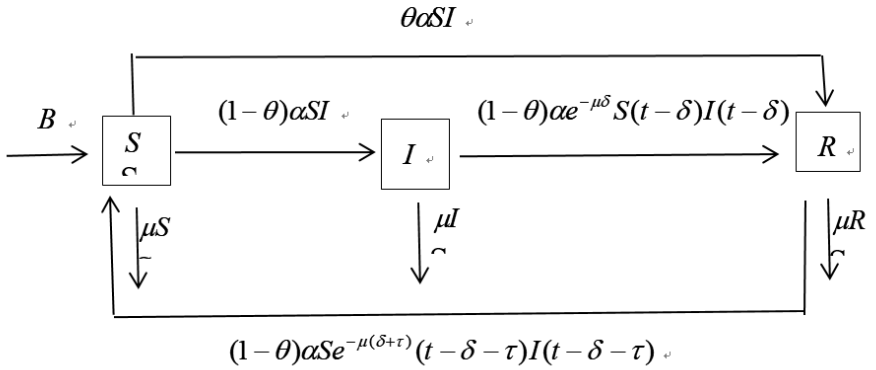


Figure 1 :The structure of emotion spreading process

Thus, the dynamic mean-field reaction rate equations can be written as:

$$\begin{cases} \dfrac{dS}{dt} = B - \alpha S(t)I(t) + (1-\theta)\alpha S(t-\delta-\tau)I(t-\delta-\tau)e^{-\mu(\delta+\tau)} - \mu S(t) \\ \dfrac{dI}{dt} = (1-\theta)\alpha S(t)I(t) - (1-\theta)\alpha S(t-\delta)I(t-\delta)e^{-\mu\delta} - \mu I(t) \\ \dfrac{dR}{dt} = \theta\alpha S(t)I(t) + (1-\theta)\alpha S(t-\delta)I(t-\delta)e^{-\mu\delta} - (1-\theta)\alpha S(t-\delta-\tau)I(t-\delta-\tau)e^{-\mu(\delta+\tau)} - \mu R(t) \end{cases}$$

$$\begin{cases} S(k^+) = (1-p)S(k) \\ I(k^+) = I(k) \\ R(k^+) = R(k) + pS(k) \end{cases} \tag{1}$$

Let's say $N(t) = S(t) + I(t) + R(t)$, it is easy to get to $N'(t) = B - \mu N(t)$ which is $\lim_{t\to\infty} N(t) \le \frac{B}{\mu}$ so $S(t), I(t), R(t)$ is the final bounded function.

3.1 Global asymptotic stability of emotionless periodic solutions

Lemma 1. Consider impulsive differential equations:

$$\begin{cases} \dfrac{dx(t)}{dt} = a - bx(t), t \ne kT \\ X(t^+) = (1-p)x(t), t = kT \end{cases} \tag{2}$$

which has a unique positive periodic solution

$$\tilde{x}(t) = \frac{a}{b}\left[1 - \frac{pe^{-b(t-kT)}}{1-(1-p)e^{-bT}}\right] \qquad kT \le t \le (k+1)T$$

Where $a > 0, b > 0, 0 < p < 1$.

Proof: It is solved by the first equation of the model:

$$x(t)=\frac{a}{b}+(x(kT)-\frac{a}{b})e^{-b(t-kT)} \qquad KT\le t\le (k+1)T$$

Let $x(kT^{+})=x_k$ the pulse stroboscopic mapping be done:

$$x_{k+1}=(1-p)\left[\frac{a}{b}+(x_k-\frac{a}{b})e^{-bT}\right]=f(x_k) \tag{3}$$

It's easy to get $f(x)=(1-p)\left[\frac{a}{b}+(x-\frac{a}{b})e^{-bT}\right]$ There is a fixed point $x*$, $x*=\frac{a(1-p)(1-e^{-bT})}{b(1-(1-p)e^{-bT})}$ Due to $|f(x)|=(1-p)e^{-(T-kT)}<1$ Therefore, the sequence determined by (3) converges to x^* and the periodic solution exists in model (2),the periodic solution is $\tilde{x}(t)=\frac{a}{b}\left[1-\frac{pe^{-b(t-kT)}}{1-(1-p)e^{-bT}}\right]$ $kT\le t\le (k+1)T, k\in Z_+$

Lemma 2. consider the following delay differential equation:

$$\frac{dx(t)}{dt}=ax(t-\tau)-bx(t)$$

Both of a, b and.τ are normal Numbers, besides $x(t)>0, t\in[-\tau,0]$.

(1) If $a<b$, then $\lim_{t\to\infty}x(t)=0$.

(2) If $a>b$, then $\lim_{t\to\infty}x(t)=+\infty$.

The existence of periodic solutions of model (1) is discussed below:

When considering the existence of emotionless equilibrium point, we know that the final stable state emotion does not exist, that is $\lim_{t\to\infty}I(t)=0$, therefore, the model becomes:

$$\begin{cases}\frac{dS}{dt}=B-\alpha S(t)I(t)+(1-\theta)\alpha S(t-\delta-\tau)I(t-\delta-\tau)e^{-\mu(\delta+\tau)}-\mu S(t), t\ne kT, k\in Z_+\\ \frac{dR}{dt}=\theta\alpha S(t)I(t)+(1-\theta)\alpha S(t-\delta)I(t-\delta)e^{-\mu\delta}-(1-\theta)\alpha S(t-\delta-\tau)I(t-\delta-\tau)e^{-\mu(\delta+\tau)}-\mu R(t), t\ne kT, k\in Z_+\end{cases}$$

$$\begin{cases}S(k^{+})=(1-p)S(k), t=kT, k\in Z_+\\ R(k^{+})=R(k)+pS(k), t=kT, k\in Z_+\end{cases} \tag{4}$$

If $\Omega=\left\{(S,R), S+R\le\frac{B}{\mu}\right\}$, Ω is the forward invariant set of the model.

The solution of (4) is $\tilde{S}(t)=\frac{B}{\mu}\left[1-\frac{pe^{-b(t-kT)}}{1-(1-p)e^{-bT}}\right]$ $kT\le t\le(k+1)T$

$\tilde{R}(t)=\frac{BP}{\mu(1-e^{-\mu T})}\left[1-\frac{pe^{-b(t-kT)}}{1-(1-p)e^{-bT}}\right]$ $kT\le t\le(k+1)T$

Lemma 3. consider the following impulsive differential inequality:

$$\begin{cases}m'(t)\le p(t)m(t)+q(t), t\ne t_k\\ m(t_k^{+})\le d_k m(t_k)+b_k, t=t_k, k=1,2,\cdots,\end{cases}$$

Where $p(t), q(t)\in C[R_+,R], d_k\ge 0$ and b_k is a constant, if

(i) Sequence $\{t_k\}$ satisfies $0\le t_0<t_1<t_2<\cdots$, and $\lim_{k\to\infty}t_k=\infty$

(ii) $m(t)\in PC'[R_+,R]$ And if $m(t)$ is continuous to the left of point $t_k(k=1,2,\cdots)$, then

$m(t)\le m(t_0)(\prod_{t_0<t_k<t}d_k)\,exp\left\{\int_{t_0}^{t}p(s)ds\right\}+\sum_{t_0<t_k<t}(\prod_{t_0<t_k<t}d_k\,exp\left\{\int_{t_0}^{t}p(s)ds\right\})\,b_k+\int_{t_0}^{t\int\left\{\int_s^t p(v)dv\right\}_0}\prod_{t_0<t_k<t}d_k\,exp$ is true.

Define $R*=\frac{(1-\theta)\alpha[B+(1-p)\alpha e^{-\mu(\delta+\tau)}\frac{B^2}{\mu^2}](1-e^{-\mu T})(1-e^{-\mu\delta})}{\mu^2[1-(1-p)e^{-\mu T}]}$

So, we get the following theorem:

Theorem 1. When $R*<1$, the emotionless periodic solution $(\tilde{S}(t),0,\tilde{R}(t))$ of the model is globally asymptotically stable.

Proof: in region Ω, obtained by the first equation of the model:

$$\frac{dS}{dt} \le B + (1-\theta)\alpha S(t-\delta-\tau)I(t-\delta-\tau)e^{-\mu(\delta+\tau)} - \mu S(t) \tag{5}$$

Consider the differential equation of impulse comparison:

$$\begin{cases} y'(t) = B + (1-\theta)\alpha y(t-\delta-\tau)I(t-\delta-\tau)e^{-\mu(\delta+\tau)} - \mu y(t), t \ne kT, k \in Z_+ \\ S(t) = (1-p)S(t), t = kT, k \in Z_+ \end{cases}$$

The solution is obtained from the differential inequality above

$$\tilde{y}(t) = \frac{B + (1-\theta)\alpha e^{-\mu(\delta+\tau)}\frac{B^2}{\mu^2}}{\mu}\left[1 - \frac{pe^{-\mu T}}{1-(1-p)e^{-\mu T}}\right] \tag{6}$$

According to the comparison theorem of differential equations, there is an integer $k_0 > 0$, so that when $t > k_0 T$, there is

$$S(t) \le \tilde{y}(t) + \varepsilon \le \frac{(B + (1-\theta)\alpha e^{-\mu(\delta+\tau)}\frac{B^2}{\mu^2})(1-e^{-\mu T})}{\mu(1-(1-P)e^{-\mu T})} + \varepsilon = S^M \tag{7}$$

From the second equation of equation (4): $I'(t) \le \mu[\frac{(1-\theta)\alpha S^M(1-e^{-\mu\delta})}{\mu} - 1]I(t)$ $R* < 1$,so $\frac{(1-\theta)\alpha S^M(1-e^{-\mu\delta})}{\mu} < 1$,And for $\forall\varepsilon$, $\lim_{t\to\infty}|S(t) - S^M(t)| = 0$,we can get $I'(t) \le \mu[\frac{(1-\theta)\alpha S^M(1-e^{-\mu\delta})}{\mu} - 1]I(t) = I(0)e^{-ct}$, then ti's easy to calculate $\lim_{t\to\infty} I(t) = 0$.

Let's say $V(t) = |S(t) - \tilde{S}(t)|$ when $t = kT$ the derivative of $V(t)$ with respect to t is

$$\begin{aligned} V'(t) &= sign|S(t) - \tilde{S}(t)||S'(t) - \tilde{S}'(t)| \\ &\le [B + (1-\theta)\alpha S(t-\delta-\tau)I(t-\delta-\tau)e^{-\mu(\delta+\tau)} - \mu S(t) - B + \mu\tilde{S}(t)] \\ &\le \eta - \mu|S(t) - \tilde{S}(t)| \le \eta - \mu V(t) \end{aligned}$$

Where $\eta = (1-\theta)\alpha\frac{B}{\mu}I(0)e^{-ct}(1 + e^{-\mu(\delta+\tau)})$ (8)

And by the the recursive formula: $V(t_k^+) = (1-p)V(t_k^-)$ (9)

Combining with the equations (8), (9) and lemma 2, we can know:

$$V(t^+) \le V(0^+)(1-p)^{[t]}e^{-\mu t} + \frac{\eta}{\mu}e^{-\mu t}[\int_0^T \prod_{s<kT<t}(1-p)e^{\mu s}\, d(\mu s) + \int_T^{2T} \prod_{s<kT<t}(1-p)e^{\mu s}\, d(\mu s) + \cdots + \int_{[t]T}^t \prod_{s<kT<t}(1-p)e^{\mu s}\, d(\mu s)]$$

$$\le V(0^+)(1-p)^{[t]}e^{-\mu t} + \frac{\eta}{\mu}e^{-\mu t}[(1-p)^{[t]}(e^{\mu T}-1) + (1-p)^{[t]-1}(e^{\mu T}-1)e^{\mu T} + \cdots + (1-p)(e^{\mu T} - 1)e^{\mu([t]-1)T} + e^{\mu t} - e^{\mu[t]T}]$$

$$= V(0^+)(1-p)^{[t]}e^{-\mu t} + \frac{\eta}{\mu}e^{-\mu t}[\frac{(1-p)^{[t]}(e^{\mu T}-1)[1 - (\frac{e^{\mu T}}{1-P})^{[t]}]}{1 - \frac{e^{\mu T}}{1-P}} + e^{\mu t} - e^{\mu[t]T}]$$

$$= V(0^+)(1-p)^{[t]}e^{-\mu t} + \frac{\eta}{\mu}[\frac{(1-p)^{[t]+1}(e^{\mu T}-1) + pe^{\mu([t]-1)T}e^{-\mu t}}{1-P-e^{\mu T}} + 1] \tag{10}$$

We know from the above equation $\lim_{t\to\infty} V(t) = 0$ thus $\lim_{t\to\infty} S(t) = \tilde{S}(t)$, and in the same way, we can prove that $\lim_{t\to\infty} R(t) = \tilde{R}(t)$.

Corollary 1. for model 2, there are the following conclusions:

(1) When $(1-\theta)\alpha[B + (1-p)\alpha e^{-\mu(\delta+\tau)}\frac{B^2}{\mu^2}] < \mu^2$, the basic reproducible number $R* < 1$, then the emotion transmission finally disappears.

(2) When $(1-\theta)\alpha[B + (1-p)\alpha e^{-\mu(\delta+\tau)}\frac{B^2}{\mu^2}] > \mu^2$ But propagator propagates time delay $\delta > \delta*$,When the pulse interrupts $p < p*$ the emotion propagation finally disappears.

$$\delta^* = \frac{1}{\mu}\ln\left(\frac{\mu^2[1-(1-p)e^{-\mu T}]}{(1-\theta)\alpha B(1-p)(1-e^{-\mu T})(1-e^{-\mu\delta})}\right)$$

$$p^* = \frac{((1-\theta)\alpha[B+(1-p)\alpha e^{-\mu(\delta+\tau)}\frac{B^2}{\mu^2}]-\mu^2)(1-e^{-\mu T})}{\mu e^{-\mu T}}$$

3.2. The persistence of the emotions, the enduring proof of the model

We define $R_* = \frac{(1-\theta)\alpha B(1-p)(1-e^{-\mu T})(1-e^{-\mu\delta})}{\mu^2[1-(1-p)e^{-\mu T}]}$.

Theorem 2. If $R_* > 1$, the emotion eventually does not disappear, but tend to a non-zero state of stability, the model has a positive periodic solution, and the model persists.

Proof: since both $S(t), I(t). R(t)$ are upper bound functions, we only need to prove that they have non-negative bounds.

If for any small positive number m, there is $t_0 > 0$, when $t > t_0$, $I(t) < m$ is not true, otherwise, when $t > t_0$, $I(t) > m$is always true

According to the first equation of model (2), it can be concluded that:

$$S'(t) \geq B - \mu S(t) - \alpha m S(t)$$

Consider the differential equation of impulse comparison:

$$\begin{cases} Z'(t) = B-(\mu+\alpha m)Z(t), t \neq kT, k \in Z_+ \\ Z(t^+) = (1-p)Z(0), t = kT,, k \in Z_+ \end{cases} \tag{11}$$

According to lemma 1, the equation has a unique positive periodic solution

$$\tilde{Z}(t) = \frac{B}{(\mu+\alpha m)}\left[1-\frac{pe^{-(\mu+\alpha m)(t-kT)}}{1-(1-p)e^{-\mu T}}\right] = \frac{B}{(\mu+\alpha m)}\left[\frac{1-e^{-\mu T}+Pe^{-\mu T}+pe^{-(\mu+\alpha m)(t-kT)}}{1-(1-p)e^{-\mu T}}\right] \tag{12}$$

According to the principle of differential comparison of impulses, $\exists t_1 > 0$, when $t \geq t_1$,there is

$$S(t) \geq \tilde{Z}(t) > \frac{B(1-p)(1-e^{-(\mu+\alpha m)T})}{(\mu+\alpha m)1-(1-p)e^{-(\mu+\alpha m)T}} = Z^* \tag{13}$$

$$R^* > 1,\text{so}\ \frac{(1-\theta)\alpha(1-e^{-\mu\delta})B(1-p)(1-e^{-(\mu+\alpha m)T})}{\mu(\mu+\alpha m)[1-(1-p)e^{-(\mu+\alpha m)T}]} = \frac{(1-\theta)\alpha(1-e^{-\mu\delta})}{\mu(\mu+\alpha m)}Z^* > 1 \tag{14}$$

is still true for any positive number m.

$$U(t) = I(t) - \int_{t-\delta}^{t}(1-\theta)\alpha e^{-\mu\delta}S(u)I(u)du$$

When $t \geq t_1$, the derivative with respect to $U(t)$

$$U'(t) = I'(t) - ((1-\theta)\alpha e^{-\mu\delta}S(t)I(t) - (1-\theta)\alpha e^{-\mu\delta}S(t-\delta)I(t-\delta))$$

$$= [(1-\theta)\alpha(1-e^{-\mu\delta})S(t) - \mu]I(t)$$

$$> \mu[\frac{(1-\theta)\alpha(1-e^{-\mu\delta})}{\mu}Z^*(t) - 1]I(t) \tag{15}$$

let $l = \min_{t\in[t_1,t_1+\delta]} I(t)$ for all of t, when it meets $t \geq t_1$, $I(t) \geq m$Otherwise, $\exists t_2 > t_1 + \delta, I(t_2) = m$,

And When $t \in [t_1, t_2]$, $I(t) \geq l$.

According to the second equation of the model (1):

$$I(t) = \int_{t-\delta}^{t}(1-\theta)\alpha e^{-\mu(t-u)}S(u)I(u)du \tag{16}$$

Combining equation (14), we know that $I(t) \geq$ （$1-\theta$）$\alpha Z * \int_{t-\delta}^{t} e^{-\mu(t-u)} I(u)du$, so we can get

$$I(t_2) \geq (1-\theta)\ \alpha Z * I(t_2)\int_{t-\delta}^{t} e^{-\mu(t_2-u)}du = \frac{(1-\theta)\alpha Z*(1-e^{-\mu\delta})}{\mu} I(t_2) \tag{17}$$

This is inconsistent with (14) and therefore the hypothesis $I(t) < m$is not valid, thus when $t \geq t_1$, $I(t) \geq m$. From (15), we get $U'(t) > 0$ This is a bounded contradiction to $U(t)$, so $I(t) < m$ is not true. Then consider the following two scenarios:

(1) there is t_1， when $t \geq t_1, I(t) \geq m$

(2) When t is sufficiently large, $I(t)$ oscillates with respect to m.

Proof: (1) has been proved below only proof (2) is valid, if case (2) is true, it exists $h > 0, t_0 > t_1 + \delta$， $I(t_0) = I(t_0 + h) = m$

And this is true for $\forall t \in [t_0, t_0 + h]$, $I(t) \leq m$Then, by the continuity of $I(t)$, $\exists g(0 < g < \delta)$, that makes $I(t) \geq \frac{m}{2}$， $t \in [t_0, t_0 + g]$.

The following are discussed under different circumstances:

1. When $h < g < \delta, I(t) \geq \frac{m}{2}$ is obviously true, $t \in [t_0, t_0 + h]$

Now we can get

$$I(t) \geq (1-\theta)\alpha\int_{t_0}^{t_0+g} e^{-\mu(t-u)} S(u)I(u)du > (1-\theta)\ \alpha Z * \frac{m}{2}e^{-\mu\delta} g = q \tag{18}$$

Let $h_2 = min\{\frac{m}{2}, q\}$ so $I(t) \geq h_2$.

2. When $g < \delta < h$ consider the following two situations:

(i)When $t \in [t_0, t_0 + \delta]$, it’s easy to get $I(t) \geq q$

(ii)When$t \in [t_0 + \delta, t_0 + h]$, Suppose $I(t) \geq q$otherwise it exists$t*$， When$t* \in [t_0 + \delta, t_0 + h]$, for $\forall t \in [t_0 + \delta, t*]$, $I(t) \geq q$， we know from formula (16):

$$I(t^*) \geq (1-\theta)\alpha\int_{t^*-\delta}^{t^*} e^{-\mu(t^*-u)} S(u)I(u)du > \frac{(1-\theta)\ \alpha Z*(1-e^{-\mu\delta})}{\mu} q$$

From (16) we can get $\lim_{t\to\infty} inf\ I(t) \geq h_2 > 0$. According to the first equation of model (1):

$$S'(t) \geq B - \mu S(t) - \alpha\frac{B}{\mu}S(t) = B - (\frac{B\alpha}{\mu} + \mu)S(t) \tag{19}$$

According to the differential comparison theorem of impulses,

$$\liminf_{t\to\infty} S(t) = \frac{B(1-p)}{\frac{B\alpha}{\mu} + \mu} = h_1 > 0 \tag{20}$$

The third equation of model (1) can be obtained

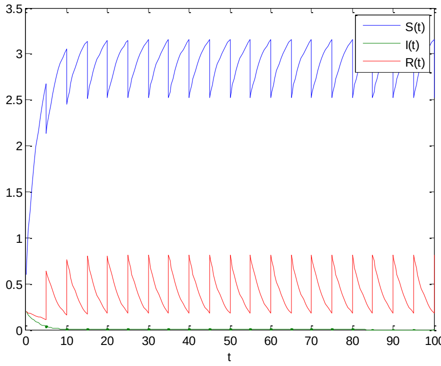
$$R(t) \geq \int_{t-\delta-\tau}^{t-\delta} (1-\theta)\alpha e^{-\mu\delta-\mu(t-\delta-\mu)u} S(u)I(u)du$$

It's easy to get from the formula (15) and $\lim_{t\to\infty} inf\ I(t) \geq h_2 > 0$

$$R(t) \geq \frac{(1-\theta)\ \alpha e^{-\mu\delta}(1-e^{-\mu\tau})h_1h_2}{\mu} = h_3 > 0 \tag{21}$$

3.3. Numerical simulation

We have introduced the spread of investor sentiment in the pulse function and double under the influence of the delay propagation analysis, give the theory of emotional spread double delay threshold, the emotions are analyzed and various groups stable state and its stability when there is no proof, below to model the influence of various parameters on the mood spread to numerical simulation of the control variable method is used to estimate the parameters.



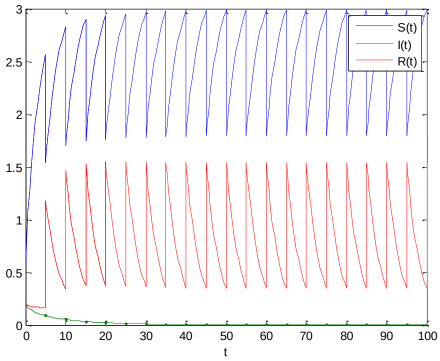
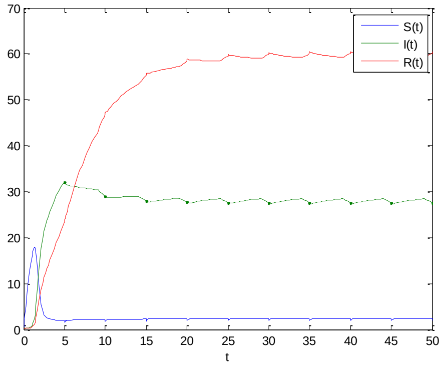


Figure 2: Emotions eventually stop spreading at p=0.2, $R*<1$ Figure 3:Emotions eventually stop spreading at p=0.4, $R*<1$



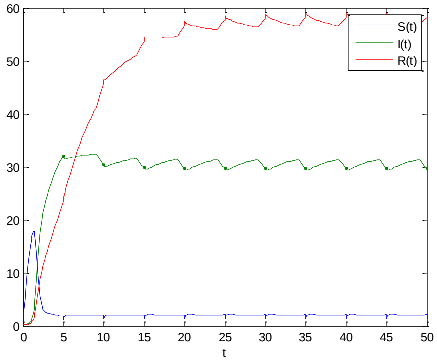


Figure 4:Emotional transmission persists and tends to stabilize at $R*>1$ Figure5: The transmission of emotions persists, the model oscillations persist at $R*>1$

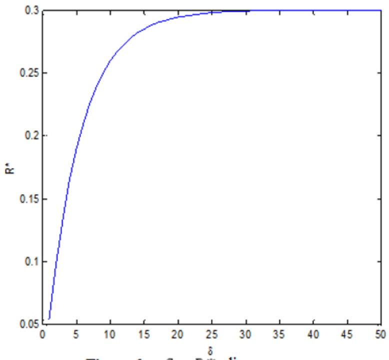


Figure 6a: $\delta - R^*$ diagram

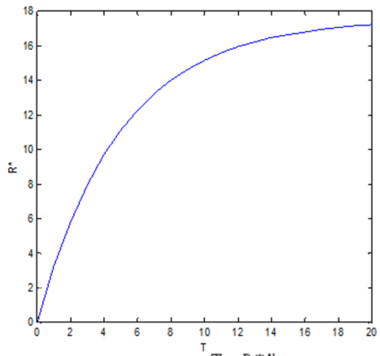


Figure6b: $T - R^*$diagram

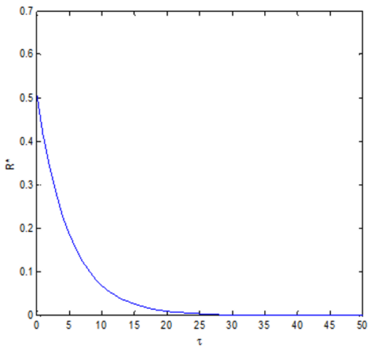


Figure 6c: $\tau - R^*$ diagram

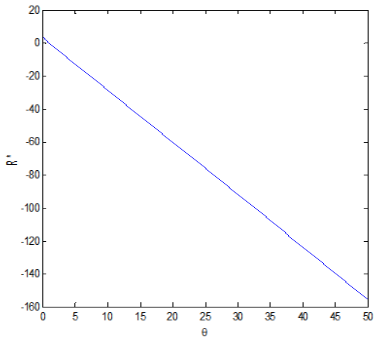


Figure 6d: $\theta - R^*$ diagram

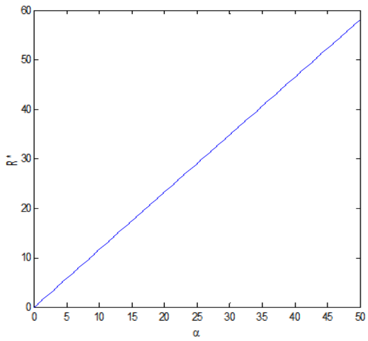


Figure 6e: $\alpha - R^*$ diagram

4. Discussion

(1) Figure1 show when $B = 1, \mu = 0.3, \alpha = 0.2, \theta = 0.3, \delta = 1, \tau = 2, \varepsilon = 0.1, p = 0.2$ $R* < 1$ emotions eventually stop spreading .

(2) Figure2 show when $B = 1, \mu = 0.3, \alpha = 0.2, \theta = 0.3, \delta = 1, \tau = 2, \varepsilon = 0.1, p = 0.4$ $R* < 1$ Emotions eventually stop spreading, and the transmitter disappears faster.

(3) Figure3 show when $B = 18, \mu = 0.6, \alpha = 0.3, \theta = 0.3, \delta = 1, \tau = 2, \varepsilon = 0.1, p = 0.4$ $R* > 1$ emotional transmission persists and tends to stabilize.

(4) Figure4 show when $B = 18, \mu = 0.6, \alpha = 0.3, \theta = 0.3, \delta = 4, \tau = 5, \varepsilon = 0.1, p = 0.4$ $R* > 1$ the transmission of emotions persists, the model oscillations persist.

(5) Figures5a-e show the influence of various parameters on emotional transmission, where Figure 5a is a relational graph of $\delta - R*$,in which δ and $R*$ is positively correlated, and the basic reproducible number $R*$ increases with the increase of propagation delay δ. Figure5b is a relational graph of $T - R*$, the pulse period T is positively correlated with the basic renewable number, and the shorter the pulse interference period, the smaller $R*$ is. Figure 5c is a relational graph of $\tau - R*$, τ and $R*$ is negatively correlated with each other. The longer the forgetting delay is, the smaller $R*$ is. Figure5d is a relational graph of $\theta - R*$, θ and $R*$ is negatively correlated with each other, the greater the probability θ that the susceptible will become resistant, the smaller the $R*$. Figure5e is a relational graph of $\alpha - R*$, α and $R*$ figure (d) is a relational graph, which is positively correlated with each other. The smaller the transmission rate α, the smaller $R*$ is.

5. Conclusion

Through model solution and image information verification of numerical simulation, we get some following conclusions:

(1) Under the impulse action of a reasonable threshold, the process of emotion transmission speeds up significantly, so adjusting the probability of impulse action（$p < p*$） can prevent the spread of bad emotions in the financial market as soon as possible.

(2) According to the relationship between the time delay and the basic reproducible number in the figure, the time-delay of transmission δ is controlled to be $\delta < \delta*$ when the emotion transmission finally disappears, and the longer the delay is resisted, the faster the emotion transmission will disappear.

(3) Strengthen the propaganda of irrational investment cases, improve the rational investment awareness of investors, so as to increase the rejection rate of bad emotions θ, reduce the transmission rate α, and achieve the purpose of making the emotional transmission disappear or become stable as soon as possible.

6.Acknowledgment

The topic selection, writing and final writing of this paper are all completed under the careful guidance of Professor. His rigorous attitude, excellent financial professional knowledge and accomplishment, and tireless guidance have benefited me a lot, which is worthy of lifelong learning. In addition, I would like to thank my senior colleagues for generously providing relevant materials and helping me solve many difficult problems in my papers. From them, I learned the importance of helping each other overcome academic difficulties.

References

1. Li, D.; Ma, J. How the government's punishment and individual's sensitivity affect the rumor spreading in online social networks. Physica A: Statistical Mechanics and its Applications 2017, 469, 284-292, doi:10.1016/j.physa.2016.11.033.
2. Afassinou, K. Analysis of the impact of education rate on the rumor spreading mechanism. Physica A: Statistical Mechanics and its Applications 2014, 414, 43-52, doi:10.1016/j.physa.2014.07.041.
3. Zhou, F.; Yao, H. Stability analysis for neutral-type inertial BAM neural networks with time-varying delays. Nonlinear Dynamics 2018, 92, 1583-1598, doi:10.1007/s11071-018-4148-7.
4. Hu, Y.; Pan, Q.; Hou, W.; He, M. Rumor spreading model considering the proportion of wisemen in the crowd. Physica A: Statistical Mechanics and its Applications 2018, 505, 1084-1094, doi:10.1016/j.physa.2018.04.056.

5. Sahafizadeh, E.; Tork Ladani, B. The impact of group propagation on rumor spreading in mobile social networks. Physica A: Statistical Mechanics and its Applications 2018, 506, 412-423, doi:10.1016/j.physa.2018.04.038.
6. Zhao, L.; Xie, W.; Gao, H.O.; Qiu, X.; Wang, X.; Zhang, S. A rumor spreading model with variable forgetting rate. Physica A: Statistical Mechanics and its Applications 2013, 392, 6146-6154, doi:10.1016/j.physa.2013.07.080.
7. Chen, G. ILSCR rumor spreading model to discuss the control of rumor spreading in emergency. Physica A: Statistical Mechanics and its Applications 2019, 522, 88-97, doi:10.1016/j.physa.2018.11.068.
8. Huo, L.a.; Wang, L.; Song, N.; Ma, C.; He, B. Rumor spreading model considering the activity of spreaders in the homogeneous network. Physica A: Statistical Mechanics and its Applications 2017, 468, 855-865, doi:10.1016/j.physa.2016.11.039.
9. Liu, Q.; Li, T.; Sun, M. The analysis of an SEIR rumor propagation model on heterogeneous network. Physica A: Statistical Mechanics and its Applications 2017, 469, 372-380, doi:10.1016/j.physa.2016.11.067.
10. Zhang, Y.; Zhu, J. Stability analysis ofI2S2Rrumor spreading model in complex networks. Physica A: Statistical Mechanics and its Applications 2018, 503, 862-881, doi:10.1016/j.physa.2018.02.087.
11. Hu, Y.; Pan, Q.; Hou, W.; He, M. Rumor spreading model with the different attitudes towards rumors. Physica A: Statistical Mechanics and its Applications 2018, 502, 331-344, doi:10.1016/j.physa.2018.02.096.
12. Gao, S.; Chen, L.; Teng, Z. Impulsive vaccination of an SEIRS model with time delay and varying total population size. Bull Math Biol 2007, 69, 731-745, doi:10.1007/s11538-006-9149-x.
13. Chen, L.-J.; Chen, X.-L.; Cai, M.; Wang, W. Complex contagions with social reinforcement from different layers and neighbors. Physica A: Statistical Mechanics and its Applications 2018, 503, 516-525, doi:10.1016/j.physa.2018.03.017.
14. Zan, Y. DSIR double-rumors spreading model in complex networks. Chaos, Solitons & Fractals 2018, 110, 191-202, doi:10.1016/j.chaos.2018.03.021.