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One input control and synchronization for generalized Lorenz-like systems

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Abstract. This paper proposes a new class of nonlinear systems called generalized Lorenz-like systems which can be used to describe many usual three-dimensional chaotic systems such as Lorenz system, Lü system, Chen system, Liu system, etc. Then the control and synchronization problems for generalized Lorenz-like system via a single input are studied and two control laws are proposed based on partial feedback linearization with asymptotically stable zero dynamics. Finally, the numerical simulations demonstrate the correctness and effectiveness of the proposed control strategies.

Keywords: Chaos synchronization; zero dynamics; generalized Lorenz-like system.

1. Introduction

In the past three decades, the topic of control and synchronization for chaotic systems has attracted increasing attentions because of its possible applications in secure communication [1-2], biomedical Engineering [3] and etc. The chaos synchronization was introduced, in 1990, by Pcora and Carroll [4], which is used to synchronize two identical chaotic systems with different initial conditions. Since then, a wide variety of methods of the control and synchronization for chaotic systems have been proposed, such as linear feedback control method [5-6], sliding mode control [7], adaptive control method [8-9], backstepping control method [10-11] and so on.

It is well known that if a nonlinear control system is partial feedback linearizable and its corresponding zero dynamics is asymptotically stable, then the control that stabilizes the linear sub-system will stabilize the original system [12-15]. In this paper, a class of generalized Lorenz-like system is introduced which can describe many usual chaotic systems such as Lorenz system, Chen system, Liu system, Lü system and etc. Our object is to realize the control and synchronization, for any given initial conditions, of generalized Lorenz-like system by one input. Two one-input control strategies are proposed for the control and synchronization, respectively, based on partial feedback linearization with asymptotically stable zero dynamics of the corresponding error systems. Note that the generalized Lorenz-like system is similar to the generalized Lorenz system in [16-18], but our system is, in fact, more generalized that can describe a much bigger class of nonlinear systems (see Remark 5).

This paper is organized as follows. In Section 2, the generalized Lorenz-like system is introduced and moreover, useful notations and problem statement is also given. The main results are presented in Section 3. Numerical simulations are shown in Section 4 to verify the effectiveness and correctness of the proposed one-input control strategies. Finally, concluding remarks are given in Section 5.

2. Preliminaries and problem statement

2.1 Zero dynamics [12-13]

Consider a single-input single-output nonlinear system

$$\Sigma: \quad \begin{cases} \dot{x} = f(x) + g(x)u \\ y = h(x) \end{cases}$$

where the state $x \in \mathrm{R}^n$, the control $u \in R$ and the entries f, g are smooth vector fields on R^n. Let $y = h(x)$ be an output of Σ with relative degree $r < n$ at some point x_0, then locally there exist a regular static state feedback $u = \alpha(x) + \beta(x)v$ and a state transformation $z = (z^1, z^2) = (\Phi^1(x), \Phi^2(x)) = \Phi(x)$, where

$z^1 = (z_1, \dots, z_r)^\top$, $z^2 = (z_{r+1}, \dots, z_n)^\top$, and Φ is a diffeomorphism, such that in the $z-$coordinates, the system Σ reads, locally,

$$\begin{aligned} \dot{z}_1 &= z_2 \\ &\vdots \\ \dot{z}_{r-1} &= z_r \\ \dot{z}_r &= v \\ \dot{z}^2 &= \eta(z^1, z^2) \\ y &= z_1 \end{aligned}$$

Definition 1. The zero dynamics of system Σ is defined by the dynamics $\dot{z}^2 = \eta(0, z^2)$ which are the internal dynamics consistent with the constraint that $y(t) \equiv 0$.

Lemma 2. If the zero dynamics of system Σ is asymptotically stable, then the control u that stabilizes the linear sub-system will stabilize the system Σ.

2.2 Generalized Lorenz-like systems

Consider a nonlinear autonomous system defined on R^3

$$\Lambda: \quad \dot{x} = Ax + f(x),$$

where the state $x = (x_1, x_2, x_3)^{\mathrm{T}}$, the smooth vector field $f(x) = (f_1(x), f_2(x), f_3(x))^{\mathrm{T}}$ is the nonlinear part of system and A is a constant matrix which is in the following form:

$$A = \begin{pmatrix} a_{11} & a_{12} & 0 \\ a_{21} & a_{22} & 0 \\ 0 & 0 & a_{33} \end{pmatrix}.$$

Definition 3. The nonlinear system Λ is called a *generalized Lorenz-like system* if it satisfies $a_{33} < 0, a_{12} \neq 0$ and the elements of vector field f satisfy $f_1(x) = \dfrac{\partial f_2(x)}{\partial x_2} = \dfrac{\partial f_3(x)}{\partial x_3} = 0$. In other words, the generalized Lorenz-like system is in the following form

$$\begin{cases} \dot{x}_1 = a_{11}x_1 + a_{12}x_2 \\ \dot{x}_2 = a_{21}x_1 + a_{22}x_2 + f_2(x_1, x_3) \\ \dot{x}_3 = a_{33}x_3 + f_3(x_1, x_2). \end{cases} \tag{1}$$

Remark 4. Many usual chaotic systems can be described by the generalized Lorenz-like system. For example, when $a_{11} < 0, a_{12} = -a_{11}$, $f_2(x_1, x_3) = lx_1x_3$ and $f_3(x_1, x_2) = hx_2^2$, it becomes Multi-wing system [16]. Moreover, it is easy to see that Lorenz system [20], Chen system [2], Liu system [21], Lü system [22], etc., can also be described by this system (1).

Remark 5. In [16-18], a similar nonlinear control system called generalized Lorenz system was introduced in which the elements of $f(x)$ was quadratic and defined by $f_1(x) = 0$, $f_2(x_1, x_3) = x_1x_3$, $f_3(x_1, x_2) = -x_1x_2$. However, for system (1), they need not to be quadratic. Moreover, we do not introduce the condition that the eigenvalues of the matrix A satisfy $-\lambda_2 > \lambda_1 > -a_{33} > 0$ which may leads to the chaotic behavior when its nonlinear part is in quadratic. In fact, unlike the generalized Lorenz system in [16-18], the constant a_{33} in system (1) may not be an eigenvalue of the linearized system at an equilibrium point. Therefore, the system (1) is more generalized than the generalized Lorenz system in [16-18].

2.3 Problem statement

In this paper, the control and synchronization for the generalized Lorenz-like system via one input is studied and the control strategies are proposed based on the partial feedback linearization with asymptotically stable zero dynamics. More precisely, we add a control variable to the second equation of (1),

$$\begin{cases} \dot{x}_1 = a_{11}x_1 + a_{12}x_2 \\ \dot{x}_2 = a_{21}x_1 + a_{22}x_2 + f_2(x_1, x_3) + u \\ \dot{x}_3 = a_{33}x_3 + f_3(x_1, x_2). \end{cases} \tag{2}$$

which is called the slave system with $a_{33} < 0, a_{12} \neq 0$ and the master system denotes the original system in variable y :

$$\begin{cases} \dot{y}_1 = a_{11}y_1 + a_{12}y_2 \\ \dot{y}_2 = a_{21}y_1 + a_{22}y_2 + f_2(y_1, y_3) \\ \dot{y}_3 = a_{33}y_3 + f_3(y_1, y_2). \end{cases} \tag{3}$$

The object of this paper is to solve the following control and synchronization problems for generalized Lorenz-like system (1) via single input:

(i) For any equilibrium point (x_1^*, x_2^*, x_3^*) of (1), find a suitable control u such that $\lim\limits_{t\to\infty}\left|x - x^*\right| = 0$, for any initial condition $(x_1(0), x_2(0), x_3(0))$;

(ii) Find a suitable control u such that $\lim\limits_{t\to\infty}\left|x - y\right| = 0$ for any initial conditions of the slave system and the master system $(x_1(0), x_2(0), x_3(0))$ and $(y_1(0), y_2(0), y_3(0))$.

3. Main results

Lemma 6. The second dimensional linear control system [23-24]

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = u, \end{cases}$$

can be globally stabilized in finite time under the feedback control law

$$u = -k_1 \text{sign}(x_1) \,|\, x_1 \,|^{\alpha_1} - k_2 \text{sign}(x_2) \,|\, x_2 \,|^{\alpha_2}.$$

where $k_1, k_2 > 0, \alpha_1 \in (0,1), \alpha_2 = \dfrac{2\alpha_1}{1+\alpha_1}.$

Theorem 7. The control problem by one input of the generalized Lorenz-like system, given by (1), can be achieved by the following control law

$$\begin{aligned} u_1 = a_{12}^{-1}(&-k_1 \text{sign}(x_1 - x_1^*) \,|\, x_1 - x_1^* \,|^{\alpha_1} - k_2 \text{sign}(a_{11}x_1 + a_{12}x_2) \,|\, a_{11}x_1 + a_{12}x_2 \,|^{\alpha_2} \\ &- (a_{11}^2 + a_{12}a_{21})x_1) - (a_{11} + a_{22})x_2 - f_2(x_1, x_3). \end{aligned}$$

Proof. Let (x_1^*, x_2^*, x_3^*) denote an equilibrium point of the generalized Lorenz-like system and the control errors are defined by $e_i^* = x_i - x_i^*$. Thus the error dynamics can be obtained in the following form:

$$\begin{cases} \dot{e}_1^* = a_{11}e_1^* + a_{12}e_2^* \\ \dot{e}_2^* = a_{21}e_1^* + a_{22}e_2^* + a_{21}x_1^* + a_{22}x_2^* + f_2(e_1^* + x_1^*, e_3^* + x_3^*) + u_1 \\ \dot{e}_3^* = a_{33}e_3^* + a_{33}x_3^* + f_3(e_1^* + x_1^*, e_2^* + x_2^*). \end{cases} \tag{4}$$

It is easy to see that the error system (4) can be partial linearized into following form

$$\begin{cases} \dot{z}_1 = z_2 \\ \dot{z}_2 = v_1 \\ \dot{z}_3 = a_{33}z_3 + a_{33}x_3^* + f_3(z_1 + x_1^*, a_{12}^{-1}(z_2 - a_{11}z_1) + x_2^*) \end{cases} \tag{5}$$

under the invertible change of coordinates

$$\begin{cases} z_1 = e_1^* \\ z_2 = a_{11}e_1^* + a_{12}e_2^* \\ z_3 = e_3^* \end{cases} \quad (6)$$

and the feedback $u_1 = a_{12}^{-1}(v_1 - a_{11}(a_{11}e_1^* + a_{12}e_2^*)) - (a_{21}e_1^* + a_{22}e_2^* + a_{21}x_1^* + a_{22}x_2^* + f_2(e_1^* + x_1^*, e_3^* + x_3^*))$. Note that the zero dynamics of (5) is given by

$$\dot{z}_3 = a_{33}z_3 + a_{33}x_3^* + f_3(x_1^*, x_2^*).$$

Since (x_1^*, x_2^*, x_3^*) is an equilibrium point, we have clearly $a_{33}x_3^* + f_3(x_1^*, x_2^*) = 0$. Therefore, the zero dynamics of (5) is given by just $\dot{z}_3 = a_{33}z_3$ that is asymptotically stable due to $a_{33} < 0$. By Lemma 6, the control law

$$v_1 = -k_1\text{sign}(z_1)\,|\,z_1\,|^{\alpha_1} - k_2\text{sign}(z_2)\,|\,z_2\,|^{\alpha_2} \quad (7)$$

stabilizes the variable z_1, z_2 in finite time T^*. According to Lemma 2, the control law (7) will also stabilize the system (5). Since that the change of coordinates (5) is invertible globally, we have $z_1 = z_2 = 0$ if and only if $e_1^* = e_2^* = 0$ and consequently, the control law

$$\begin{aligned} u_1 &= a_{12}^{-1}(v_1 - a_{11}(a_{11}e_1^* + a_{12}e_2^*)) - (a_{21}e_1^* + a_{22}e_2^* + a_{21}x_1^* + a_{22}x_2^* + f_2(e_1^* + x_1^*, e_3^* + x_3^*)) \\ &= a_{12}^{-1}(-k_1\text{sign}(x_1 - x_1^*)\,|\,x_1 - x_1^*\,|^{\alpha_1} - k_2\text{sign}(a_{11}x_1 + a_{12}x_2)\,|\,a_{11}x_1 + a_{12}x_2\,|^{\alpha_2} \\ &\quad - (a_{11}^2 + a_{12}a_{21})x_1) - (a_{11} + a_{22})x_2 - f_2(x_1, x_3) \end{aligned}$$

will stabilize the error systems (4). □

Theorem 8. The synchronization problem of the generalized Lorenz-like system, given by (1), can be achieved by the following control law

$$\begin{aligned} u_2 = a_{12}^{-1}(&-k_1\text{sign}(x_1 - y_1)\,|\,x_1 - y_1\,|^{\alpha_1} \\ &- k_2\text{sign}(a_{11}x_1 + a_{12}x_2 - a_{11}y_1 - a_{12}y_2)\,|\,a_{11}x_1 + a_{12}x_2 - a_{11}y_1 - a_{12}y_2\,|^{\alpha_2} \\ &- (a_{11}^2 + a_{12}a_{21})(x_1 - y_1)) - (a_{11} + a_{22})(x_2 - y_2) - (f_2(x_1, x_3) - f_2(y_1, y_3)). \end{aligned}$$

Proof. It is easy to see that the output $h(x) = x_1$ has relative degree 2 and clearly, under the change of coordinates $z = \Phi(x)$ in the form

$$\begin{cases} z_1 = x_1 \\ z_2 = a_{11}x_1 + a_{12}x_2 \\ z_3 = x_3 \end{cases} \quad (8)$$

and the feedback $u_2 = a_{12}^{-1}(v_2 - a_{11}(a_{11}x_1 + a_{12}x_2)) - (a_{21}x_1 + a_{22}x_2 + f_2(x_1, x_3))$, the slave system (2) can be transformed into the following form

$$\begin{cases} \dot{z}_1 = z_2 \\ \dot{z}_2 = v_2 \\ \dot{z}_3 = a_{33}z_3 + f_3(z_1, a_{12}^{-1}(z_2 - a_{11}z_1)). \end{cases} \quad (9)$$

By the same change of coordinates $w = \Phi(y)$, the master system can be transformed into the form

$$\begin{cases} \dot{w}_1 = w_2 \\ \dot{w}_2 = (a_{12}a_{21} - a_{11}a_{22})w_1 + (a_{11} + a_{22})w_2 + a_{12}f_2(w_1, w_3) \\ \dot{w}_3 = a_{33}w_3 + f_3(w_1, a_{12}^{-1}(w_2 - a_{11}w_1)). \end{cases} \quad (10)$$

Define the errors by $e_i = z_i - w_i$, for $1 \le i \le 3$, and then the error dynamics reads

$$\begin{cases} \dot{e}_1 = e_2 \\ \dot{e}_2 = \tilde{v}_2 \\ \dot{e}_3 = a_{33}e_3 + f_3(z_1, a_{12}^{-1}(z_2 - a_{11}z_1)) - f_3(w_1, a_{12}^{-1}(w_2 - a_{11}w_1)) \end{cases} \tag{11}$$

where $\tilde{v}_2 = v_2 - ((a_{12}a_{21} - a_{11}a_{22})w_1 + (a_{11} + a_{22})w_2 + a_{12} f_2(w_1, w_3))$. By Lemma 6, the control law

$$\tilde{v}_2 = -k_1 \mathrm{sign}(e_1) | e_1 |^{\alpha_1} - k_2 \mathrm{sign}(e_2) | e_2 |^{\alpha_2} \tag{12}$$

stabilizes the variable e_1, e_2 in finite time T which follows $z_1 = w_1, z_2 = w_2$, for any $t \geq T$, and consequently we have $f_3(z_1, a_{12}^{-1}(z_2 - a_{11}z_1)) - f_3(w_1, a_{12}^{-1}(w_2 - a_{11}w_1)) = 0$. Thus, the zero dynamics of the error system (11) is given by $\dot{e}_3 = a_{33}e_3$ which is, obviously, asymptotically stable due to $a_{33} < 0$. According to Lemma 2, the control law (12) will also stabilize the system (11), i.e., $\lim_{t\to\infty} |z - w| = 0$ that is equivalent to $\lim_{t\to\infty} |x - y| = 0$ for any initial conditions. The control law that achieve the synchronization problem is given by

$$\begin{aligned} u_2 &= a_{12}^{-1}(-k_1 \mathrm{sign}(e_1) | e_1 |^{\alpha_1} - k_2 \mathrm{sign}(e_2) | e_2 |^{\alpha_2} \\ &\quad - (a_{11}^2 + a_{12}a_{21})e_1 - (a_{11} + a_{22})(e_2 - a_{11}e_1) - a_{12}(f_2(z_1, z_3) - f_2(w_1, w_3))) \\ &= a_{12}^{-1}(-k_1 \mathrm{sign}(x_1 - y_1) | x_1 - y_1 |^{\alpha_1} \\ &\quad - k_2 \mathrm{sign}(a_{11}x_1 + a_{12}x_2 - a_{11}y_1 - a_{12}y_2) | a_{11}x_1 + a_{12}x_2 - a_{11}y_1 - a_{12}y_2 |^{\alpha_2} \\ &\quad - (a_{11}^2 + a_{12}a_{21})(x_1 - y_1)) - (a_{11} + a_{22})(x_2 - y_2) - (f_2(x_1, x_3) - f_2(y_1, y_3)). \end{aligned}$$

4. Numerical simulations

In order to verify the effectiveness of proposed controller design, we consider the following three dimensional autonomous chaotic system introduced in [25]:

$$\begin{cases} \dot{x}_1 = a_{11}x_1 + a_{12}x_2 \\ \dot{x}_2 = a_{21}x_1 + a_{22}x_2 - x_1x_3 \\ \dot{x}_3 = a_{33}x_3 + x_1^2. \end{cases} \tag{14}$$

When the parameters of systems (14) are given by $a_{11} = -15, a_{12} = 20, a_{21} = 20, a_{22} = -1$ and $a_{33} = -8$, the system (14) is chaotic [25]. Obviously, this system belongs to the generalized Lorenz-like system with $f_2(x_1, x_3) = -x_1x_3$ and $f_3(x_1, x_2) = x_1^2$.

4.1 Control to the equilibria

Solving the equations $\dot{\boldsymbol{x}}_1 = \dot{\boldsymbol{x}}_2 = \dot{\boldsymbol{x}}_3 = \mathbf{0}$ in (14), we obtain the three equilibria of the system (14) as $\boldsymbol{O}(\mathbf{0}, \mathbf{0}, \mathbf{0}), \boldsymbol{E}_1(\sqrt{\mathbf{154}}, \mathbf{0.75}\sqrt{\mathbf{154}}, \mathbf{19.25}), \boldsymbol{E}_2(-\sqrt{\mathbf{154}}, -\mathbf{0.75}\sqrt{\mathbf{154}}, \mathbf{19.25})$. By Theorem 7, for any equilibrium point $\boldsymbol{x}^*$ of system (14), the states $(\boldsymbol{x}_1, \boldsymbol{x}_2, \boldsymbol{x}_3)$ can be controlled to $\boldsymbol{x}^*$ from any initial condition by the following control strategy

$$\begin{aligned} u_1 &= a_{12}^{-1}(-k_1 \mathrm{sign}(x_1 - x_1^*) | x_1 - x_1^* |^{\alpha_1} - k_2 \mathrm{sign}(a_{11}x_1 + a_{12}x_2) | a_{11}x_1 + a_{12}x_2 |^{\alpha_2} \\ &\quad - ({a_{11}}^2 + a_{12}a_{21})x_1) - (a_{11} + a_{22})x_2 + x_1x_3. \end{aligned}$$

For the numerical simulations, we assume that the initial condition is given by $(\boldsymbol{x}_1(\mathbf{0}), \boldsymbol{x}_2(\mathbf{0}), \boldsymbol{x}_3(\mathbf{0})) = (-\mathbf{1}, \mathbf{2}, \mathbf{3})$ .We choose values for the constants $\boldsymbol{k}_1 = \mathbf{2}, \boldsymbol{k}_2 = \mathbf{3}$, , $\boldsymbol{\alpha}_1 = \mathbf{1/3}, \boldsymbol{\alpha}_2 = \mathbf{1/2}$ and choose $\boldsymbol{E}_1(\sqrt{\mathbf{154}}, \mathbf{0.75}\sqrt{\mathbf{154}}, \mathbf{19.25})$ as the target equilibrium point. The simulation results are shown in Fig.1.

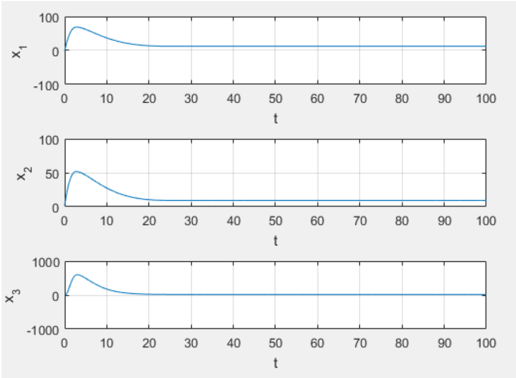

Fig.1 Chaos control at equilibrium point E_1

4.2 Synchronization between two identical generalized Lorenz-like systems

Assume that the slave system and the master system are taken, respectively, as follows

$$\text{slave system:} \quad \begin{cases} \dot{x}_1 = a_{11}x_1 + a_{12}x_2, \\ \dot{x}_2 = a_{21}x_1 + a_{22}x_2 - x_1x_3 + u_2, \\ \dot{x}_3 = a_{33}x_3 + x_1^2, \end{cases} \tag{15}$$

$$\text{master system:} \quad \begin{cases} \dot{y}_1 = a_{11}y_1 + a_{12}y_2, \\ \dot{y}_2 = a_{21}y_1 + a_{22}y_2 - y_1y_3, \\ \dot{y}_3 = a_{33}y_3 + y_1^2, \end{cases} \tag{16}$$

By Theorem 8, for any initial conditions, the above two systems are globally synchronized by the control law

$$\begin{aligned} u_2 = a_{12}^{-1}(&-k_1\mathrm{sign}(x_1 - y_1)|x_1 - y_1|^{\alpha_1} \\ &- k_2\mathrm{sign}(a_{11}x_1 + a_{12}x_2 - a_{11}y_1 - a_{12}y_2)|a_{11}x_1 + a_{12}x_2 - a_{11}y_1 - a_{12}y_2|^{\alpha_2} \\ &- (a_{11}^2 + a_{12}a_{21})(x_1 - y_1)) - (a_{11} + a_{22})(x_2 - y_2) + (x_1x_3 - y_1y_3). \end{aligned}$$

For the numerical simulations, we assume that the initial condition is given by $(\boldsymbol{x_1(0)}, \boldsymbol{x_2(0)}, \boldsymbol{x_3(0)}) = (\boldsymbol{-1}, \boldsymbol{2}, \boldsymbol{3})$ and $(\boldsymbol{y_1(0)}, \boldsymbol{y_2(0)}, \boldsymbol{y_3(0)}) = (\boldsymbol{1}, \boldsymbol{1}, \boldsymbol{1})$. The values of the constants are chosen by $\boldsymbol{k_1} = \boldsymbol{2}, \boldsymbol{k_2} = \boldsymbol{3}, \boldsymbol{\alpha_1} = \boldsymbol{1/3}, \boldsymbol{\alpha_2} = \boldsymbol{1/2}$. Fig.2 and Fig.3 display the state response and synchronization errors of systems (14) and (15). It can be seen that the synchronization errors converge to zero rapidly.

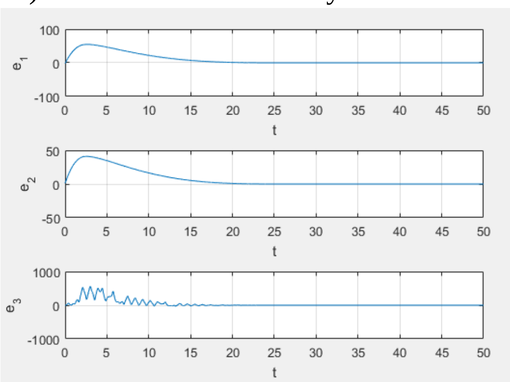

Fig.3 The error state time responses

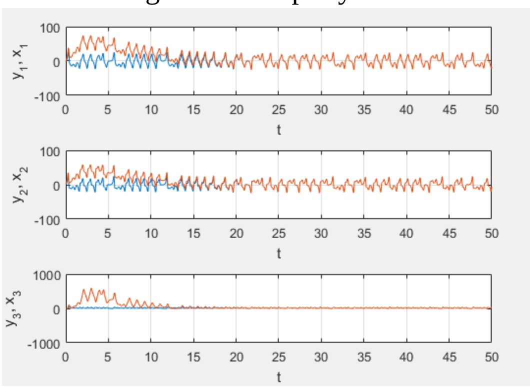

Fig.4 synchronization systems

5. Concluding remark

In this paper, we proposed a class of nonlinear control systems called generalized Lorenz-like systems which can be used to describe many usual chaotic systems such as Lorenz system, Chen system, Liu system, Lü system, etc. For this class of systems, one input control laws which achieved the control and synchronization problems has been proposed based on partial feedback linearization with stable zero dynamics. Finally, the numerical simulations are provided to show the effectiveness and correctness of the proposed control strategies.

6. Acknowledgements

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