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Image encryption based on fractional-order memristive neuron system and DNA coding technology

Fanqi Meng [1,2], Xiaoqin Zeng[2], Zuolei Wang [1*]

[1]*School of Mathematics and Statistics, Yancheng Teachers University, Yancheng, P.R. China*
[2] *Computer and Information Engineering College, Hohai University, Nanjing, P.R. China*
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Abstract. In this paper, we investigate the color image encryption based on fractional-order memristive neuron system and DNA coding technology. The proposed encryption method consists of two stages: permutation and diffusion. Some numerical examples are exploited to demonstrate the feasibility of the encryption algorithm, and the security analysis shows that the algorithm has high security and good resistance to statistical attacks.

Keywords: color image encryption, fractional-order, neuron system, NDA coding

1 Introduction

In the 1980s, researchers found that the inherent randomness, sensitivity to initial values and long-term unpredictability of chaotic systems are just similar to the basic principles of cryptography. They proposed that chaotic theory be applied to cryptography and formed a new cryptography. Fridrich and Pecora et al. [1-2] proposed image pixel scrambling algorithm based on chaotic cryptography theory in the 1990s. Chaotic theory was used to encrypt and decrypt digital images. Then, in the 21st century, more and more chaotic encryption schemes were proposed by researchers to make image encryption technology based on chaotic theory. It has developed rapidly [3-4].

In the field of image encryption, chaotic image encryption by using one-dimensional and multi-dimensional chaotic sequences to scramble image pixel positions are applied in Refs. [5-14]. Recently, researchers have found that DNA sequences can be used to design encryption algorithms. Some image encryption schemes based on NDA sequences are proposed. The main idea is to use operator and DNA sequences to transform the pixel values of the diffused part. The algorithm has the characteristics of less calculation times and low power consumption and can be combined with many chaotic encryption technologies [15-16]. Wu X et al. [17] used DNA coding in combination with 1D chaotic mapping for color image decryption. Wang X et al. [18] proposed that the confusing image will be represented by DNA sequence according to the coding rule, and the extended Hamming distance is calculated to perturb the secret key to generate the key stream related to the initial image; Wei X et al [19] Designing a DNA sequence-based color image encryption algorithm for hyperchaotic systems; Maddodi et al. [20] proposed to first apply DNA sequence permutation and replacement, and then update the parameters in the chaotic neural network to improve the randomness of chaotic sequences to achieve image encryption; Zhao P et al. [21] proposed an image encryption scheme based on high-dimensional chaotic systems and DNA sequences.

Based on the above analysis of motivation, this paper will study the image encryption based on fractional-order memristive neural system and DNA coding technology, it has a great value in practical applications. Finally, some examples are used to numerically simulate the feasibility and security of the encryption algorithm.

The rest of this paper is organized as follows. Section 2 describes some basic definitions and fractional-order neural system. In Sect. 3, an color encryption algorithm based on fractional order neuron system and DNA sequence is designed. The feasibility and security of the encryption algorithm are verified by numerical experiments in Sect. 4. In Sect. 5, some conclusions are proposed.

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[*] Corresponding author. *E-mail address*: wangzuolei1971@163.com.

2 Preliminaries

In this section, we introduce the Caputo fractional order definition and fractional-order HR neuron system with memristor.

2.1 Definitions

Definition 1. [22]**.** The Caputo fractional derivative of order q for function $f(t)$ is defined by

$$D^q f(t) = \frac{1}{\Gamma(m-q)} \int_0^t \frac{f^m(\tau)}{(t-\tau)^{q-n+1}} d\tau,$$

where $m \in Z^+,\ t \geq 0,\ \mathrm{m}-1 < q < m$.

Moreover, when $0 < q < 1$,

$$D^q f(t) = \frac{1}{\Gamma(1-q)} \int_0^t f'(\tau)(t-\tau)^{-q}\, d\tau.$$

Definition 2. [22]**.** Mittag-Leffler function is defined by

$$E_q(t) = \sum_{m=0}^{\infty} \frac{t^m}{\Gamma(mq+1)}$$

where $q > 0$ and $z \in C$.

2.2 Fractional-order HR neuron system with memristor

The mathematical HR neuronal model under electromagnetic radiation with four differential equations can be described as follows [23]:

$$\begin{cases} \dot{x}_1 = ax_1^2 - bx_1^3 + x_2 - x_3 + I_{ext} - 0.2W(x_4)x_1, \\ \dot{x}_2 = c - dx_1^2 - x_2, \\ \dot{x}_3 = r(S(x_1 + 1.6) - x_3), \\ \dot{x}_4 = x_1 - kx_4, \end{cases} \tag{1}$$

where $x_1,\ x_2,\ x_3,\ I_{ext}$ describes the membrane potential, slow current, adaption current, and external input current, respectively. The variable x_4 expresses the magnetic flux which across the membrane of the neuron. $W(x_4)x_1$ calculates memory related the conductance of memristor. When the flux is changed, $W(x_4)x_1$ can estimate the effect of membrane potential, and $W(x_4) = \alpha + 3\beta x_4^2$ [24]. The parameters a, b, c, d, r, S and k are real constants.

Refer to the above model, the new fractional-order HR system with memristor according to (1) is described as

$$\begin{cases} D^q x_1 = ax_1^2 - bx_1^3 + x_2 - x_3 + I_{ext} - 0.2W(x_4)x_1, \\ D^q x_2 = c - dx_1^2 - x_2, \\ D^q x_3 = r(S(x_1 + 1.6) - x_3), \\ D^q x_4 = x_1 - kx_4\,, \end{cases} \tag{2}$$

Usually, in order to obtain the chaos generation, we set $a = 3,\ b = 1,\ c = 1,\ d = 5,\ \alpha = 0.4,\ \beta = 0.02,\ r = 0.009,\ S = 4,\ k = 0.5, q = 0.9$, than the simulation is done with the initial value (1, -1, 3, -3) to system (2).

To analyse the dynamical characteristics of the system (2), when q is fixed at 0.9. Fig.1 shows the bifurcation diagram, where the function of I_{ext} drawn at the maximum of x_1.

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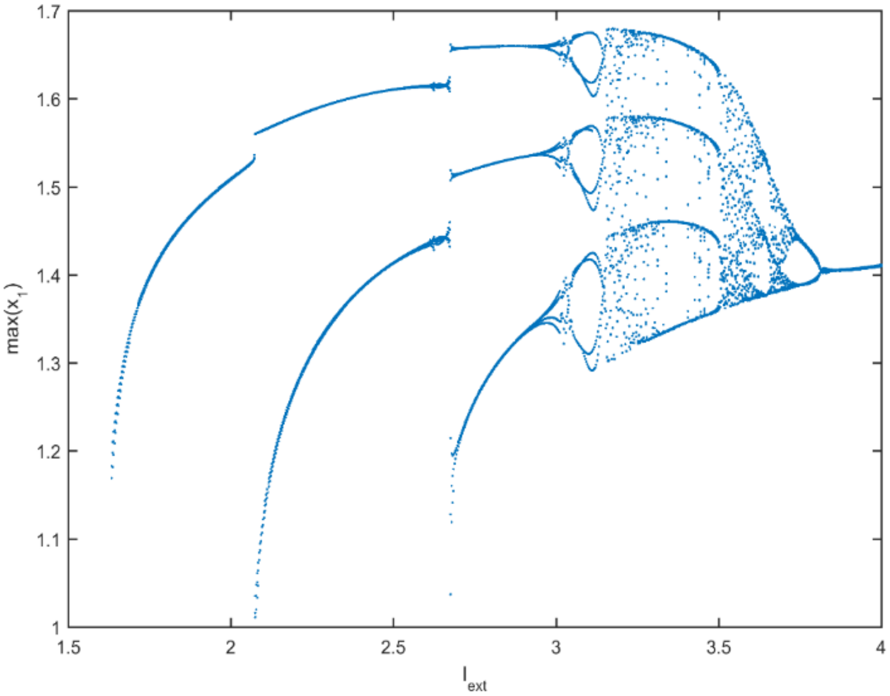

Fig. 1 Bifurcation diagram of a single fractional-order HR neuronal model under electromagnetic radiation, when $q = 0.9, 1 \leq I_{ext} \leq 4$

From the bifurcation diagram, we can see that when I = 3.6, the system (2) is a chaotic discharge state, as shown in Figure 2.

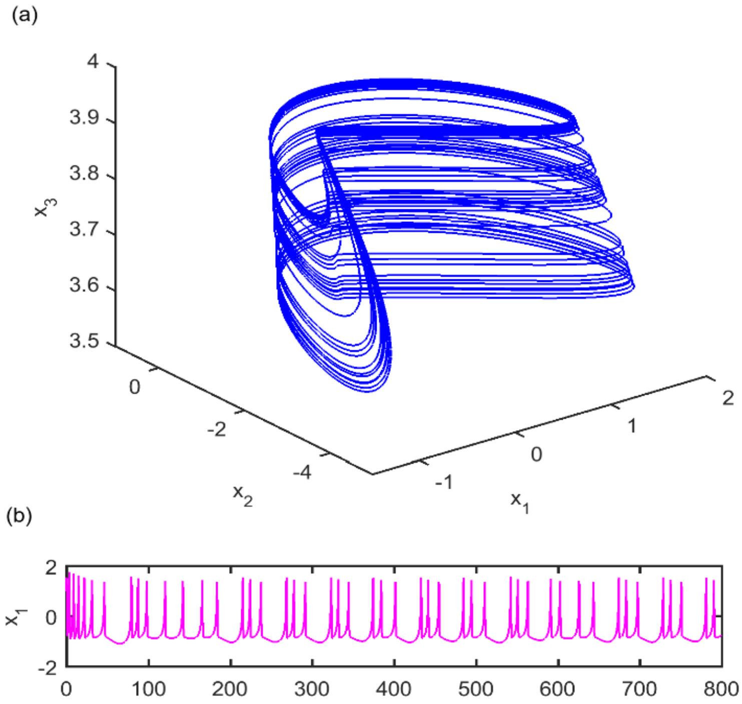

Fig. 2 Chaotic behaviour of the fractional-order HR neuronal model under electromagnetic radiation when $I_{ext} = 3.6$.

3 Image encryption algorithm

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In this section, according to the chaotic sequences of the state variables produced in section 2 with DNA coding, an image encryption method is put forward. Take the Lena image with the size of $M \times N$ and the pixel from 0 to 255 as an example. The steps of encryption are as follows:

Step 1. The plaintext color image is read to obtain an image size of $M \times N$, and the color image is decomposed into three components of R, G and B, and each of the components represents a pixel value matrix R_matrix， G_matrix and B_matrix of size $M \times N$, respectively.

Step 2. The chaotic random arrays $\left(x_1(i),\ x_2(i),\ x_3(i),\ x_4(i)\right)$ $(i = 1,2,3\cdots)$ generated by fractional-order memristive neuron system are merged into random sequence S with the size of $M \times N$. Then the range of the modified sequence is calculated by formula (3) in [0,255].

$$S\_l = (S \times 10^4)\ mod\ 255 \tag{3}$$

Finally, the S_l is transformed into a random matrix S_matrix of the same size as the plaintext color image.

Step 3. Divide R_matrix， G_matrix， B_matrix， S_matrix into equal blocks $R\_matrix(i)$， $G\_matrix(i)$， $B\_matrix(i)$， $S\_matrix(i)$. According to chaotic sequence $\left(x_1(i),\ x_2(i),\ x_3(i),\ x_4(i)\right)$, the following definitions are given:

(a) $x_1(i)$ represents DNA coding for each image $R\_matrix(i)$,$G\_matrix(i)$, $B\_matrix(i)$.

(b) $x_2(i)$ represents DNA coding for random matrix $S\_matrix(i)$.

(c) $x_3(i)$ denotes the DNA logic operation rules used in the calculation of each block.

(d) $x_4(i)$ denotes the decoding rules after each block operation. DNA coding rules and DNA operation rules are shown in Table 1-4.

Table 1 DNA coding rules

	1	2	3	4	5	6	7	8
A	00	00	10	10	01	01	11	11
C	01	10	00	11	00	11	01	10
G	10	01	11	00	11	00	10	01
T	11	11	01	01	10	10	00	00

Table 2 DNA addition Rule

	A	G	C	T
A	A	G	C	T
G	G	C	T	A
C	C	T	A	G
T	T	A	G	C

Table 3 DNA subtraction Rule

	A	T	C	G
A	A	G	C	T
T	T	A	G	C
C	C	T	A	G
G	G	A	T	A

Step 4. $R\_matrix(i)$， $G\_matrix(i)$， $B\_matrix(i)$， $S\_matrix(i)$ are coded according to the corresponding DNA coding methods (defined by $x_1(i)$ and $x_2(i)$). Then, according to DNA logic operation rules (defined by $x_3(i)$), $R\_matrix(i)$、$G\_matrix(i)$ and $B\_matrix(i)$ are calculated to get $R\_matrix(i)$ ciphertext matrix block, $G\_matrix(i)$ ciphertext matrix block and $B\_matrix(i)$ ciphertext matrix block.

Table 4 DNA XOR Rule

	A	T	G	C
A	A	T	G	C
T	T	A	C	G
G	G	C	A	T
C	C	G	T	A

Step 5. Converting $R\_matrix(i)$ ciphertext matrix block, $G\_matrix(i)$ ciphertext matrix block and $B\_matrix(i)$ ciphertext matrix block into pixel value matrix using decoding rules defined by $x_4(i)$, and then merged into R_matrix ciphertext matrix, G_matrix ciphertext matrix and B_matrix ciphertext matrix.

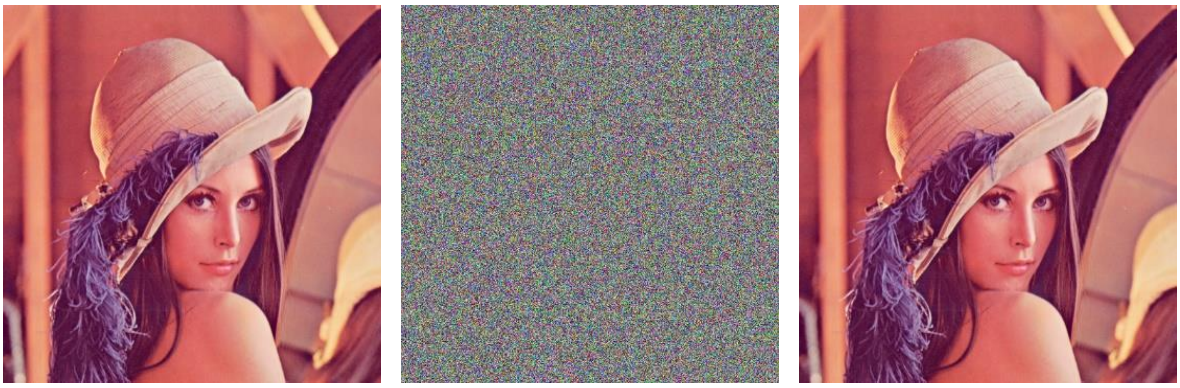
Step 6. R_matrix ciphertext matrix, G_matrix ciphertext matrix and B_matrix ciphertext matrix are merged into color ciphertext images.

Decryption of image is the inverse of the encryption process including DNA inverse decoding, DNA inverse operation, DNA inverse coding.

4 Simulations

In this section, we give some experimental results to illustrate the feasibility and security of the encryption algorithm obtained in the above subsection by MATLAB program.

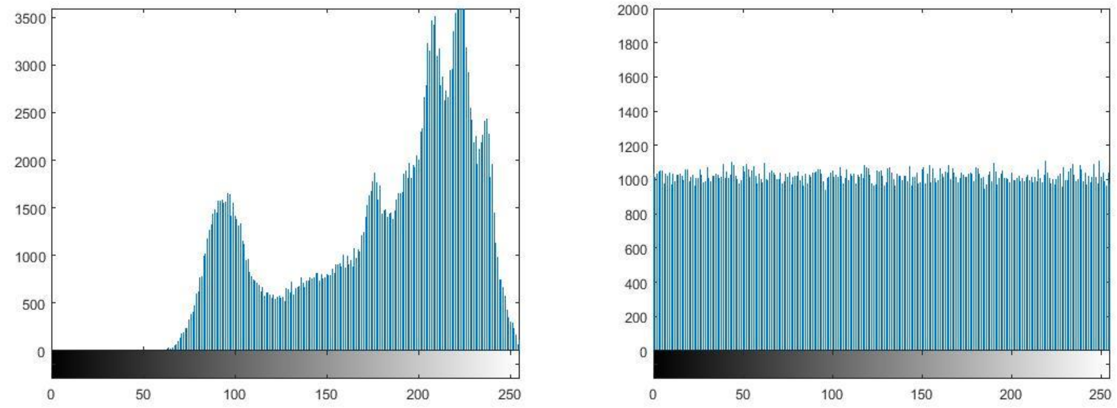
We use the initial values and parameters in Section 2.2, the typical Lena image is simulated in Figure 3, from which we know that the Lena image can be encrypted, and the encrypted image can be completely restored.



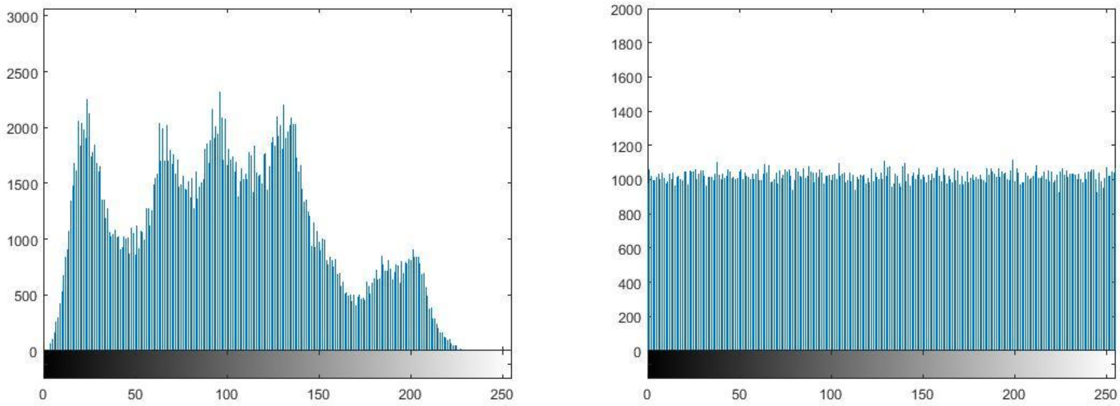
（a）Plaintext image （b）Encrypted image （c）Decrypted image

Fig. 3 The encryption and decryption results of Lena image

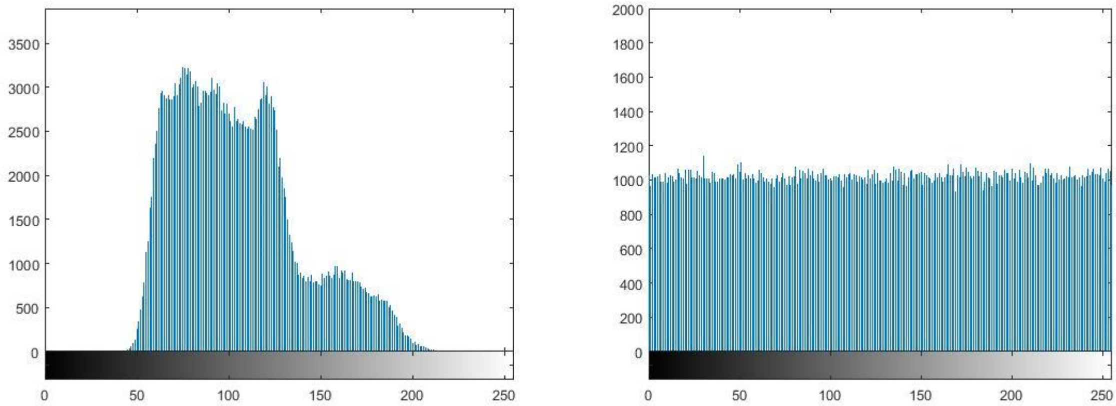
The key of our image encryption algorithm is composed of 9 parameters and 4 initial values, which can provide enough key space. On the other hand, image histogram is an important analysis standard of image encryption. In order to resist statistical attacks, a good encryption method is needed to obtain uniform histogram images which are obviously different from ordinary images. We performed statistical analysis by calculating the R, G and B component histograms of the color image lena (Fig. 4), respectively. From Figure 4, we can see that the histograms of the encrypted image are more evenly distributed, which is quite different from the histograms of the ordinary image. This means that the redundancy of the ordinary image is successfully hidden after encrypting.



（a）Plaintext image R component （b）R Component after Encryption



（c）Plaintext image G component （d）G Component after Encryption



（e）Plaintext image B component （f）B Component after Encryption

Fig. 4 Histogram of Lena image before and after encryption

5 Conclusion

In this paper, based on fractional-order memristive HR neuron system and DNA coding technology, a color image encryption scheme is proposed. The encryption algorithm not only has fast encryption speed and large key space, but also can resist certain statistical attacks by histogram analysis, which can ensure the security of digital image in the process of preservation and transmission.

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