ACADEMIC
World Academic Union

ISSN 1746-7659, England, UK
Journal of Information and Computing Science
Vol. 14, No. 2, 2019, *pp*.083-093

Energy Dependence of Modified Hindmarsh-Rose Neuron under Periodic Disturbance

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(*Received* January 10, *2019*, *accepted* March 20, *2019*)

Abstract: Due to the complex environment which neurons are located in, Hindmarsh-Rose neuron model is modified by introducing a new variable and linear coupling. Via numerical simulations, multiple modes of electric activities of the addressed neuron model can be observed by changing external forcing current. To explore the energy transition between different electric activity modes, Hamilton energy is calculated when the proposed neuron is disturbed by periodic signal. It is found that the energy is closely dependent on the electric activity mode caused by external forcing current. An interesting phenomenon is also obtained that the Hamilton energy is delayed by external forcing current, which means that the neuron plays an important role in energy coding.

Keywords: Hindmarsh-Rose neuron; Hamilton energy; periodic disturbance

1. Introduction

The biological nervous system is composed of a large number of nerve cells (neurons) connected to each other and it is provided with complex dynamic behavior. To study the dynamics of neuron accurately, on the basis of electrophysiological experiment of neuron, Hodgkin and Huxley established the well-known Hodgkin-Huxley (HH) model [1] in theory and revealed electrochemical mechanism of neurophysiological activity. Thereafter, to better describe rich discharge patterns of different neurons, some noted neurophysiological models have been improved or proposed, such as FitzHugh-Nagumo (FHN) model [2], Hindmarsh-Rose (HR) model [3], Morris-Lecar (ML) model [4], Chay model [5]. Some electric activities of above mentioned neuron model or improved neuron models have been discussed and the dynamical behaviors have been presented [6-14]. For example, several resonant behaviors different from the classical deterministic oscillators were reported [6]. Song [7] investigated the dynamic behaviors influenced by noise and pointed out the response of FitzHugh-Nagumo neuron to noise. The bifurcation of the Hindmarsh-Rose neuron model in a two-dimensional parameter space was discussed [8]. A Hindmarsh-Rose neuron model with nonlinear reset process is presented and the equilibrium point or the limit cycle of the proposed system is analyzed from qualitative aspect [9]. Bifurcations of invariant sets in a five-dimensional parameter space were studied by setting appropriate system parameters in a five-dimensional parameter space [10]. Three classes of Morris-Lecar neuron to sinusoidal inputs and synaptic pulselike stimuli with deterministic and random interspike intervals were studied and it was found that two class of neurons showed similar evolutions properties, which was different from another class of neuron [11]. Parameter regions for different firing patterns in the Chay neural model were obtained by analyzing the electric activities [12]. The transitions between different electric activity modes in Chay neuronal system were explored by depolarizing current [13] and different types of bursting in it were surveyed [14].

With the development of neurodynamics, more complicated dynamical behaviors have been found in neurons or neuron networks [15-25]. For instance, diverse behaviors of time-delay HR neuron was observed with external forcing current increasing [16] and the effect of external forcing current on electric activity of neuron under magnetic flow was discussed [17]. By analyzing the pattern formation of neurons, a result is obtained that the electric activity modes could be adjusted by altering the external forcing current [21, 22]. Dynamical characteristic in an isolated neuron with memristive synapses was investigated, which confirmed that the electrical activity mode can be controlled by synapse current [23]. By analyzing numerical simulations, complex dynamical behaviors of time-delay fractional-order coupled HR neurons under electromagnetic radiation were presented [24].The dynamics of a system of two coupled Fitzhugh-Nagumo neuron system was investigated and a narrow region of parameter space of particular interest, rich with chaotic and multistable dynamics was identified[25].

Existing results [26, 27] suggest that, with the change of neuron's electrical activity, Hamilton energy neuron holding varied. This is because the energy is dependent on the discharge modes of the neuron while the mode transition of electric activities in neuron is relative to energy encoding and energy metabolism [28, 29]. By defining a Hamilton energy function, the energy shift induced by transition of electric activity mode in Hindmarsh-Rose neuron was detected [26]. Further investigation verified that the membrane potential of a neuron is dependent on the transmembrane current [30-32].

Inspired by above mentioned results, considering the electromagnetic environment in which the neurons are located. A 4D neuron model is addressed by introducing magnetic flux as a new variable into Hindmarsh-Rose neuron model. And the electric activity of the proposed model and energy dependence on the mode are discussed under periodic disturbance. Other parts of this paper are arranged as follows. In Section 2, a 4D neuron model is described and the preliminary about Hamiton energy function is given. Section 3 depicts some numerical simulations to illustrate the electric activity mode and energy dependent on the mode of the proposed neuron model. Conclusions are drawn in Section 4.

2. Model description and preliminary

In this section, introduce magnetic flux as a new variable and use linear coupling, HR neuron model [3] can be modified as

$$\begin{cases}\dot{x} = y - ax^3 + bx^2 - z + I_{ext} - \alpha x - \beta w \\ \dot{y} = c - dx^2 - y \\ \dot{z} = r[s(x+1.6) - z] \\ \dot{w} = x - k_1 w\end{cases}, \tag{1}$$

where x, y , z are the membrane potential, the slow current for recovery variable, and the adaption current, respectively. I_{ext} is the external forcing current. α, β, k_1 are fixed parameters describing the interaction between membrane potential x and the new variable w.

Because of wide existence of biological electricity, almost all neurons are disturbed by external forcing current. In this paper, we assume that neuron model (1) is disturbed by external forcing current

$$I_{ext} = I + Asin(\omega t + \phi), \tag{2}$$

which is a periodic signal with amplitude A, angular frequency ω and initial phase ϕ. I is a constant. Then neuron model (1) can be rewritten as

$$\begin{cases}\dot{x} = y - ax^3 + bx^2 - z - \alpha x - \beta w + I + Asin(\omega t + \phi) \\ \dot{y} = c - dx^2 - y \\ \dot{z} = r[s(x+1.6) - z] \\ \dot{w} = x - k_1 w\end{cases}. \tag{3}$$

As we all know, the electric activity of neuron relies on the energy release and supply, that is to say, energy storage of neuron is dependent on the external forcing and energy release is related to the electric mode. Thence, it is necessary to explore the energy transition accompanied by electric activity mode of neuron, which is induced by changing the external forcing currents. According to Helmholtz theorem [32], dynamical equations of a neuron can be regarded as a sum of conservative field and dissipative field, that is

$$f(\cdot) = f_c(\cdot) + f_d(\cdot), \tag{4}$$

where $f_c(\cdot)$ is the conservative field consisting of full rotation and $f_d(\cdot)$ is the dissipative field involving the divergence. Therefore, neuron system (3) can be broken down into

$$f_c(x,y,z,w) = J(x,y,z,w)\nabla H \begin{pmatrix} y - z - \beta w + I + A\sin(\omega t + \phi) \\ c - dx^2 \\ rs(x+K) \\ x \end{pmatrix} \tag{5a}$$

and

$$f_d(x,y,z,w) = R(x,y,z,w)\nabla H = \begin{pmatrix} -ax^3 + bx^2 - \alpha x \\ -y \\ rz \\ -k_1 w \end{pmatrix}, \tag{5b}$$

where $J(\cdot)$ is an antisymmetric array satisfying Jacobi's closure condition and H is Hamiton energy function defined as

$$\nabla H^T f_c(x,y,z) = 0, \tag{6a}$$

$$\nabla H^T f_d(x,y,z) = dH/dt = \dot{H}. \tag{6b}$$

The Hamilton energy function can be gained as

$$H = \frac{2}{3}dx^3 - 2cx + \beta x^2 + rs(x+K)^2 + (y - z - \beta wx + I + A\sin(wt+\phi))^2, \tag{7}$$

and the change of H over time t can be calculated by

$$\begin{aligned}\dot{H} = &(2dx^2 - 2c + 2\beta x + 2rs(x+K))(ax^2 - bx^3 + \alpha x)\\ &-2(y - z - \beta w + I + A\sin(wt+\phi))y\\ &+2(y - z - \beta w + I + A\sin(wt+\phi))rz\\ &-2\beta(y - z - \beta w + I + A\sin(wt+\phi))k_1 w\end{aligned}.$$

3. Numerical simulations

In this section, system parameters are chosen as $a = 1$, $b = 3$, $c = 1$, $d = 5$, $r = 0.006$, $s = 4$, $\alpha = 0.004$, $\beta = 0.012$, $k_1 = 6.2$, and initial values of (3) are taken as x =-1.5, y =0.7, z =0.9, w=0.2. Fourth Runge-Kutta algorithm is used to resolve system (3). Time step is selected as Δh=0.01. Periodic disturbance is imposed on the proposed neuron to study the response and energy transition of it.

Without loss of generality, it is assumed that amplitude $A \neq 0$. To explain the reason for the production of multiple modes of neuron system, Hamiltonian function is applied, which provides a starting point in investigating the mechanism for the dynamics of chaotic systems. Therefore, in following discussions, as a useful tool, Hamilton energy corresponding to the electric activity mode is calculated and the main work is carried out for two cases.

Case 1 Initial phase $\phi = 0$

Firstly, take constant part I as parameter, the electric activities of system (3) along with corresponding Hamilton energy are calculated using Eq. (7) and given in Fig.1, which indicates that, if amplitude A and angular frequency ω are fixed as constants, while I is selected as different values ($I = 1.3$ and $I = 4$), multiple electric activity modes can appear in neuron system (3) and the energy dependent on the electric activity mode is closely related to it. It means that the energy varies with the change of membrane potential. Furthermore, it is obvious to see that higher energy can be reached under quiescent state while lower energy is for spiking or bursting state.

Secondly, fix ω and I, take amplitude A as different values, the electric activity of neuron system (3) and corresponding Hamilton energy are calculated and detailed results are presented in Fig.2, which confirmed that, whether amplitude is small or large, electric activities of neuron system (3) can alternate between different modes. Simultaneously, the Hamilton energy also changes with fluctuation. Furthermore, it is found that the alternation of energy shows certain delay with the change of amplitude of imposed signal.

Finally, take I and A as constants, while angular frequency ω is chosen as different values, the electric activity of neuron system (3) and Hamilton energy dependent on the discharge mode are calculated and drawn in Fig.3. Figure 3 verifies the multiple modes of neuron system (3) as well as energy transition between different modes. Therefore, by altering the angular frequency of periodic forcing current, suitable electric activities can also be achieved.

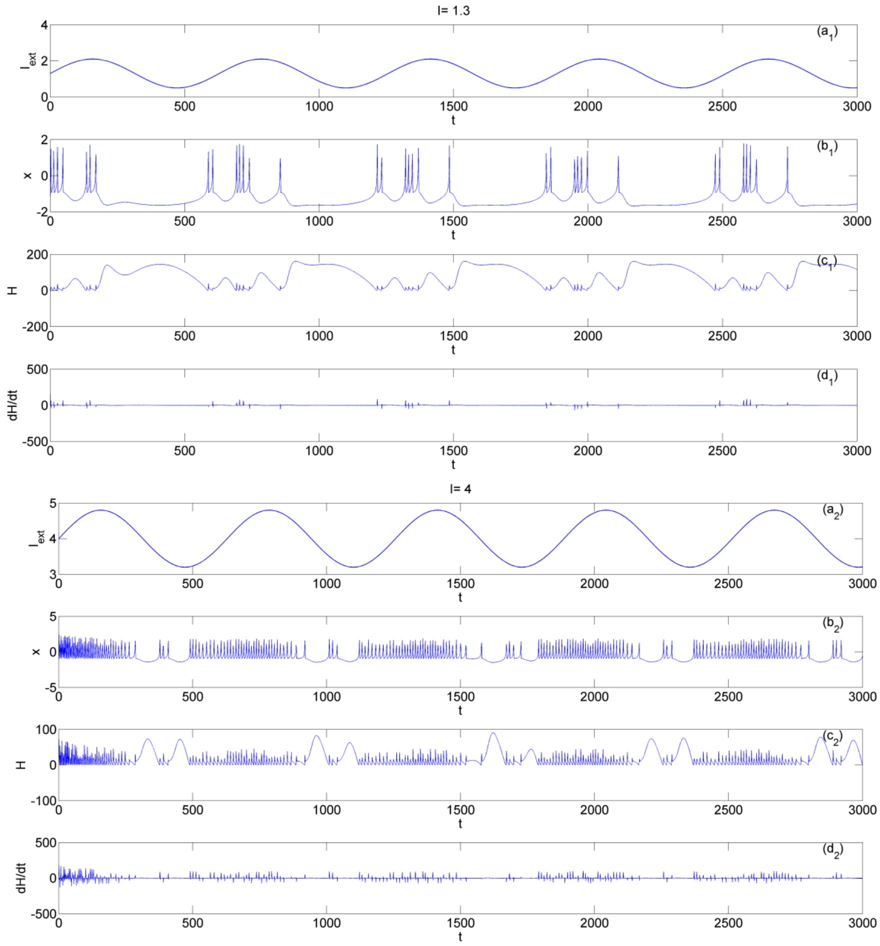

Fig.1 Electric activity of neuron system (3) and energy dependence of it for $\alpha = 0.004$, $\beta = 0.012$, $\omega = 0.01$, $A = 0.8$, and (a1,b1,c1,d1) I=1.3, (a2,b2,c2,d2) I=4.0.

Figs.1-3 suggest that when initial phase $\phi = 0$, constant part I, angular frequency ω and amplitude A all can make neuron system (3) appear multiple dynamics modes and result in energy transition between different modes.

Case 2 Initial phase $\phi \neq 0 (0 \leq \phi < 2\pi)$.

In this section, initial phase $\phi \neq 0$ is considered involving three examples for $\phi = \pi/4$, $\phi = \pi/2$ and $\phi = \pi$, respectively.

At first, take $\phi = \pi/4$, for different amplitude A, figure 4 illustrates that, both large amplitude and small amplitude can make neuron system (3) appear multiple electric activity modes along with energy transition. Higher energy is approached under quiescent state while spiking or bursting states make the neuron hold lower energy, which is similar to that for $\phi = 0$.

To further probe the dynamical behaviors of membrane potential in neuron system (3) and the energy transition with it, Figs.5-6 give the simulation results for $\phi = \pi/2$ and $\phi = \pi$, respectively.

From Figs.5-6, it is easy to know that, whether the amplitude is small or large, it can make neuron system (3) behave multiple electric activities with energy transition for low angular frequency (Fig.5) and high angular frequency (Fig.6).

From Figs.4-6, we know that, when initial phase $\phi \neq 0$, both angular frequency ω and amplitude A can make neuron system (3) show multiple electric activities along with energy transition.

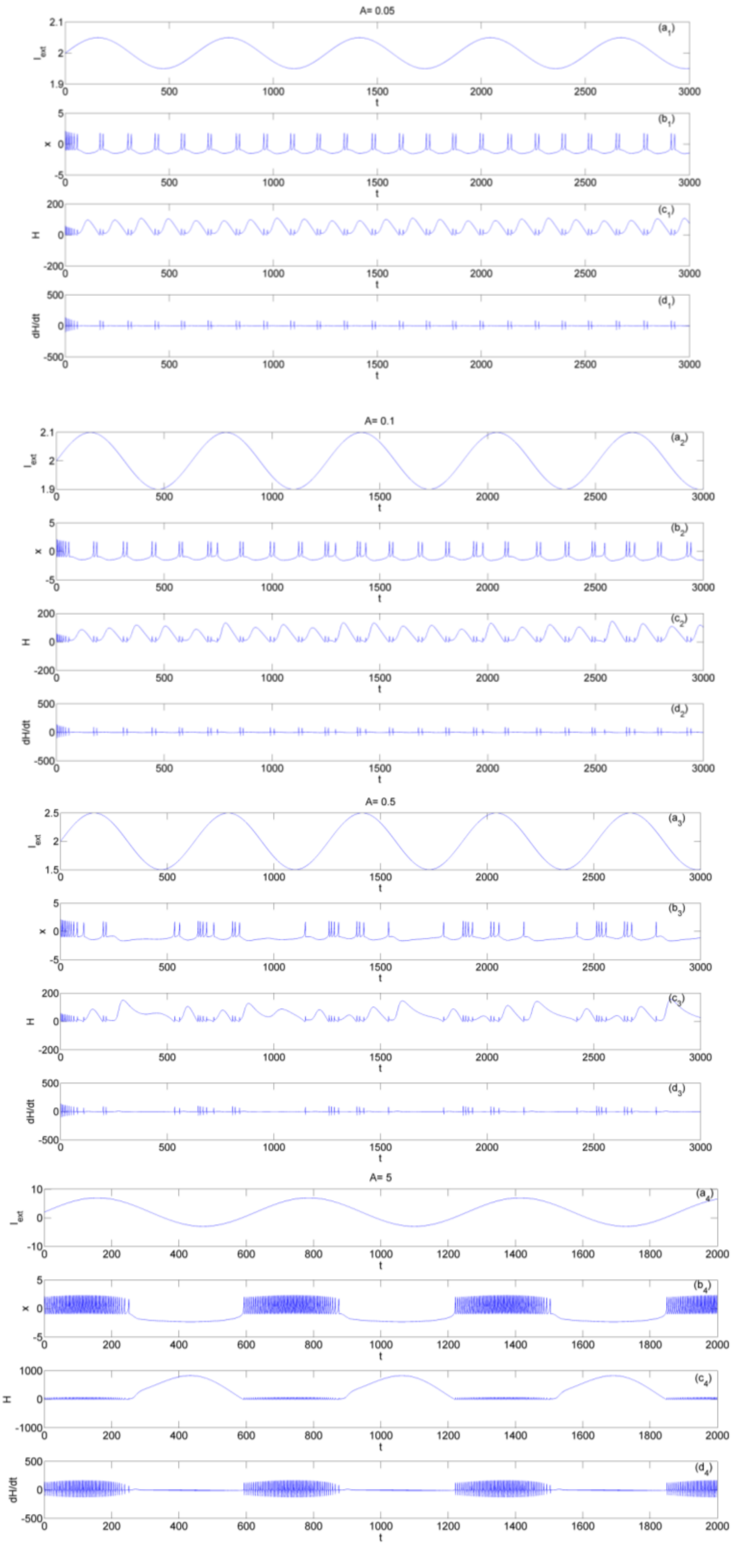

Fig.2 Electric activity of neuron system (3) and energy dependence of it for $\alpha = 0.004$, $\beta = 0.012$, $\omega = 0.01$, $I = 2.0$ and different values of A, (a1,b1,c1,d1) $A = 0.05$, (a2,b2,c2,d2) $A = 0.1$, (a3,b3,c3,d3) $A = 0.5$, (a4,b4,c4,d4) $A = 5$.

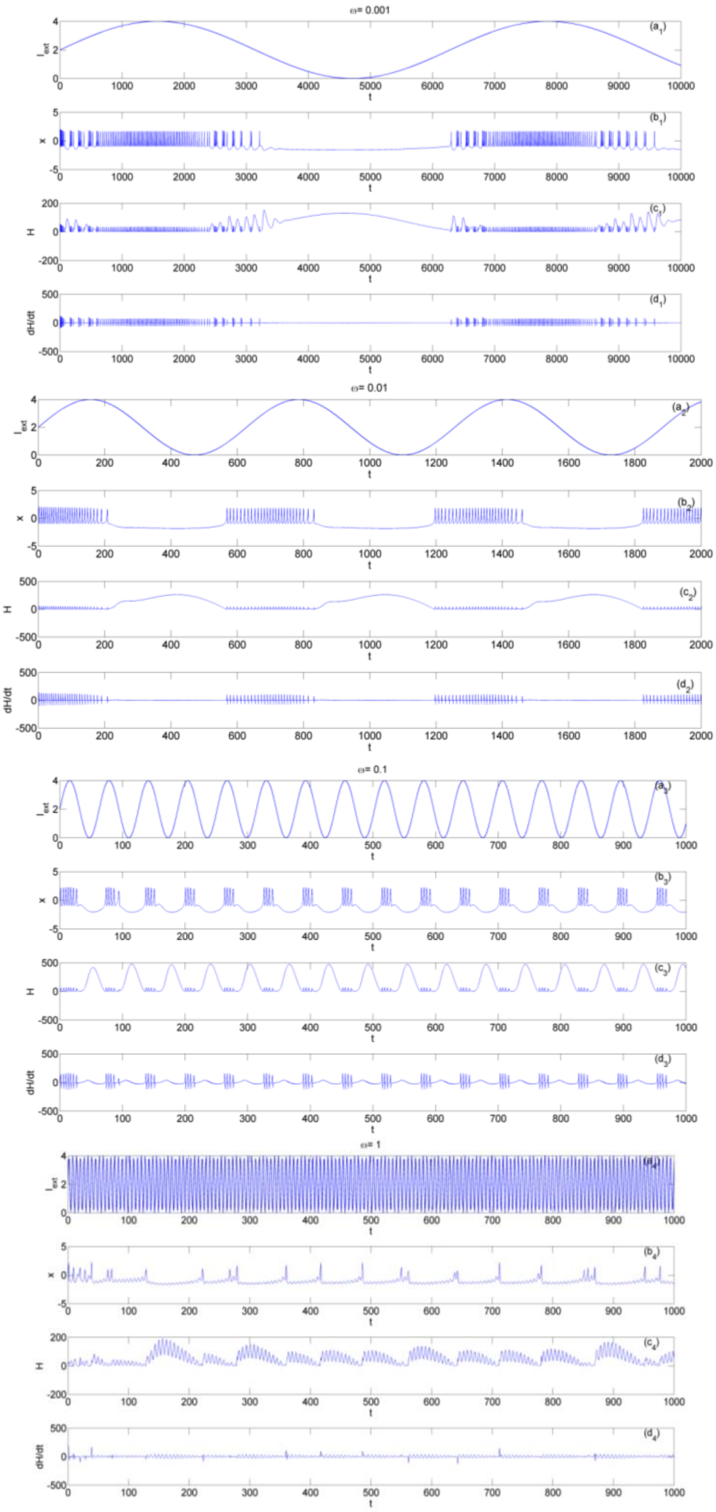

Fig.3 Electric activity of neuron system (3) and energy dependence of it for $\alpha = 0.004$, $\beta = 0.012$, $A = 2.0$, $I = 2.0$ and different values of ω, (a1, b1, c1, d1) $\omega = 0.001$, (a2, b2, c2, d2) $\omega = 0.01$, (a3, b3, c3, d3) $\omega = 0.1$, (a4, b4, c4, d4) $\omega = 1$.

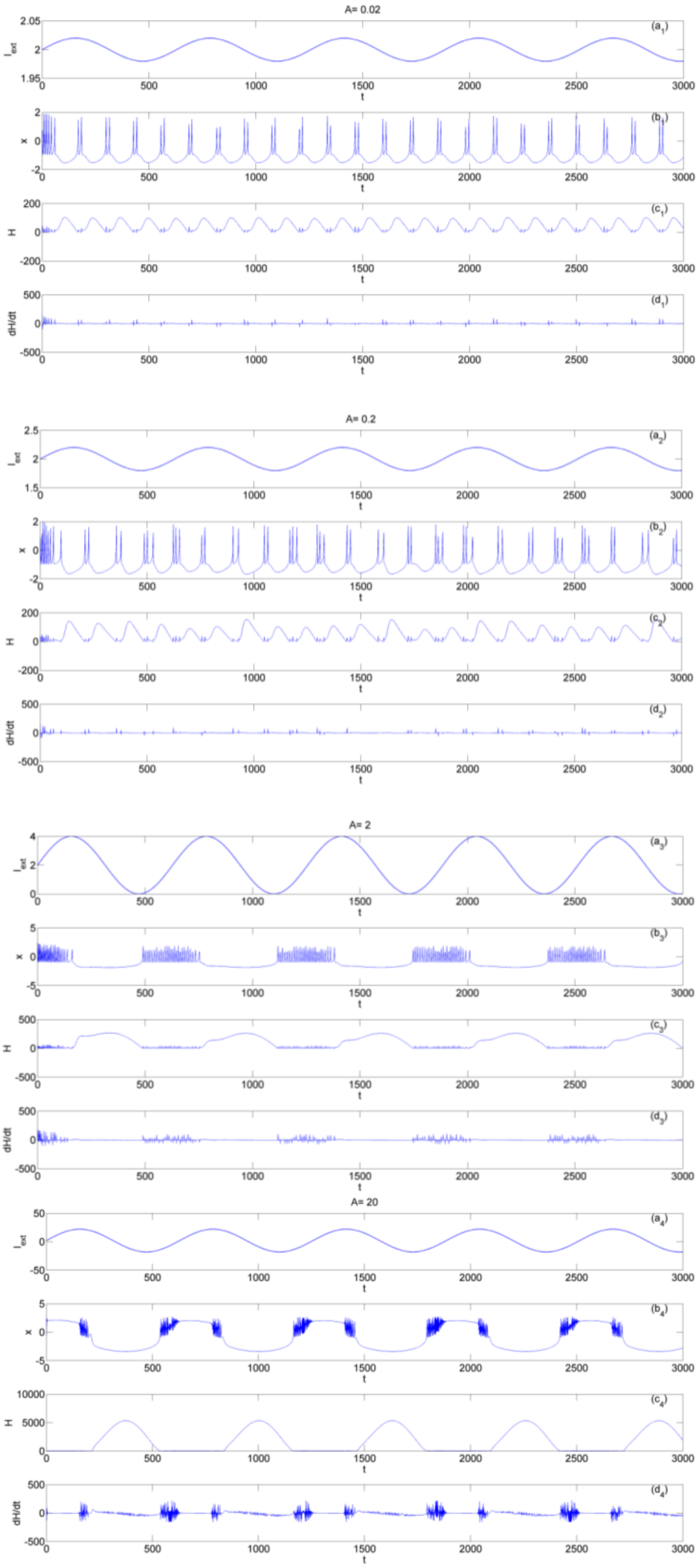

Fig.4 Electric activity of neuron system (3) and energy dependent on it for $\alpha = 0.004$, $\beta = 0.012$, $I = 2.0$, $\phi = \pi/4$, $\omega = 0.01$, and different values of A. (a1, b1, c1, d1) $A = 0.02$, (a2, b2, c2, d2) $A = 0.2$, (a3, b3, c3, d3) $A = 2$, (a4, b4, c4, d4) $A = 20$.

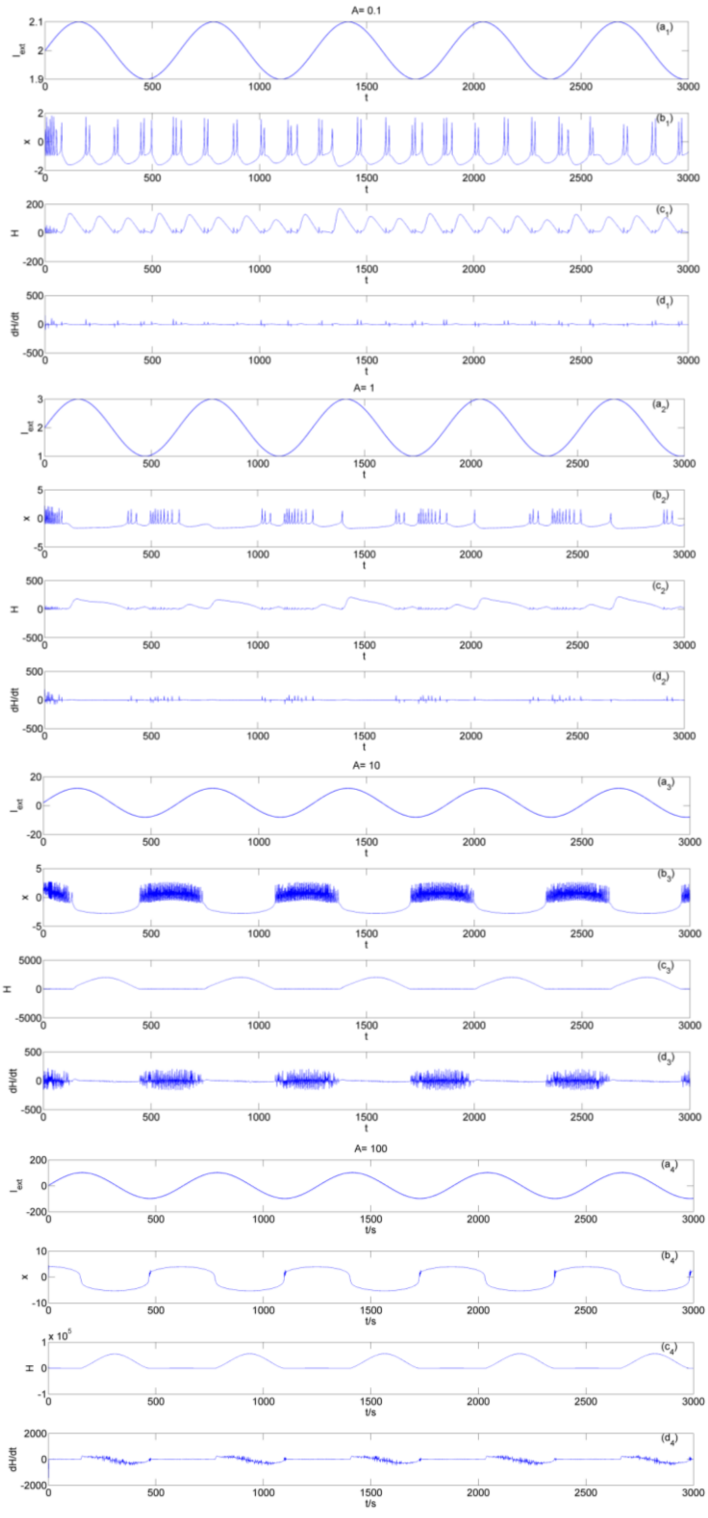

Fig.5 Electric activity of neuron system (3) and energy dependent on it for $\alpha = 0.004$, $\beta = 0.012$, $I = 2.0$, $\phi = \pi/2$, $\omega = 0.01$ and different values of A, (a1,b1,c1,d1) $A = 0.1$, (a2,b2,c2,d2) $A = 1$, (a3,b3,c3,d3) $A = 10$, (a4,b4,c4,d4) $A = 100$.

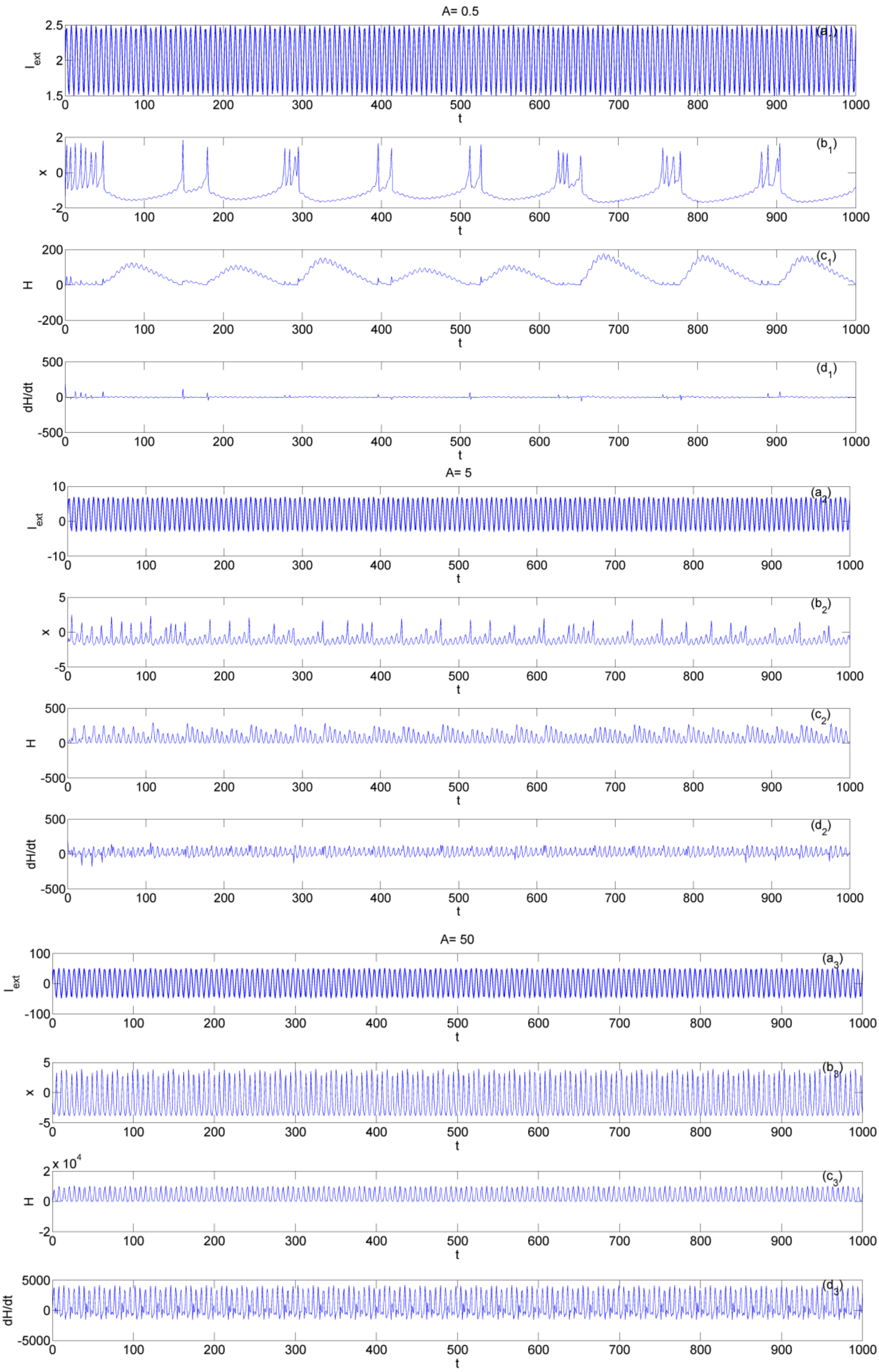

Fig.6 Electric activity of neuron system (3) and energy dependent on it for $\alpha = 0.004$, $\beta = 0.012$, $I = 2.0$, $\phi = \pi$, $\omega = 1$ and different values of A, (a1, b1, c1, d1) $A = 0.5$, (a2, b2, c2, d2) $A = 5$, (a3, b3, c3, d3)$A = 50$.

4. Conclusions

Based on HR neuron model, a modified HR neuron model is proposed. The electric activities of it are discussed and energy transition with electric activity is analyzed when the addressed neuron model is disturbed by external periodic signal. Following results are obtained.

a) Whether initial phase $\phi = 0$ or not, external periodic signal can lead to multiple electric activity modes of the proposed neuron system.

b) The Hamilton energy is closely dependent on the electric activity mode caused by selecting suitable amplitude or angular frequency of periodic signal. Higher energy is approached under quiescent state while spiking or bursting states make the neuron hold lower energy.

c) The Hamilton energy is delayed by external forcing current, which means that the neuron plays an important role in energy coding.

d) The electric activity of neuron and energy transition can be controlled by selecting appropriate parameters of the external periodic signal.

As the energy of neurons is closely related to many characteristics of itself. These research results may be beneficial for further exploring the dynamic behaviors of neuron system and the mechanism behind these dynamic behaviors.

Acknowledgments

This work is supported by National Natural Science Foundation of China (Grant Nos. 11872327 and 51777180).

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