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Diffusion of Investor Sentiment Considering Hesitating and Forgetting Mechanism

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Abstract: Excessively optimistic and negative investor sentiment will affect the stability of stock market. In this paper, we investigate investor sentiment diffusion considering hesitating and forgetting mechanism on homogeneous network. Different from previous studies, we introduce the exponent form forgetting rate into SEIR model and figure out the basic reproduction number and existence of equilibrium point. Meantime, the locally asymptotic stability and global stability of internal equilibrium point are established. Finally, we illustrate the impact of forgetting rate on the investor sentiment diffusion through carrying out numerical simulations.

Keywords: Investor sentiment, SEIR model, Forgetting mechanism.

1. Introduction

According to individual investors report released by the Shenzhen stock exchange in 2015, China's securities market is dominated by small and medium investors. Minority investors who lack risk aversion are emotionally vulnerable and their abilities to judge the market are not accurate [1,2]. Their overconfidence tends to result in their incorrect judgment of the market. However, classical economic theory believes that investors' economic behavior is " rational", which obviously does not apply to China's securities market. In China's securities market, even if the individual investor's personal education level, educational background and life experience are different, the individual investors' sentiments agree with common option gradually when investors are concerned about the same event due to the spreading of investor emotion infection. It is more convenient for investors to communicate with other people because of the development of social media which also accelerates the diffusion of investor sentiment infection process. In addition, behavioral economics shows that investors' emotions and psychological factors affect stock market prices and stability to a large extent [3]. Therefore, it is helpful to study the mechanism and ways of investor's emotional transmission in order to ensure the stable development of the stock market.

Several researchers have carried out relevant works to uncover the mechanism of investors' emotion spreading. Shiller et al utilized infectious diseases spreading model and rumor model to establish interest transmission model, explaining why investors were interested in a particular asset in the financial market [4]. Lux et al described the herd behavior of investors and the process of mutual imitation and contagion with nonlinear dynamics method [5]. Through the simulation and causality of investor psychology and stock market, Shang-Jun Y found investors' psychology may seriously affect the stability of stock market and the trend intensity of stock market is linearly related to investor psychology [6]. The stock market can also be seen as a typical complex network of different types of investors. Garas studied SIR model of crisis propagation in country-based economic network, which shows that economic crisis in developed and developing countries can spread to the whole world through the network, causing global economic crisis [7]. Xu-Chong G used multi-agent technology and complex network theory to analyze the small-world network, rule network and real-world market emotion transmission model [8]. Yuan-Yuan M built SIR epidemic model to demonstrate the crisis communication in the stock market [9]. What is more, many researchers have improved the classic SIS and SIRS infectious disease models [10]. For instance, Tchuenche et al. considered the effects of unstable birth and death rates on the population which was more realistic [11]. Jun-Hong L et al considered the non-linear transmission process of infection rate and cure rate [12-14].

However, there are great differences between traditional infectious diseases spreading and investor sentiment diffusion. Firstly, the spreading of infectious diseases is unconscious because infected people cannot stop disease or virus spreading. Obviously, in the process of investor sentiment contagion, people's psychological factors such as forgetting, interests and so on, have a great influence on the transmission of emotion. Meanwhile, because the development of network media makes people accept a large amount of information every day, forgetting has become an important factor in the process of information and sentiment diffusion. In previous studies, the forgetting rate was regarded as a constant parameter, but the forgetting speed of human beings should be ‘fast before slow’, so the constant forgetting rate does not accord with the actual situation. Considering Ebbinghaus's forgetting data curve, the index forgetting rate fits the actual situation. Besides, previous studies neglected the population diversity. Compared with SIR model, SEIR model can illustrate the investor emotion contagion more reasonably. This is because people are more likely to stay the sidelines before participating in the stock market. For example, if the market keeps going on, several investors may be optimistic and attempt to persuade their friends or relatives who may hesitate for a while rather than engaging in the stock market immediately. Hence, a dynamic model of investor sentiment diffusion is proposed considering hesitating and forgetting mechanism [15]. The organization of this paper is managed as follows. In the section 2, we establish the mean-filed equation about investor sentiment diffusion. In section 3, we study the model by calculating the basic reproduction number and analyzing the stability of equilibrium point. In the section 4, numerical simulation of $SEIR$ model is given to display the diffusion of investor sentiment [16]. Finally, we conclude the paper in section 5.

2. SEIR mode

The investors are divided into four groups: S,E,$I$and R, which stand for the investors who have adequate funding but do not open accounts(potential investor), the investors who have opened accounts but hesitate to transact(hesitate investor), the investors who are excitedly and actively spread emotion(active investor), and investors who are calm and do not spread investor sentiment(removed or calm investor) respectively. The process of $SEIR$ investor sentiment spreading is shown in Fig.1.

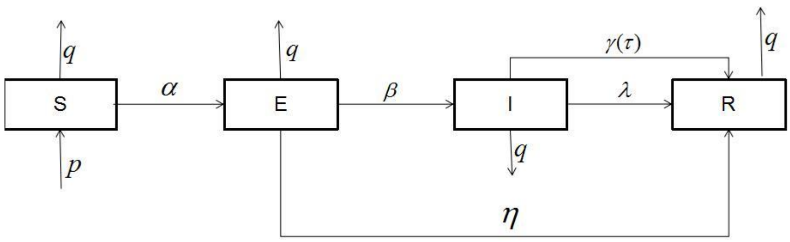


Fig. 1: The structure of $SEIR$ investor sentiment diffusion model.

When potential investor(S) meets his neighboring node(I), because of his knowledge of the stock market, the former open his stock account with probability $\alpha$and do not instantly begin stock transaction. According to the attitude of investor sentiment, hesitate investor(E) starts stock market transactions with probabilityβ. On the contrary, hesitate investor (E) who disagrees with the sentiment exits the stock market with probabilityη. The active investor(I) exit the stock market with probability λ. As time goes on, the investor will forget the emotion, calm down gradually and exit the stock market with probability $\gamma(\tau)$ which we define as the forgetting rate. In this paper, we use exponential form to represent forgetting rate[17,18]:

$$\gamma(\tau)=\begin{cases} a-e^{-b\tau}, & 0<a-e^{-b\tau}<1 \\ 1, & a-e^{-b\tau}\geq 1 \end{cases} \tag{1}$$

a,b are parameters which embody the feature of forgetting curve. When $\tau=0$, $\gamma(0)=a-1$ is constant which we define as the initial forgetting rate. When$\tau\neq 0$, the forgetting rate changes over time. If the value of $a-1$ is larger, the investor sentiment is less attractive to investors and investors are less interested in spreading it. Therefore, a reflects the investor’s interest on investor sentiment. b that decides forgetting curve’s shape denotes the forgetting speed which means that forgetting speed becomes larger as b gets larger [19]. Denote by $S(t)$, $E(t)$,$I(t)$, $R(t)$ the density of potential investors, hesitate investors, active investors, and calm investors at time $t$respectively, they satisfy the normalization condition: $S(t)+E(t)+I(t)+R(t)=1$. Based on the above condition of hypothesis, the system dynamics equations are described as follows:

$$\begin{aligned}
\frac{dS(t)}{dt} &= p - \frac{a\bar{k}S(t)I(t)}{1+\delta I(t)} - pS(t) \\
\frac{dE(t)}{dt} &= \frac{a\bar{k}S(t)I(t)}{1+\delta I(t)} - (\eta+\beta+p)E(t) \\
\frac{dI(t)}{dt} &= \beta E(t) - (\lambda+\gamma+p)I(t) \\
\frac{dR(t)}{dt} &= \eta E(t) + (\lambda+\gamma)I(t) - pR(t)
\end{aligned} \tag{2}$$

We make the following assumptions:

(1)The inflow rate is equal to the outflow rates, thus $p = q$.

(2)The stock network is a homogeneous network and the average degree of freedom is $\bar{k}$.

(3)Considering that individual contact ability is limited, we use a saturated incidence:

$$\frac{\alpha\bar{k}S(t)I(t)}{1+\delta I(t)}.[20]$$

3. Theoretical analysis

3.1. Basic reproduction number

Following van den Driessche and Watmough[21], we note that only compartments $E(t)$,$I(t)$ are involved in the calculation of R_0 . We define $F$is the rate of appearance of new infections and V is the rate of transfer of individuals out of the two compartments. Based on the system equation, we can obtain:

$$F = \begin{bmatrix} 0 & \alpha\bar{k} \\ 0 & 0 \end{bmatrix} \tag{3}$$

and

$$V = \begin{bmatrix} \eta+\beta+p & 0 \\ -\beta & \lambda+\gamma+p \end{bmatrix} \tag{4}$$

Hence,

$$R_0 = \rho(F \cdot V^{-1}) = \frac{\alpha\bar{k}}{(\eta+\beta+p)(\lambda+\gamma+p)} \tag{5}$$

where ρ represents the spectral radius of the matrix $F \cdot V^{-1}$.

3.2. The stability of equilibrium points

According to the spreading dynamic equation, we calculate equilibrium (S, E, I, R). We discover that the model possesses two equilibrium point: $E_0 = (1,0,0,0)$, $E_* = (X, Y, U, V)$, where

$$\begin{aligned} X &= \frac{p\beta\delta + (\eta+\beta+p)(\lambda+\gamma+p)}{\beta(\alpha\bar{k}+\delta p)} \\ Y &= \frac{p(\alpha\beta\bar{k} - (\eta+\beta+p)(\lambda+\gamma+p))}{\delta(\alpha\bar{k}+\delta p)(\eta+\beta+p)} \\ U &= \frac{p(\alpha\beta\bar{k} - (\eta+\beta+p)(\lambda+\gamma+p))}{(\alpha\bar{k}+\delta p)(\eta+\beta+p)(\lambda+\gamma+p)} \\ V &= \frac{p\eta(\alpha\beta\bar{k} - (\eta+\beta+p)(\lambda+\gamma+p))}{\beta(\alpha\bar{k}+\delta p)(\eta+\beta+p)(\lambda+\gamma+p)} \end{aligned} \tag{6}$$

The Jacobian matrix of the system is

$$J = \begin{bmatrix} -\frac{\alpha\bar{k}I(t)}{1+\delta I(t)} - p & 0 & \frac{\alpha\delta\bar{k}S(t)I(t)}{(1+\delta I(t))^2} - \frac{\alpha\bar{k}S(t)}{1+\delta I(t)} & 0 \\ \frac{\alpha\bar{k}I(t)}{1+\delta I(t)} & -(\eta+\beta+p) & -\frac{\alpha\delta\bar{k}S(t)I(t)}{(1+\delta I(t))^2} + \frac{\alpha\bar{k}S(t)}{1+\delta I(t)} & 0 \\ 0 & \beta & -(\lambda+\gamma+p) & 0 \\ 0 & \eta & \lambda+\gamma & -p \end{bmatrix} \tag{7}$$

Theorem 1 The equilibrium point E_0 is locally asymptotically stable if $R_0 < 1$ and unstable if $R_0 > 1$.

Proof. The jacobian matrix at the equilibrium point E_0 is:

$$J(E_0) = \begin{bmatrix} -p & 0 & -\alpha\bar{k} & 0 \\ 0 & -(\beta+\eta+p) & \alpha\bar{k} & 0 \\ 0 & \beta & -(\lambda+\gamma+p) & 0 \\ 0 & \eta & \lambda+\gamma & -p \end{bmatrix} \tag{8}$$

We describe the characteristic equation of matrix $J(E_0)$ as

$$\left|J(E_0) - \mu\right| = \begin{bmatrix} -p-\mu & 0 & -\alpha\bar{k} & 0 \\ 0 & -(\beta+\eta+p)-\mu & \alpha\bar{k} & 0 \\ 0 & \beta & -(\lambda+\gamma+p)-\mu & 0 \\ 0 & \eta & \lambda+\gamma & -p-\mu \end{bmatrix} \tag{9}$$

From the above model, we can obtain two characteristic values $\mu_{1,2} = -p < 0$. Then we can establish a polynomial to judge the others characteristic roots of Jacobi matrix$J(E_0)$.

$$\phi_2\mu^2 + \phi_1\mu + \phi_0 = 0$$

where$\phi_2 = 1 > 0, \phi_1 = [(\eta+\beta+p) + (\lambda+\gamma+p)] > 0, \phi_0 = (\eta+\beta+p)(\lambda+\gamma+p) - \alpha\beta\bar{k}$.

If $R_0 < 1$ we can obtain $\phi_2 > 0$, $\phi_1 > 0$, $\phi_0 > 0$. According to the Routh-Hureitz stability judgement, the equilibrium point E_0 is locally asymptotically stable if $R_0 < 1$ and unstable if $R_0 > 1$.

Theorem 2 If $R_0 < 1$,the equilibrium point E_0 is globally asymptotic stable.

Proof. We definite a Lyapunov function $L(t) = a_1E(t) + a_2I(t)$

$$
\begin{aligned}
\frac{dL(t)}{dt} &= a_1 \frac{dE(t)}{dt} + a_2 \frac{dI(t)}{dt} \\
&= a_1[\alpha\bar{k}\frac{S(t)I(t)}{1+\delta I(t)} - (\eta+\beta+p)E(t)] + a_2[\beta E(t) - (\lambda+\gamma+p)I(t)] \\
&= a_1\alpha\bar{k}\frac{S(t)}{1+\delta I(t)} I(t) - a_2(\lambda+\gamma+p)I(t) + a_2\beta E(t) - a_1(\eta+\beta+p)E(t) \\
&\le a_1\alpha\bar{k}\frac{S(t)}{1+\delta I(t)} - a_2(\lambda+\gamma+p) + a_2\beta - a_1(\eta+\beta+p) \\
&\le a_1\alpha\bar{k} - a_2(\lambda+\gamma+p) + a_2\beta - a_1(\eta+\beta+p) \\
&= a_2(\lambda+\gamma+p)(\frac{a_1}{a_2}\frac{\alpha\bar{k}}{\lambda+\gamma+p} - 1) + a_2(\eta+\beta+p)(\frac{a_2}{a_1}\frac{\beta}{\eta+\beta+p} - 1)
\end{aligned}
\tag{10}
$$

We put $\frac{a_1}{a_2} = \frac{\beta}{\eta+\beta+p}$ into the formula, then we have

$$
\frac{dL(t)}{dt} \le a_2(\lambda+\gamma+p)(R_0 - 1) \tag{11}
$$

If $R_0 < 1$, then it is clear that we have $\frac{dL(t)}{dt} < 0$.

Theorem 3 if $\frac{(\alpha\bar{k}-\lambda-\gamma)\beta}{p(p+\beta+\eta+\lambda+\gamma)+\eta(\lambda+\gamma)} > 1$, E_* is locally asymptotically stable.

Proof. The Jacobi matrix at the equilibrium point E_* can be written as:

$$
J(E_*) = \begin{bmatrix}
-\frac{\alpha\bar{k}U}{1+\delta U} - p & 0 & \frac{\alpha\delta\bar{k}XU}{(1+\delta U)^2} - \frac{\alpha\bar{k}X}{1+\delta U} & 0 \\
\frac{\alpha\bar{k}U}{1+\delta U} & -(\eta+\beta+p) & -\frac{\alpha\delta\bar{k}XU}{(1+\delta U)^2} + \frac{\alpha\bar{k}X}{1+\delta U} & 0 \\
0 & \beta & -(\lambda+\gamma+p) & 0 \\
0 & \eta & \lambda+\gamma & -p
\end{bmatrix} \tag{12}
$$

Then, we proof $J(E_*)$ is the negative definite matrix. Considering the order principal minor determinant of $J(E_*)$:

$$D_1 = \left| -\frac{\alpha \bar{k} U}{1+\delta U} - p \right| = -\frac{\alpha \bar{k} U}{1+\delta U} - p < 0,$$

$$D_2 = \begin{vmatrix} \left| -\frac{\alpha \bar{k} U}{1+\delta U} - p \right| & 0 \\ \frac{\alpha \bar{k} U}{1+\delta U} & -(\eta+\beta+p) \end{vmatrix} = D_1(-(\eta+\beta+p)) > 0,$$

$$D_3 = \begin{vmatrix} -\frac{\alpha \bar{k} U}{1+\delta U} - p & 0 & \frac{\alpha\delta \bar{k} X U}{(1+\delta U)^2} - \frac{\alpha \bar{k} X}{1+\delta U} \\ \frac{\alpha \bar{k} U}{1+\delta U} & -(\eta+\beta+p) & -\frac{\alpha\delta \bar{k} X U}{(1+\delta U)^2} + \frac{\alpha \bar{k} X}{1+\delta U} \\ 0 & \beta & -(\lambda+\gamma+p) \end{vmatrix} \tag{13}$$

$$= \frac{(\alpha\bar{k}+\delta p)(-p^2+(-\beta-\eta-\lambda-\gamma)p+(\alpha\bar{k}-\lambda-\gamma)\beta-\eta(\lambda+\gamma))p(\beta+\eta+p)(\lambda+\gamma+p)}{\alpha\bar{k}(p^2+((\delta+1)p+\eta+\lambda+\gamma)p+(\lambda+\gamma)(\beta+\eta)}$$

$$D_4 = J(E_*) = -pD_3$$

While $\frac{(\alpha\bar{k}-\lambda-\gamma)\beta}{p(p+\beta+\eta+\lambda+\gamma)+\eta(\lambda+\gamma)} > 1$,based on the determination of negative definite matrix, $J(E_*)$ is negative definite matrix and the equilibrium point E_* is locally asymptotically stable.

4. Numerical stimulation

In this section, theoretical model is validated by using numerical simulation. This study is based on the homogeneous network thus we assume the number of individuals is $N = 100000$.

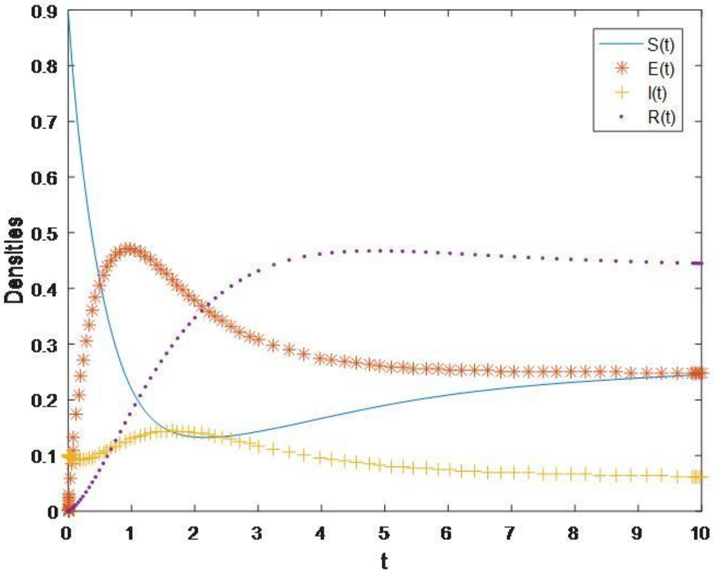


Fig. 2: The density of four groups over time at E_*.

Firstly, we assume there is only one active investor in the network and the others are potential investors. In the network, the average degree is 50 and the initial forgetting parameters $a = 1.2, b = 0.1$. Because we will study how the forgetting rate affects the dynamics behaviors of system, we control all the parameters of model are equal to 0.4 which can satisfy the local asymptotic stability condition of E_* in Fig.2. The quantity of $S(t)$ witnesses a dramatic decrease initially and increases slightly while other groups increasing to some extend and reach the stable state at the same time generally.

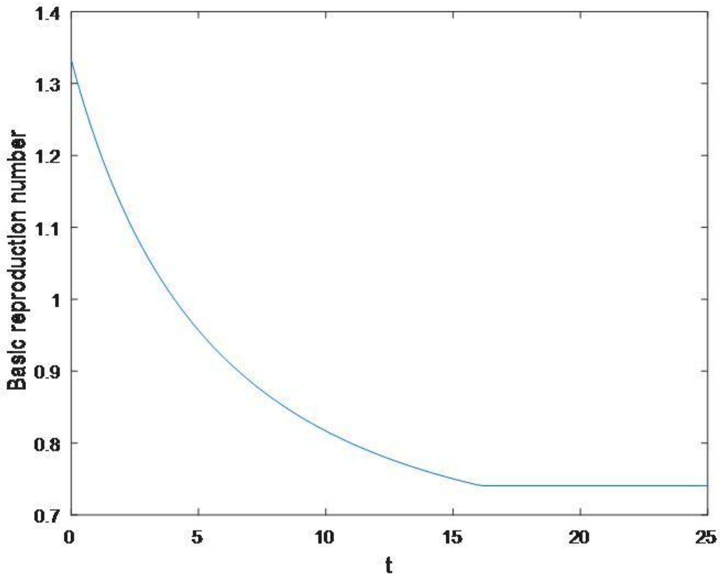
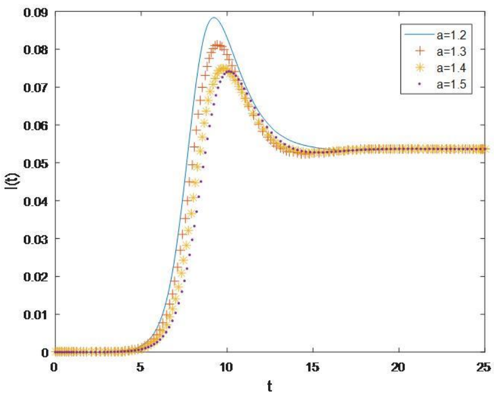
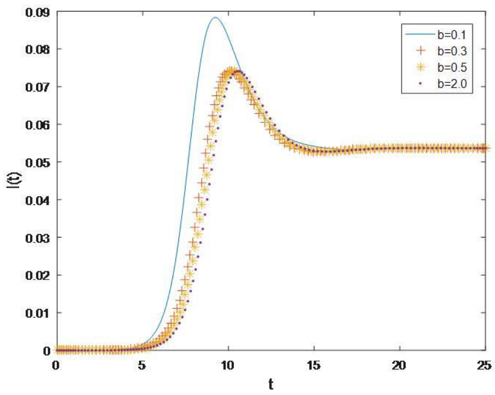


Fig. 3: The basic reproduction number changes over time when $a = 1.2$,$b = 0.1$,$\bar{k} = 10$.

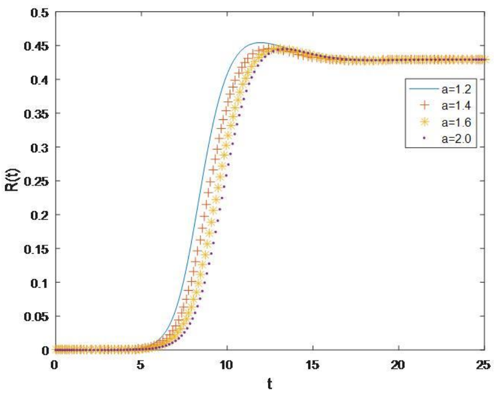
In the Fig.3, we obtain the basic reproduction number become less than 1 over time and then remain constant by decreasing the average degree from 50 to 10. This means even though there are large numbers of excited spreaders in the stock network in the beginning, the group of investor sentiment spreader will be extinct finally.



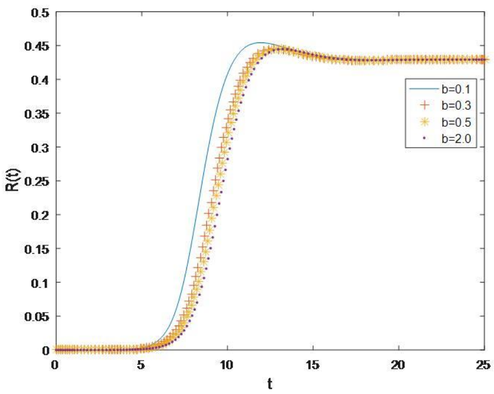
(a)



(c)



(b)

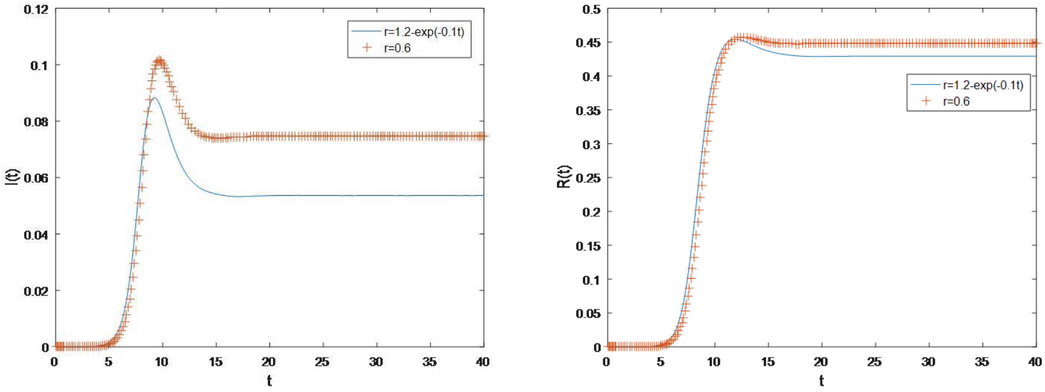


(d)

Fig. 4: (a),(c) represents the densities of $I(t)$ over time with different a and b.

(b),(d) shows the densities of $R(t)$ over time with different a and b

The above Fig.4 shows how the density of $I(t)$, $R(t)$ changes over time with different a, b,respectively. Different a, $b$affect the dynamical process, however they do not affect the final steady state of each class. When the value of a reaches to the critical value, it has little influence on the densities of $I(t)$ and $R(t)$. When it comes to the value of b, the same situation happened. It implies we can control the investor sentiment diffusion through controlling the value of a and b. What is more, it is very reasonable that the officials or some institutions can create some hot news to attract the public's attention in order to increase the value of a,b and decrease the effect of investor emotion contagion simultaneously.



(a) (b)

Fig. 5: (a),(b) represents the densities of $I(t)$,$R(t)$ over time with different forgetting rate respectively.

Furthermore, we study the influence of the forgetting mechanism. In the Fig.5, we compare the variable forgetting rate with constant rate through studying how the densities of $I(t)$, $R(t)$ change over time respectively. The density of $I(t)$ arrives to higher peak with constant forgetting rate than variable forgetting rate. The steady state of $I(t)$,$R(t)$ with the variable forgetting rate is smaller than the state with constant.

5. Conclusions

Considering the population diversity, we propose new SEIR model according with the hesitating and forgetting mechanism. Because previous studies neglect the feature of forgetting, we introduce the exponent form forgetting rate and obtain the mean-field equations. we have the following conclusions:

- Different initial forgetting parameters have little effect active investor group and calm investor group. However, the smaller initial forgetting parameters determines the higher peak of these groups. It is obvious that when the initial forgetting parameters reaches the corresponding critical level, their impact on emotional transmission becomes weaker.
- Under the same condition, the form of forgetting rate has little influence on the initial stage of investment emotion transmission, but when the forgetting rate is exponential, the active group will arrive at the small scale finally. This also shows that forgetting has an important impact on the spread of investment sentiment. Therefore, we can calculate the value of forgetting parameters in real world and create some top news to control these values in order to minimize the effect of investor emotion diffusion.

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