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# Hermite Wavelet based Galerkin Method for the Numerical Solutions of One Dimensional Elliptic Problems

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**Abstract** Wavelet analysis is a recently developed mathematical tool in applied mathematics. In this paper, we proposed the wavelet based Galerkin method for the numerical solution of one dimensional elliptic problems using Hermite wavelets. Here, Galerkin bases are constructed Hermite functions which are orthonormal bases and these are assumed bases elements which allow us to obtain the numerical solutions of the elliptic problems. The obtained numerical solutions are compared with the existing numerical methods and exact solution. Some of the test problems are considered to demonstrate the applicability and validity of the purposed method.

**Keywords:** Hermite wavelets; Galerkin method; Elliptic problems; Numerical solution.

## 1. Introduction

One dimensional elliptic problem occurs frequently in the fields of engineering and science. In most cases, we do not always find the exact solutions for these equations via analytical methods. In this case, it is very meaningful to give the high precision numerical solutions for this kind of problem by numerical methods. Recently, some of the numerical methods are used for the numerical solutions of boundary value problems. For example, Haar wavelet method [1], Legendre wavelet collocation method [2], Wavelet-Galerkin method [3], Laguerre wavelet Galerkin method [4] etc.

Wavelets theory is a new and has been emerging tool in applied mathematical research area. Its applications have been applied in a wide range of engineering disciplines; particularly, signal analysis for waveform representation and segmentations, time-frequency analysis and fast algorithms for easy implementation. Wavelets have generated significant interest from both theoretical and applied researchers over the last few decades. The concepts for understanding wavelets were provided by Meyer, Mallat, Daubechies, and many others, [5]. Since then, the number of applications where wavelets have been used has exploded. In areas such as approximation theory and numerical solutions of differential equations, wavelets are recognized as powerful weapons not just tools.

In general it is not always possible to obtain exact solution of an arbitrary differential equation. This necessitates either discretization of differential equations leading to numerical solutions, or their qualitative study which is concerned with deduction of important properties of the solutions without actually solving them. The Galerkin method is one of the best known methods for finding numerical solutions of differential equations and is considered the most widely used in applied mathematics [6]. Its simplicity makes it perfect for many applications. The wavelet-Galerkin method is an improvement over the standard Galerkin methods. The advantage of wavelet-Galerkin method over finite difference or finite element method has lead to tremendous applications in science and engineering. An approach to study differential equations is the use of wavelet function bases in place of other conventional piecewise polynomial trial functions in finite element type methods.

In this paper, we developed Hermite wavelet-Galerkin method (HWGM) for the numerical solution of differential equations and are based on expanding the solution by Hermite wavelets with unknown coefficients. The properties of Hermite wavelets together with the Galerkin method are utilized to evaluate the unknown coefficients and then a numerical solution of the differential equation is obtained.

The organization of the paper is as follows. In section 2, Preliminaries of Hermite wavelets are given. Hermite wavelet-Galerkin method of solution for the elliptic problem is given in section 3. In section 4 Numerical results are presented. Finally, conclusions of the proposed work are discussed in section 5.

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## 2. Preliminaries of Hermite wavelets

Wavelets constitute a family of functions constructed from dialation and translation of a single function $\psi(x)$ called mother wavelet. When the dialation parameter $a$ and translation parameter $b$ varies continuously, we have the following family of continuous wavelets [7, 8]:

$$\psi_{a,b}(x) = |a|^{-1/2}\,\psi\left(\frac{x-b}{a}\right), \ \forall \ a,b \in R, \ a \neq 0.$$

If we restrict the parameters $a$ and $b$ to discrete values as $a = a_0^{-k},\ b = n b_0 a_0^{-k},\ \ a_0 > 1,\ \ b_0 > 0.$ We have the following family of discrete wavelets

$$\psi_{k,n}(x) = |a|^{1/2}\,\psi(a_0^k x - n b_0),\ \forall\ a,\ b \in R,\ a \neq 0,$$

where $\psi_{k,n}$ form a wavelet basis for $L^2(R)$. In particular, when $a_0 = 2$ and $b_0 = 1$,then $\psi_{k,n}(x)$ forms an orthonormal basis. Hermite wavelets are defined as

$$\psi_{n,m}(x) = \begin{cases} \dfrac{2^{\frac{k}{2}}}{\sqrt{\pi}}\tilde{H}_m(2^k x - 2n + 1), & \dfrac{n-1}{2^{k-1}} \le x < \dfrac{n}{2^{k-1}} \\ 0, & \text{otherwise} \end{cases} \tag{1}$$

$$\tilde{H}_m = \sqrt{\frac{2}{\pi}}\,H_m(x) \tag{2}$$

where $m = 0,1,\ldots,M-1$. In eq. (2) the coefficients are used for orthonormality. Here $H_m(x)$ are the second Hermite polynomials of degree m with respect to weight function $W(x) = \sqrt{1-x^2}$ on the real line $R$ and satisfies the following reccurence formula $H_0(x) = 1,\ H_1(x) = 2x$,

$$H_{m+2}(x) = 2xH_{m+1}(x) - 2(m+1)H_m(x), \text{ where } m = 0,1,2,\ldots. \tag{3}$$

For $k = 1$ & $n = 1$ in (1) and (2), then the Hermite wavelets are given by

$$\psi_{1,0}(x) = \frac{2}{\sqrt{\pi}},$$

$$\psi_{1,1}(x) = \frac{2}{\sqrt{\pi}}(4x-2),$$

$$\psi_{1,2}(x) = \frac{2}{\sqrt{\pi}}(16x^2 - 16x + 2),$$

$$\psi_{1,3}(x) = \frac{2}{\sqrt{\pi}}(64x^3 - 96x^2 + 36x - 2),$$

$$\psi_{1,4}(x) = \frac{2}{\sqrt{\pi}}(256x^4 - 512x^3 + 320x^2 - 64x + 2)$$

And so on.

### 2.1 Function approximation

We would like to bring a solution function $u(x)$ under Hermite space by approximating $u(x)$ by elements of Hermite wavelet bases as follows,

$$u(x) = \sum_{n=1}^{\infty}\sum_{m=0}^{\infty} c_{n,m}\,\psi_{n,m}(x) \tag{4}$$

where $\psi_{n,m}(x)$ is given in eq. (1).

We approximate $u(x)$ by truncating the series represented in Eq. (4) as,

$$u(x) = \sum_{n=1}^{2^{k-1}} \sum_{m=0}^{M-1} c_{n,m}\, \psi_{n,m}(x) \tag{5}$$

where $c$ and $\psi$ are $2^{k-1} M \times 1$ matrix.

**2.2 Convergence analysis**

**Theorem 1.** A continuous function $u(x)$ in $H^2[0,1)$ defined on $[0,1)$ be bounded, then the Hermite wavelets expansion of $u(x)$ converges to it [9].

## 3. Method of solution

Consider the differential equation is of the form,

$$\frac{\partial^2 u}{\partial x^2} + \alpha \frac{\partial u}{\partial x} + \beta u = f(x) \tag{6}$$

With boundary conditions:

$$u(0) = a, \quad u(1) = b \tag{7}$$

Where $\alpha$ , $\beta$ are functions of $x$ or $u$ or constants and $f(x)$ is function of $x$ or constant. Write the equation (6) as

$$R(x) = \frac{\partial^2 u}{\partial x^2} + \alpha \frac{\partial u}{\partial x} + \beta u - f(x) \tag{8}$$

where $R(x)$ is the residual of the eq. (6). When $R(x) = 0$ for the exact solution, $u(x)$ only which will satisfy the boundary conditions.

Consider the trail series solution of the differential equation (6), $u(x)$ defined over [0, 1) can be expanded as a modified Hermite wavelet, satisfying the given boundary conditions which is involving unknown parameter as follows,

$$u(x) = \sum_{n=1}^{2^{k-1}} \sum_{m=0}^{M-1} c_{n,m}\, \psi_{n,m}(x) \tag{9}$$

where $c_{n,m}$'s are unknown coefficients to be determined.

Accuracy in the solution is increased by choosing higher degree Hermite wavelet polynomials.

Differentiating eq. (9) twice with respect to $x$ and substitute the values of $\frac{\partial^2 u}{\partial x^2}, \frac{\partial u}{\partial x}, u$ in eq. (8). To find $c_{n,m}$'s we choose weight functions as assumed bases elements and integrate on boundary values together with the residual to zero [10]. i.e. $\int_0^1 \psi_{1,m}(x) R(x)\, dx = 0$, $m = 0, 1, 2, \ldots\ldots$ then we obtain a system of linear equations, on solving this system, we get unknown parameters. Then substitute these unknowns in the trail solution, numerical solution of eq. (6) is obtained.

## 4. Numerical Experiment

**Problem 4.1** First, consider the second order equation [11],

$$\frac{\partial^2 u}{\partial x^2} + u = x^2, \quad 0 \le x \le 1 \tag{10}$$

With boundary conditions:

$$u(0) = 0, \quad u(1) = 0 \tag{11}$$

The implementation of the eq. (10) as per the method explained in section 3 is as follows:
The residual of eq. (10) can be written as:

$$R(x) = \frac{\partial^2 u}{\partial x^2} + u - x^2 \tag{12}$$

Now choosing the weight function $w(x) = x(1-x)$ for Hermite wavelet bases to satisfy the given boundary conditions (11), i.e.

$$\psi(x) = w(x) \times \psi(x)$$

$$\psi_{1,0}(x) = \psi_{1,0}(x) \times x(1-x) = \frac{2}{\sqrt{\pi}} x(1-x) \quad ,$$

$$\psi_{1,1}(x) = \psi_{1,1}(x) \times x(1-x) = \frac{2}{\sqrt{\pi}} (4x-2) x(1-x)$$

$$\psi_{1,2}(x) = \psi_{1,2}(x) \times x(1-x) = \frac{2}{\sqrt{\pi}} (16x^2 - 16x + 2) x(1-x)$$

Assuming the trail solution of (10) for $k=1$ and $m=3$ is given by

$$u(x) = c_{1,0}\psi_{1,0}(x) + c_{1,1}\psi_{1,1}(x) + c_{1,2}\psi_{1,2}(x) \tag{13}$$

Then the eq. (13) becomes

$$u(x) = c_{1,0}\frac{2}{\sqrt{\pi}} x(1-x) + c_{1,1}\frac{2}{\sqrt{\pi}}(4x-2)x(1-x) + c_{1,2}\frac{2}{\sqrt{\pi}}(16x^2-16x+2)x(1-x) \tag{14}$$

Differentiating eq. (14) twice w.r.t. $x$ we get,

$$\frac{\partial u}{\partial x} = c_{1,0}\frac{2}{\sqrt{\pi}}(1-2x) + c_{1,1}\frac{2}{\sqrt{\pi}}(-12x^2+12x-2) + c_{1,2}\frac{2}{\sqrt{\pi}}(-64x^3+96x^2-36x+2) \tag{15}$$

$$\frac{\partial^2 u}{\partial x^2} = c_{1,0}\frac{2}{\sqrt{\pi}}(-2) + c_{1,1}\frac{2}{\sqrt{\pi}}(-24x+12) + c_{1,2}\frac{2}{\sqrt{\pi}}(-192x^2+192x-36) \tag{16}$$

Using eq. (14) and (16), then eq. (12) becomes,

$$R(x) = c_{1,0}\frac{2}{\sqrt{\pi}}(-2) + c_{1,1}\frac{2}{\sqrt{\pi}}(-24x+12) + c_{1,2}\frac{2}{\sqrt{\pi}}(-192x^2+192x-36) +$$
$$\left(c_{1,0}\frac{2}{\sqrt{\pi}}x(1-x) + c_{1,1}\frac{2}{\sqrt{\pi}}(4x-2)x(1-x) + c_{1,2}\frac{2}{\sqrt{\pi}}(16x^2-16x+2)\right) - x^2$$
$$\Rightarrow R(x) = c_{1,0}\frac{2}{\sqrt{\pi}}(-x^2+x-2) + c_{1,1}\frac{2}{\sqrt{\pi}}(-4x^3+6x^2-26x+12) +$$
$$c_{1,2}\frac{2}{\sqrt{\pi}}(-16x^4+32x^3-210x^2+194x-36) - x^2 \tag{17}$$

This is the residual of eq. (10). The "weight functions" are the same as the bases functions. Then by the weighted Galerkin method, we consider the following:

$$\int_0^1 \psi_{1,m}(x) R(x)\,dx = 0, \; m = 0, 1, 2 \tag{18}$$

For $m = 0, 1, 2$ in eq. (18),

$$\int_0^1 \psi_{1,0}(x)R(x)\,dx = 0, \; \int_0^1 \psi_{1,1}(x)R(x)\,dx = 0, \; \int_0^1 \psi_{1,2}(x)R(x)\,dx = 0$$

$$\Rightarrow \quad (-0.3820)c_{1,1} + (0)c_{1,2} + (0.4487)c_{1,3} - 0.0564 = 0 \tag{19}$$

$$(0)c_{1,1} - (0.9943)c_{1,2} + (0)c_{1,3} - 0.0376 = 0 \tag{20}$$

$$(0.4487)c_{1,0} + (0)c_{1,1} - (2.3686)c_{1,2} + 0.0591 = 0 \tag{21}$$

We have three equations (19) – (21) with three unknown coefficients i.e. $c_{1,0}$ , $c_{1,1}$ and $c_{1,2}$ . By solving this system of algebraic equations, we obtain the values of $c_{1,0} = -0.1522$, $c_{1,1} = -0.0378$ and $c_{1,2} = -0.0039$. Substituting these values in eq. (14), we get the numerical solution; these results and absolute error = $|u_a(x) - u_e(x)|$ (where $u_a(x)$ and $u_e(x)$ are numerical and exact solutions respectively) are presented in table 1 and figure 1 in comparison with exact solution of eq. (10) is $u(x) = \dfrac{\sin(x) + 2\sin(1-x)}{\sin(1)} + x^2 - 2$.

Table 1: Comparison of numerical solution and exact solution of the problem 4.1

| x | Numerical solution | | | | Exact solution | Absolute Error | | |
|---|---|---|---|---|---|---|---|---|
| | FDM | LWGM* for $k=1$ & $M=5$ | HWGM for $k=1$ & $m=3$ | HWGM for $k=1$ & $M=5$ | | FDM | LWGM $(k=1$ & $m=5)$ | HWGM $(k=1$ & $m=5)$ |
| 0.1 | -0.009481 | -0.0095247 | -0.009536 | -0.009562 | -0.009555 | 7.40e-05 | 3.00e-05 | 7.00e-06 |
| 0.2 | -0.018768 | -0.0188780 | -0.018895 | -0.018904 | -0.018897 | 1.29e-04 | 1.94e-05 | 7.00e-06 |
| 0.3 | -0.027466 | -0.0276135 | -0.027643 | -0.027636 | -0.027635 | 1.69e-04 | 2.14e-05 | 1.00e-06 |
| 0.4 | -0.034990 | -0.0351476 | -0.035179 | -0.035171 | -0.035180 | 1.90e-04 | 3.28e-05 | 9.00e-06 |
| 0.5 | -0.040564 | -0.0407250 | -0.040734 | -0.040734 | -0.040759 | 1.95e-04 | 3.41e-05 | 2.50e-05 |
| 0.6 | -0.043233 | -0.0434003 | -0.043369 | -0.043425 | -0.043416 | 1.83e-04 | 1.56e-05 | 9.00e-06 |
| 0.7 | -0.041869 | -0.0420382 | -0.041974 | 0.042031 | -0.042025 | 1.56e-04 | 2.79e-05 | 6.00e-06 |
| 0.8 | -0.035186 | -0.0353301 | -0.035273 | -0.035294 | -0.035302 | 1.16e-04 | 2.57e-05 | 8.00e-06 |
| 0.9 | -0.021752 | -0.0218298 | -0.021544 | -0.021803 | -0.021815 | 6.30e-05 | 1.48e-05 | 1.20e-05 |

*LWGM : Laguerre wavelet-Galerkin method

| x | Haar solution [11] | HWGM | Exact solution | Absolute Error | |
|---|---|---|---|---|---|
| | | | | Haar solution | HWGM |
| 0.125 | -0.0121 | -0.0119 | -0.0119 | 0.0002 | 0.000 |
| 0.375 | -0.0340 | -0.0334 | -0.0334 | 0.0006 | 0.000 |
| 0.625 | -0.0440 | -0.0435 | -0.0435 | 0.0005 | 0.000 |
| 0.875 | -0.0261 | -0.0259 | -0.0259 | 0.0002 | 0.000 |

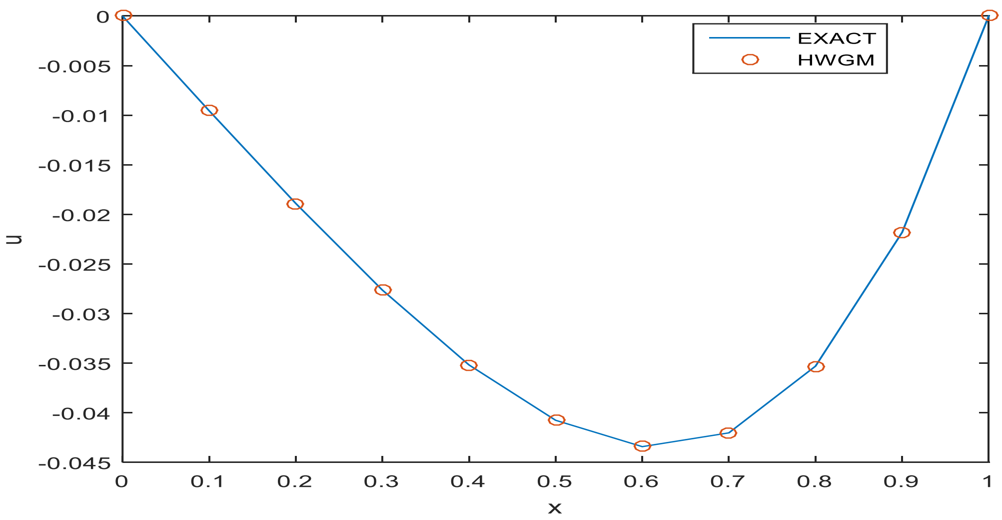


Fig. 1: Comparison of numerical and exact solutions of the problem 4.1.

**Problem 4.2** Next, consider another differential equation [1]

$$\frac{\partial^2 u}{\partial x^2} - \frac{\partial u}{\partial x} = -1, \quad 0 \le x \le 1 \tag{22}$$

With boundary conditions:

$$u(0) = 0, \quad u(1) = 0 \tag{23}$$

which has the exact solution $u(x) = x - \left(\frac{e^x - 1}{e - 1}\right)$.

By applying the method explained in the section 3, we obtain the constants $c_{1,0} = 0.4387$, $c_{1,2} = 0.0361$ and $c_{1,3} = 0.0023$. Substituting these values in eq. (14) we get the numerical solution. Obtained numerical solutions are compared with exact and other existing method solutions are presented in table 2 and figure 2.

Table 2: Comparison of numerical solution and exact solution of the problem 4.2

| x | Numerical solution | | | | Exact solution | Absolute Error | | |
|---|---|---|---|---|---|---|---|---|
| | FDM | LWGM* for $k=1$ & $M=5$ | HWGM for $k=1$ & $m=3$ | HWGM for $k=1$ & $M=5$ | | FDM | LWGM $(k=1$ & $m=5)$ | HWGM $(k=1$ & $m=5)$ |
| 0.1 | 0.037255 | 0.038699 | 0.038816 | 0.038824 | 0.038793 | 1.54e-03 | 9.04e-05 | 3.10e-05 |
| 0.2 | 0.068235 | 0.071137 | 0.071109 | 0.071158 | 0.071149 | 2.91e-03 | 1.20e-05 | 9.00e-06 |
| 0.3 | 0.092313 | 0.096385 | 0.096359 | 0.096414 | 0.096389 | 4.08e-03 | 5.00.e-06 | 1.00e-06 |
| 0.4 | 0.108799 | 0.113756 | 0.113748 | 0.113779 | 0.113769 | 4.97e-03 | 1.30e-05 | 1.00e-05 |
| 0.5 | 0.116933 | 0.122449 | 0.122457 | 0.122457 | 0.122459 | 5.53e-03 | 1.00e-05 | 2.00e-06 |
| 0.6 | 0.115881 | 0.121538 | 0.121569 | 0.121541 | 0.121546 | 5.66e-03 | 8.00e-06 | 5.00e-06 |
| 0.7 | 0.104724 | 0.110016 | 0.110056 | 0.110018 | 0.110020 | 5.30e-03 | 4.00.e-06 | 2.00e-06 |
| 0.8 | 0.082451 | 0.086755 | 0.086792 | 0.086758 | 0.086764 | 4.31e-03 | 9.00e-06 | 6.00e-06 |
| 0.9 | 0.047950 | 0.050539 | 0.050538 | 0.050541 | 0.050545 | 2.59e-03 | 6.00e-06 | 4.00e-06 |

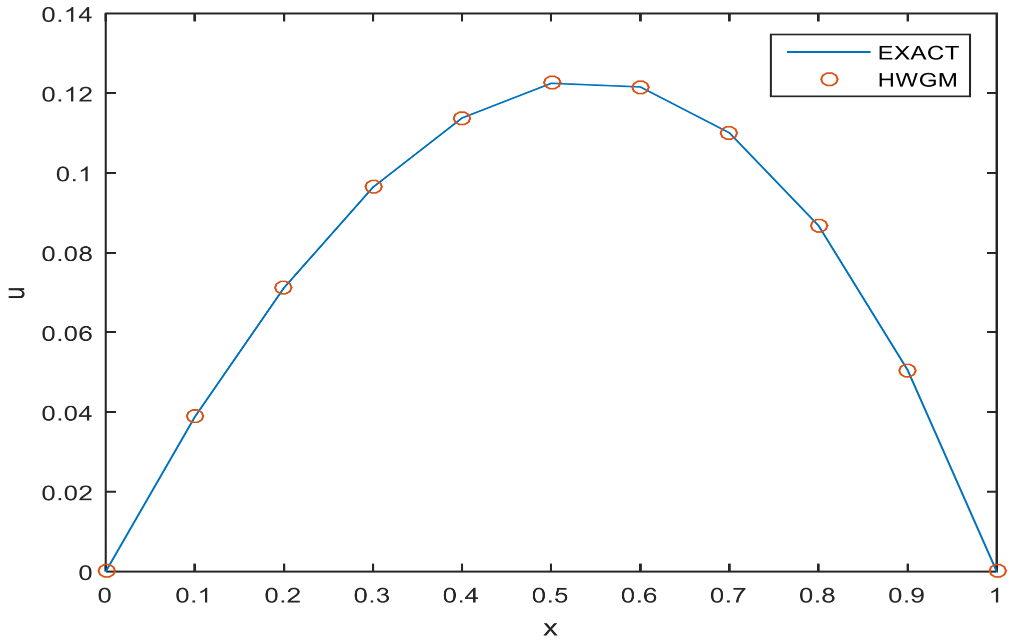


Fig. 2: Comparison of numerical and exact solutions of the problem 4.2.

**Problem 4.3** Finally, consider the singular boundary value problem [12],

$$\frac{\partial^2 u}{\partial x^2}+\frac{1}{x}\frac{\partial u}{\partial x}+u=x^2-x^3-9x+4,\quad 0\le x\le 1 \tag{24}$$

With boundary conditions:

$$u(0)=0,\quad u(1)=0 \tag{25}$$

which has the exact solution $u(x)=x^2-x^3$

As in the previous example, we obtained the constants $c_{1,0}=0.4429$, $c_{1,2}=0.2217$ and $c_{1,3}=-0.0001$. Substituting these values in eq. (14) we get the numerical solution. Obtained numerical solutions are compared with exact and other existing method solutions are presented in table 3 and figure 3.

Table 3: Comparison of numerical solution and exact solution of the problem 4.3

| **x** | **Numerical solution** | | | | **Exact solution** | **Absolute Error** | | |
|---|---|---|---|---|---|---|---|---|
| | **FDM** | **LWGM for** $k=1\ \&\ M=5$ | **HWGM for** $k=1\ \&\ m=3$ | **HWGM for** $k=1\ \&\ M=5$ | | **FDM** | **LWGM** $(k=1\ \&\ m=5)$ | **HWGM** $(k=1\ \&\ m=5)$ |
| 0.1 | -0.014709 | 0.009482 | 0.008949 | 0.008988 | 0.009000 | 2.37e-02 | 4.82e-04 | 5.10e-05 |
| 0.2 | -0.013726 | 0.032067 | 0.031941 | 0.032156 | 0.032000 | 4.57e-02 | 6.70e-05 | 5.90e-05 |
| 0.3 | -0.002584 | 0.063049 | 0.062954 | 0.063277 | 0.063000 | 6.56e-02 | 4.90e-05 | 4.60e-05 |
| 0.4 | 0.015387 | 0.096028 | 0.095977 | 0.096101 | 0.096000 | 8.06e-02 | 2.80e-05 | 2.30e-05 |
| 0.5 | 0.036564 | 0.125018 | 0.124996 | 0.125015 | 0.125000 | 8.84e-02 | 1.80e-05 | 4.00e-06 |
| 0.6 | 0.056572 | 0.144061 | 0.144008 | 0.144363 | 0.144000 | 8.74-02 | 6.01e-05 | 8.00e-06 |
| 0.7 | 0.070066 | 0.147039 | 0.147009 | 0.147409 | 0.147000 | 7.69e-02 | 3.90e-05 | 9.00e-06 |
| 0.8 | 0.070568 | 0.128037 | 0.128003 | 0.128241 | 0.128000 | 5.74e-02 | 3.70e-05 | 3.00e-06 |
| 0.9 | 0.050294 | 0.081075 | 0.080996 | 0.081132 | 0.081000 | 3.07e-02 | 7.50e-05 | 4.00e-06 |

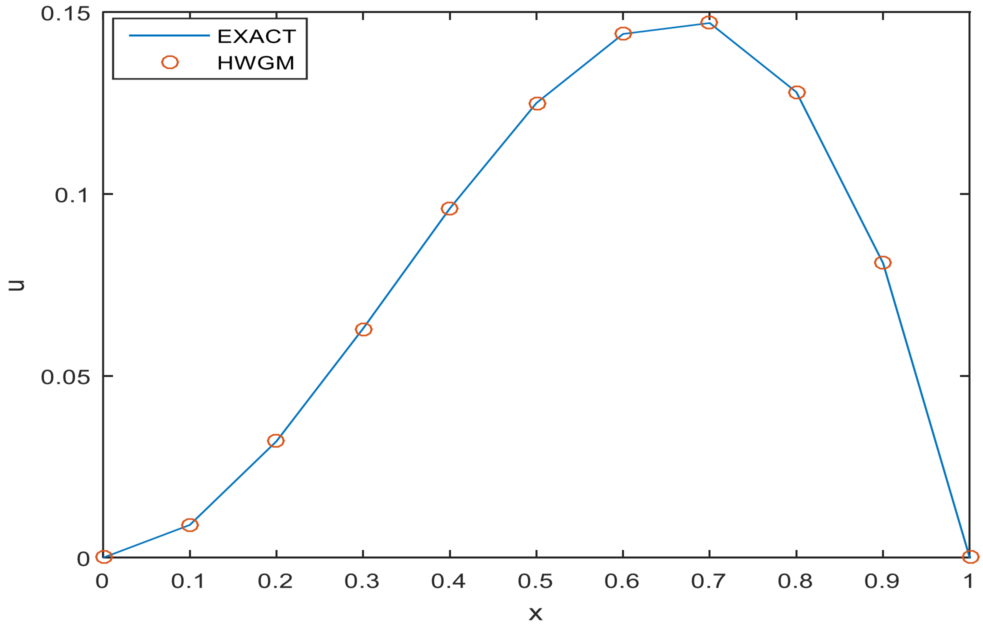


Fig. 3: Comparison of numerical and exact solution of the problem 4.3.

**Problem 4.4** Finally, consider another singular boundary value problem [13]

$$\frac{\partial^2 u}{\partial x^2} + \frac{8}{x}\frac{\partial u}{\partial x} + xu = x^5 - x^4 + 44x^2 - 30x, \quad 0 \le x \le 1 \tag{26}$$

With boundary conditions:

$$u(0) = 0, \quad u(1) = 0 \tag{27}$$

which has the exact solution $y(x) = -x^3 + x^4$.

By applying the method explained in the section 3, we obtain the constants and substituting these values in eq. (14) we get the numerical solution. Obtained numerical solutions are compared with exact and other existing method solutions are presented in table 4 and figure 4.

Table 4: Comparison of numerical solution and exact solution of the problem 4.4

| x | Numerical solution | | Exact solution | Absolute error | |
|---|---|---|---|---|---|
| | FDM | HWGM | | FDM | HWGM |
| 0.1 | 0.024647 | -0.000900 | -0.000900 | 2.55e-02 | 0 |
| 0.2 | 0.024538 | -0.006401 | -0.006400 | 3.09e-02 | 1.00e-06 |
| 0.3 | 0.016024 | -0.018904 | -0.018900 | 3.40e-02 | 4.00e-06 |
| 0.4 | -0.000072 | -0.038407 | -0.038400 | 3.83e-02 | 7.00e-06 |
| 0.5 | -0.022021 | -0.062512 | -0.062500 | 4.05e-02 | 1.20e-05 |
| 0.6 | -0.045926 | -0.086417 | -0.086400 | 4.05e-02 | 1.70e-05 |
| 0.7 | -0.065532 | -0.102920 | -0.102900 | 3.74e-02 | 2.00e-05 |
| 0.8 | -0.072190 | -0.102420 | -0.102400 | 3.02e-02 | 2.00e-05 |
| 0.9 | -0.054840 | -0.072914 | -0.072900 | 1.81e-02 | 1.40e-05 |

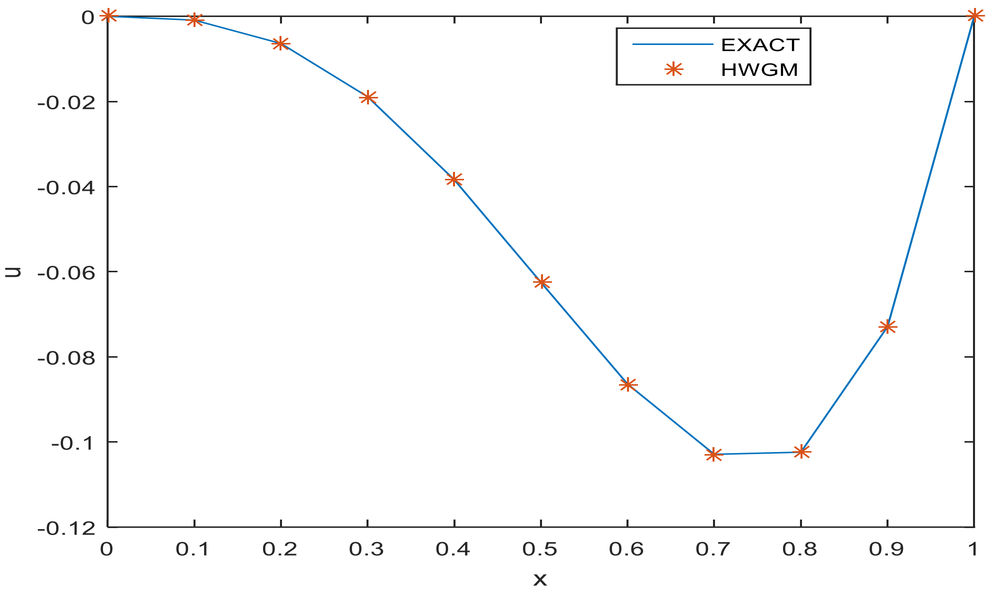


Table 4: Comparison of numerical solution and exact solution of the teat problem 4.4.

## 5. Conclusions

In this paper, we made an attempt for obtaining numerical solution of one dimensional elliptic problem by Galerkin method using modified Hermite wavelets. From the above tables and figures, which we observed that the comparison of the numerical solutions obtained using proposed method is better than FDM, LWGM and nearer to the exact solution. As increasing the values of $M$, we get more accuracy in the numerical solution which represented in the above tables. Hence, the proposed method is effective for solving differential equations.

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