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Stability Analysis of Fuzzy Hopfield Neural Networks with Time-varying Delays

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Abstract. In this paper, the problem of asymptotic stability for Takagi-Sugeno (T-S) fuzzy Hopfield neural networks with time-varying delays is studied. Based on the Lyapunov functional method, considering the system with uncertainties or without uncertainties, new delay-dependent stability criteria are derived in terms of Linear Matrix Inequalities (LMIs) that can be calculated easily by the LMI Toolbox in MATLAB. The proposed approach does not involve free weighting matrices and can provide less conservative results than some existing ones. Besides, numerical examples are given to show the effectiveness of the proposed approach.

Keywords: asymptotic stability; T-S fuzzy model; Hopfield neural networks; time-varying delay

1. Introduction

Hopfield neural networks (HNNs) were first introduced by Hopfield [1]. The dynamic behavior of HNNs has been widely studied due to their potential applications in signal processing, combinatorial optimization and pattern recognition [2-4]. These applications are mostly dependent on the stability of the equilibrium of neural networks. Thus, the stability analysis is a necessary step for the design and applications of neural networks. Sometimes, neural networks have to be designed such that there is only global stable equilibrium. For example, when a neural network is applied to solve the optimization problem, it must have unique equilibrium which is globally stable.

Both in biological and artificial neural networks, the interactions between neurons are generally asynchronous which inevitably result in time delays. Time-delay is often the main factor of instability and poor performance of neural network systems [5]. Therefore, lots of efforts have been made on stability analysis of neural networks with time-varying delays in recent years [6-9]. The free-weighting matrix method was proposed to investigate the delay-dependent stability [10], and some less conservative delay-dependent stability criteria for systems with time-varying delay were presented [11-16]. However, Researchers have realized that too many slack variables introduced will make the system synthesis complicated, lead to a significant increase in the computational burden, and cannot result in less conservative results indeed [17-19]. In practical systems, there always are some uncertain elements, and these uncertainties may come from unknown internal or external noise, environmental influence, and so on. Hence, it has been the focus of intensive research in recent years [10], [12], [20].

It is well-known that the T-S fuzzy models have been very important in academic research and practical applications, and the fuzzy logic theory has shown to be an efficient method to dealing with the analysis and synthesis issues for complex nonlinear systems [21-24]. Very recently, some results have been produced in the study of stability analysis of T-S fuzzy Hopfield neural networks systems with time-varying delays [25-27], To the best of our knowledge, the robust stability problem for uncertain fuzzy HNNs with time-varying interval delays has not been fully investigated, which remains as an open and challenging issue.

In this paper, the problem of stability analysis for T-S fuzzy HNNs with time-varying delays is considered. Based on Jensen integral inequality and some important Lemma, new sufficient conditions are derived in terms of LMIs. By constructing a Lyapunov-Krasovskii function without free-weighting matrices approach, the proposed criteria in this paper are much less conservative than some existing results. Numerical examples are given to show the applicability of the obtained results. The rest of this paper is arranged as follows. Section 2 gives problem statement and some preliminaries used in later sections. Section 3 presents our main results. Section 4 provides the numerical examples and Section 5 concludes the paper.

2. Problem Statement and Preliminaries

In this brief, we will consider the following HNNs with uncertainties represented by a T-S fuzzy model, and the i th rule of the T-S fuzzy model is of the following form:

Plant rule i :

IF $z_1(t)$ *is* M_1^i *and* $z_2(t)$ *is* $M_2^i, \cdots,$ *and* $z_n(t)$ *is* M_n^i

$$\textbf{THEN} \quad \dot{x}(t) = -(A_i + \Delta A_i(t))x(t) + (B_i + \Delta B_i(t)) f(x(t)) + (C_i + \Delta C_i(t)) f(x(t-d(t))) \tag{1}$$
$$x(t) = \varphi(t), t \in [-h_2, 0], i = 1, 2, \cdots q,$$

where $M_j^i (j = 1, 2, \cdots, n)$ is the fuzzy set, $z(t) = [z_1(t), z_2(t), \cdots, z_n(t)]$ is the premise variable vector, $x(t) \in \mathbf{R}^n$ is the system state variable, the time delay $0 \le h_1 \le d(t) \le h_2$ is the time-varying delay with an upper bound of h_2, $\dot{d}(t) \le \mu$ and q is the number of *IF-THEN* rules. $\Delta A_i(t)$, $\Delta B_i(t)$ and $\Delta C_i(t)$ are unknown matrices that represent the time-varying parameter uncertainties and are assumed to be admissible if the following assumption is satisfied.

Assumption 1 [30]:

$$[\Delta A_i(t)\ \Delta B_i(t)\ \Delta C_i(t)] = H_i \Delta_i(t) [E_{1i}\ E_{2i}\ E_{3i}] \tag{2}$$

where H_i, E_{1i}, E_{2i} and E_{3i} are given real constant matrices. The class of parametric uncertainties $\Delta_i(t)$ that satisfy

$$\Delta_i(t) = \left[I - F_i(t) J\right]^T F_i(t) \tag{3}$$

is said to be admissible, where J is also a known matrix satisfying

$$I - JJ^T > 0 \tag{4}$$

and $F_i(t)$ denotes unknown time-varying matrix functions. It is assumed that all elements $F_i(t)$ are Lebesgue measurable satisfying

$$F_i^T(t) F_i(t) \le I, \quad \forall t \in \mathbf{R} \tag{5}$$

To obtain our main results, we introduce the following lemmas.

Lemma 1 [28]: Let M, P, Q be the given matrices such that $Q > 0$, then

$$\begin{bmatrix} P & M^T \\ M & -Q \end{bmatrix} < 0 \Leftrightarrow P + M^T Q^{-1} M < 0$$

Lemma 2 [17]: For any constant matrix $M \in \mathbf{R}^{m \times m}$, $M = M^T > 0, \gamma > 0$ is a scalar, $\omega : \mathbf{R} \to \mathbf{R}^m$ is a vector function, then the following inequality holds:

$$(\int_0^\gamma \omega(s) ds)^T M (\int_0^\gamma \omega(s) ds) \le \gamma \int_0^\gamma \omega^T(s) M \omega(s) ds$$

Lemma 3 [18] For any scalars $W_1 \ge 0, W_2 \ge 0, d(t)$ is a continuous function and satisfies $h_1 < d(t) < h_2$, then

$$\frac{W_1}{d(t) - h_1} + \frac{W_2}{h_2 - d(t)} \ge \min\left\{\frac{3W_1 + W_2}{h_2 - h_1}, \frac{W_1 + 3W_2}{h_2 - h_1}\right\}$$

Lemma 4 [29] Assume that $\Delta_i(t)$ is given by (2)-(5). Given matrices $\Psi_i = \Psi_i^T, M_i$ and E_i of appropriate dimensions, the inequality

$$\Psi_i + M_i \Delta_i(t) N_i + N_i^T \Delta_i^T(t) M_i^T < 0 \tag{6}$$

holds for all $F(t)$ satisfies $F^T(t) F(t) \le I$. Then, the following inequality

$$\Psi_i + M_i F_i(t) N_i + (M_i F_i(t) N_i)^T < 0 \tag{7}$$

holds if and only if there exists a scalar $\varepsilon > 0$ satisfying

$$\begin{bmatrix} \Psi & M & \varepsilon N^T \\ * & -\varepsilon I & \varepsilon J^T \\ * & * & -\varepsilon I \end{bmatrix} < 0. \tag{8}$$

Using a standard fuzzy inference method, the system (1) is inferred as follows:

$$\dot{x}(t) = \sum_{i=1}^{q} \mu_i(z(t))[-(A_i + \Delta A_i(t))x(t) + (B_i + \Delta B_i(t)) f(x(t)) \ + (C_i + \Delta C_i(t)) f(x(t-d(t)))] \tag{9}$$

where

$$\mu_i(z(t)) = \frac{w_i(z(t))}{\sum_{j=1}^{q} w_j(z(t))}, w_i(z(t)) = \prod_{j=1}^{n} M_j^i(z(t)), \tag{10}$$

from the fuzzy sets theory, we have $\mu_i(z(t)) \geq 0, \ \sum_{i=1}^{q} \mu_i(z(t)) = 1$.

3. Main Results

3.1. Time-varying delay systems without uncertainties

Theorem 1[31]**.** For given scalars $0 \leq h_1 < h_2$ and $h_{12} = h_2 - h_1$, system (9) is asymptotically stable if there exist matrices $P > 0$, $Q_i > 0 (i = 1, 2, 3)$, $R_1 > 0$, $R_2 > 0$ with appropriate dimensions such that the following LMIs hold:

$$\Phi_{i,j} = \begin{bmatrix} \Phi_{00i} + \Phi_{0j} & \Phi_{12i} & \Phi_{13i} \\ * & -R_1 & 0 \\ * & * & -R_2 \end{bmatrix} < 0, i = 1, 2, \cdots q, \ j = 1, 2, 3, 4. \tag{11}$$

where

$$\Phi_{00i} = \begin{bmatrix} \Psi_{00i} & 0 & 0 & 0 & PB_i & PC_i \\ * & -Q_1 & 0 & 0 & 0 & 0 \\ * & * & -Q_2 & 0 & 0 & 0 \\ * & * & * & -(1-\mu)Q_3 & 0 & 0 \\ * & * & * & * & 0 & 0 \\ * & * & * & * & * & 0 \end{bmatrix} \tag{12}$$

$$\Psi_{00i} = -A_i^T P - PA_i + Q_1 + Q_2 + Q_3 \tag{13}$$

$$\Phi_{12i} = \begin{bmatrix} -\sqrt{h_1} A_i^T R_1 \\ 0 \\ 0 \\ 0 \\ \sqrt{h_1} B_i^T R_1 \\ \sqrt{h_1} C_i^T R_1 \end{bmatrix} \qquad \Phi_{13i} = \begin{bmatrix} -\sqrt{h_{12}} A_i^T R_2 \\ 0 \\ 0 \\ 0 \\ \sqrt{h_{12}} B_i^T R_{12} \\ \sqrt{h_{12}} C_i^T R_{12} \end{bmatrix} \tag{14}$$

$$\Phi_{01}=\begin{bmatrix} -\frac{R_1}{h_1} & 0 & 0 & \frac{R_1}{h_1} & 0 & 0 \\ * & -\frac{3R_1}{h_1}-\frac{3R_2}{h_{12}} & 0 & \frac{3R_1}{h_1}+\frac{3R_2}{h_{12}} & 0 & 0 \\ * & * & -\frac{R_2}{h_{12}} & \frac{R_2}{h_{12}} & 0 & 0 \\ * & * & * & -\frac{4R_1}{h_1}-\frac{4R_2}{h_{12}} & 0 & 0 \\ * & * & * & * & 0 & 0 \\ * & * & * & * & * & 0 \end{bmatrix} \tag{15}$$

$$\Phi_{02}=\begin{bmatrix} -\frac{R_1}{h_1} & 0 & 0 & \frac{R_1}{h_1} & 0 & 0 \\ * & -\frac{3R_1}{h_1}-\frac{R_2}{h_{12}} & 0 & \frac{3R_1}{h_1}+\frac{R_2}{h_{12}} & 0 & 0 \\ * & * & -\frac{3R_2}{h_{12}} & \frac{3R_2}{h_{12}} & 0 & 0 \\ * & * & * & -\frac{4R_1}{h_1}-\frac{4R_2}{h_{12}} & 0 & 0 \\ * & * & * & * & 0 & 0 \\ * & * & * & * & * & 0 \end{bmatrix} \tag{16}$$

$$\Phi_{03}=\begin{bmatrix} -\frac{3R_1}{h_1} & 0 & 0 & \frac{3R_1}{h_1} & 0 & 0 \\ * & -\frac{R_1}{h_1}-\frac{3R_2}{h_{12}} & 0 & \frac{R_1}{h_1}+\frac{3R_2}{h_{12}} & 0 & 0 \\ * & * & -\frac{R_2}{h_{12}} & \frac{R_2}{h_{12}} & 0 & 0 \\ * & * & * & -\frac{4R_1}{h_1}-\frac{4R_2}{h_{12}} & 0 & 0 \\ * & * & * & * & 0 & 0 \\ * & * & * & * & * & 0 \end{bmatrix} \tag{17}$$

$$\Phi_{04}=\begin{bmatrix} -\frac{3R_1}{h_1} & 0 & 0 & \frac{3R_1}{h_1} & 0 & 0 \\ * & -\frac{R_1}{h_1}-\frac{R_2}{h_{12}} & 0 & \frac{R_1}{h_1}+\frac{R_2}{h_{12}} & 0 & 0 \\ * & * & -\frac{3R_2}{h_{12}} & \frac{3R_2}{h_{12}} & 0 & 0 \\ * & * & * & -\frac{4R_1}{h_1}-\frac{4R_2}{h_{12}} & 0 & 0 \\ * & * & * & * & 0 & 0 \\ * & * & * & * & * & 0 \end{bmatrix} \tag{18}$$

Proof. Choose a Lyapunov-Krasovskii functional candidate as follows:

$$V(x_t) = V_1(x_t) + V_2(x_t) + V_3(x_t) + V_4(x_t)$$

where

$$V_1(x_t) = x^T(t)Px(t)$$

$$V_2(x_t) = \int_{t-h_1}^{t} x^T(s)Q_1x(s)ds + \int_{t-h_2}^{t} x^T(s)Q_2x(s)ds$$

$$V_3(x_t) = \int_{t-d(t)}^{t} x^T(s)Q_3x(s)ds$$

$$V_4(x_t) = \int_{h_1}^{0}\int_{t+\theta}^{t} \dot{x}^T(s)R_1\dot{x}(s)dsd\theta + \int_{-h_2}^{-h_1}\int_{t+\theta}^{t} \dot{x}^T(s)R_2\dot{x}(s)dsd\theta$$

Then, the time derivative of $V(x_t)$ along the trajectory of system (9) yields

$$\dot{V}_1(x_t) = 2x^T(t)P\dot{x}(t) \tag{19}$$

$$\dot{V}_2(x_t) = x^T(t)(Q_1+Q_2)x(t) - x^T(t-h_1)Q_1x(t-h_1) - x^T(t-h_2)Q_2x(t-h_2) \tag{20}$$

$$\dot{V}_3(x_t) = x^T(t)Q_3x(t) - (1-\mu)x^T(t-d(t))Q_3x(t-d(t)) \tag{21}$$

$$\dot{V}_4(x_t) = \dot{x}^T(t)(h_1R_1 + h_{12}R_2)\dot{x}(t) - \int_{t-h_1}^{t} \dot{x}^T(s)R_1\dot{x}(s)ds - \int_{t-h_2}^{t-h_1} \dot{x}^T(s)R_2\dot{x}(s)ds \tag{22}$$

By using Lemma 2 and Lemma 3, we have

$$\begin{aligned}
&-\int_{t-h_1}^{t} \dot{x}^T(s)R_1\dot{x}(s)ds \\
&= -\int_{t-d(t)}^{t} \dot{x}^T(s)R_1\dot{x}(s)ds - \int_{t-h_1}^{t-d(t)} \dot{x}^T(s)R_1\dot{x}(s)ds \\
&\le -\{[(\int_{t-d(t)}^{t} \dot{x}(s)ds)^T R_1 \int_{t-d(t)}^{t} \dot{x}(s)ds]/d(t) \\
&\quad + [(\int_{t-h_1}^{t-d(t)} \dot{x}(s)ds)^T R_1 \int_{t-h_1}^{t-d(t)} \dot{x}(s)ds]/(h_1-d(t))\} \le -\max\{\frac{W_1+3W_2}{h_1}, \frac{3W_1+W_2}{h_1}\}
\end{aligned} \tag{23}$$

where

$$W_1 = \begin{bmatrix} x(t) \\ x(t-d(t)) \end{bmatrix}^T \begin{bmatrix} R_1 & -R_1 \\ * & R_1 \end{bmatrix} \begin{bmatrix} x(t) \\ x(t-d(t)) \end{bmatrix}$$

$$W_2 = \begin{bmatrix} x(t-d(t)) \\ x(t-h_1) \end{bmatrix}^T \begin{bmatrix} R_1 & -R_1 \\ * & R_1 \end{bmatrix} \begin{bmatrix} x(t-d(t)) \\ x(t-h_1) \end{bmatrix}$$

and

$$\begin{aligned}
&-\int_{t-h_2}^{t-h_1} \dot{x}^T(s)R_2\dot{x}(s)ds \\
&= -\int_{t-d(t)}^{t-h_1} \dot{x}^T(s)R_2\dot{x}(s)ds - \int_{t-h_2}^{t-d(t)} \dot{x}^T(s)R_2\dot{x}(s)ds \\
&\le -\{[\int_{t-d(t)}^{t-h_1} \dot{x}(s)ds)^T R_2 \int_{t-d(t)}^{t-h_1} \dot{x}(s)ds]/(d(t)-h_1) \\
&\quad + [\int_{t-h_2}^{t-d(t)} \dot{x}(s)ds)^T R_2 \int_{t-h_2}^{t-d(t)} \dot{x}(s)ds]/(h_2-d(t))\} \le -\max\{\frac{W_3+3W_4}{h_{12}}, \frac{3W_3+W_4}{h_{12}}\}
\end{aligned} \tag{24}$$

where

$$W_3 = \begin{bmatrix} x(t-h_1) \\ x(t-d(t)) \end{bmatrix}^T \begin{bmatrix} R_2 & -R_2 \\ * & R_2 \end{bmatrix} \begin{bmatrix} x(t-h_1) \\ x(t-d(t)) \end{bmatrix} \quad W_4 = \begin{bmatrix} x(t-d(t)) \\ x(t-h_2) \end{bmatrix}^T \begin{bmatrix} R_2 & -R_2 \\ * & R_2 \end{bmatrix} \begin{bmatrix} x(t-d(t)) \\ x(t-h_2) \end{bmatrix}$$

It can be shown from (11),(19)-(24) and Lemma 1 that

$$\dot{V}(x_t) = \dot{V}_1(x_t) + \dot{V}_2(x_t) + \dot{V}_3(x_t) + \dot{V}_4(x_t) \le \sum_{i=1}^{q} \mu_i(z(t))\xi^T(t)\Phi_{i,j}\xi(t) \le 0, \quad j = 1,2,3,4. \tag{25}$$

where $\xi^T(t) = [x(t)\ x(t-h_1)\ x(t-h_2)\ x(t-d(t)\ f(x(t))\ f(x(t-d(t)))]$

Hence, system (9) is asymptotically stable. This completes the proof. When there is no fuzzy and no uncertainties in (9), the system is reduced to

$$\dot{x}(t) = -Ax(t) + Bf(x(t)) + Cf(x(t-d(t))) \tag{26}$$

Corollary 1 For given scalars $0 \le h_1 < h_2$ and $h_{12} = h_2 - h_1$, system (26) is asymptotically stable if there exist matrices $P > 0$, $Q_i > 0 (i = 1,2,3)$, $R_1 > 0$, $R_2 > 0$ with appropriate dimensions such that the following LMIs hold:

$$\Phi_j = \begin{bmatrix} \Phi_{00} + \Phi_{0j} & \Phi_{12} & \Phi_{13} \\ * & -R_1 & 0 \\ * & * & -R_2 \end{bmatrix} < 0, \ j = 1,2,3,4. \tag{27}$$

where

$$\Phi_{00} = \begin{bmatrix} \Psi_{00} & 0 & 0 & 0 & PB & PC \\ * & -Q_1 & 0 & 0 & 0 & 0 \\ * & * & -Q_2 & 0 & 0 & 0 \\ * & * & * & -(1-\mu)Q_3 & 0 & 0 \\ * & * & * & * & 0 & 0 \\ * & * & * & * & * & 0 \end{bmatrix}, \quad \Psi_{00} = -A^T P - PA + Q_1 + Q_2 + Q_3, \text{ and } \Phi_{0j} \text{ are}$$

defined as in Theorem **1**.

3.2. Time-varying delay systems with uncertainties

Now, we shall discuss the feasible robust stability criteria for time-varying delay systems with uncertainty.

Theorem 2. For given scalars $h_2 > h_1 \ge 0$ ($h_{12} = h_2 - h_1$) and $\varepsilon > 0$, the system(9) is robust stability if there exist matrices $P > 0, Q_i > 0 (i = 1,2,3), R_1 > 0, R_2 > 0$ and J of appropriate dimensions and scalar $\varepsilon > 0$ such that the following LMIs hold:

$$\begin{bmatrix} \Phi_{i,j} & M_i & \varepsilon N_i^T \\ * & -\varepsilon I & \varepsilon J^T \\ * & * & -\varepsilon I \end{bmatrix} < 0, \ i = 1,2,\cdots,q, \qquad j = 1,2,3,4 \tag{28}$$

where $\Phi_{i,j}$ is defined in (11), and

$$M_i = \begin{bmatrix} PH_i & 0 & 0 & 0 & 0 & 0 & \sqrt{h_1} H_i^T R_1^T & \sqrt{h_{12}} H_i^T R_2^T \end{bmatrix}^{\mathrm{T}},$$

$$N_i = \begin{bmatrix} -E_{1i}^T & 0 & 0 & 0 & E_{2i}^T & E_{3i}^T & 0 & 0 \end{bmatrix}$$

Proof. Assume that inequalities (28) hold, from Lemma 1 and Lemma 4,

$$\Phi_{i,j} + M_i F_i(t) N_i + [M_i F_i(t) N_i]^T < 0 \qquad (i = 1,2,\cdots,q;\ j = 1,2,3,4)$$

hold. From (26), it can be verified that

$$\sum_{i=1}^{q} \mu_i(z(t))\xi^T(t)\left\{\Phi_{i,j} + M_i\Delta_i(t)N_i + [M_i\Delta_i(t)N_i]^T\right\}\xi^T(t) \le 0.$$

Hence, system (9) is robust stability from theorem 2. When there is no fuzzy in (9), the system is reduced to

$$\dot{x}(t) = -(A + \Delta A)x(t) + (B + \Delta B) f(x(t)) + (C + \Delta C) f(x(t-d(t))) \tag{29}$$

Corollary 2 For given scalars $0 \le h_1 < h_2$ and $h_{12} = h_2 - h_1$, system (29) is asymptotically stable if there exist matrices $P > 0$, $Q_i > 0 (i = 1,2,3)$, $R_1 > 0$, $R_2 > 0$ and J with appropriate dimensions and scalar

$\varepsilon > 0$ such that the following LMIs hold:

$$\begin{bmatrix} \Phi_j & M & \varepsilon N^T \\ * & -\varepsilon I & \varepsilon J^T \\ * & * & -\varepsilon I \end{bmatrix} < 0, \; j = 1,2,3,4, \tag{30}$$

where Φ_j is defined in (27), and

$$M = \begin{bmatrix} PH & 0 & 0 & 0 & 0 & 0 & \sqrt{h_1} H^T R_1^T & \sqrt{h_{12}} H^T R_2^T \end{bmatrix}^{\mathrm{T}},$$
$$N = \begin{bmatrix} -E_1^T & 0 & 0 & 0 & E_2^T & E_3^T & 0 & 0 \end{bmatrix}$$

Remark 1: when $J = 0$, it will reduced to the system in [30].

4. Numerical Examples

In this section, three numerical examples are given to illustrate the effectiveness of the proposed methods.

Example 1 In this example, we consider the DNNs (9) with

$$A_1 = \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}, \; B_1 = \begin{bmatrix} -0.4 & 0.3 \\ 0.4 & -0.6 \end{bmatrix}, \; C_1 = \begin{bmatrix} -0.5 & 0.1 \\ -0.2 & -0.5 \end{bmatrix} \; A_2 = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}, B_2 = \begin{bmatrix} -0.9 & 0.4 \\ 0.5 & -0.7 \end{bmatrix}, C_2 = \begin{bmatrix} -0.7 & 0.6 \\ -0.3 & -0.1 \end{bmatrix}$$
$$\mu = 2, \; \Delta A_i = \Delta B_i = \Delta C_i = 0, \; i = 1,2,$$

The time-varying delays are taken as $d(t) = 0.1 + \sin^2 t$ and the activation function is described by $f(x) = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$, the membership function is $\mu_1(z(t)) = \sin^2 x_1$, $\mu_2(z(t)) = \cos^2 x_1$, using MATLAB LMI Toolbox to solve the LMIs in theorem 1, some positive definite feasible matrices are given as follows

$$P = \begin{bmatrix} 0.3777 & -0.1747 \\ -0.1747 & 1.2997 \end{bmatrix}, Q_1 = \begin{bmatrix} 0.3915 & -0.2512 \\ -0.2512 & 0.9871 \end{bmatrix},$$
$$Q_2 = \begin{bmatrix} 0.3077 & -0.2371 \\ -0.2371 & 1.0812 \end{bmatrix}, Q_3 = \begin{bmatrix} 0.0263 & -0.0238 \\ -0.0238 & 0.1124 \end{bmatrix}$$
$$R_1 = \begin{bmatrix} 0.5338 & -0.2606 \\ -0.2606 & 0.5518 \end{bmatrix}, R_2 = \begin{bmatrix} 0.1196 & -0.0438 \\ -0.0438 & 0.4276 \end{bmatrix}$$

and the state trajectories of the systems with different initial conditions are showed as follows (Figs. 1-3)

Figs.1-3 show that the state trajectories of the systems are converging to zero with different initial state, that is to say, system (9) is asymptotically stable when theorem 1 holds.

Example 2 In this example, we consider the DNNs (27) and corollary 1 with

$$A = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}, \; B = \begin{bmatrix} 0.7 & 0.8 \\ -0.5 & 0.3 \end{bmatrix}, \; C = \begin{bmatrix} 0.2 & 0.2 \\ 0.1 & -0.2 \end{bmatrix}, \mu = 2.$$

The activation function is described by $f(x) = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$, the maximum allowable upper bound of h_2 with given h_1 is showed in Table 1.

Table 1: Maximum allowable upper bound of h_2 with given h_1

h_1	**0.0001**	**0.001**	**0.01**	**0.05**	**0.1**
Muralisankar et al. [15]	<16.17	<16.17	<16.17	<16.17	16.1614
Wu et al.[20]	<11.08	<11.08	<11.08	<11.08	11.0727
Corollary 1	16.4021	15.8384	14.1293	11.3221	9.4548

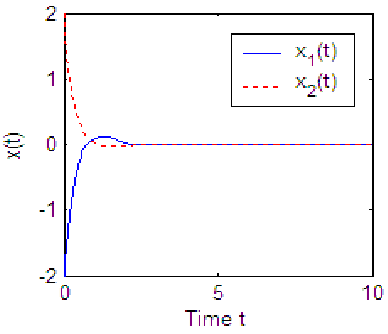

Fig. 1: The state trajectories with $x(0)=\begin{bmatrix}-2 & 2\end{bmatrix}^T$

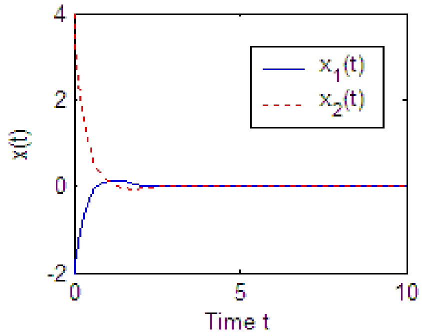

Fig. 2: The state trajectories with $x(0)=\begin{bmatrix}-2 & 4\end{bmatrix}^T$

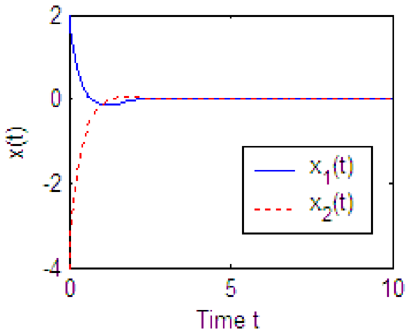

Fig. 3: The state trajectories with $x(0)=\begin{bmatrix}2 & -4\end{bmatrix}^T$

According to the Table 1, this example shows that our results are better than those results discussed in [15,20] when h_1 is small enough, although free-weighting matrix approach is adopted in [15,20].

Example 3 In this example, we consider the DNNs (11) and Theorem 2 with

$$A_1=\begin{bmatrix}2 & 0\\ 0 & 3\end{bmatrix}, A_2=\begin{bmatrix}3 & 0\\ 0 & 2\end{bmatrix}, B_1=\begin{bmatrix}0.7 & 0.8\\ -0.5 & 0.3\end{bmatrix}, B_2=\begin{bmatrix}-0.9 & 0.4\\ 0.5 & -0.7\end{bmatrix}, C_1=\begin{bmatrix}0.2 & 0.2\\ 0.1 & -0.2\end{bmatrix},$$

$$C_2=\begin{bmatrix}0.3 & 0.6\\ 0.2 & -0.4\end{bmatrix}, F_i(t)=\begin{bmatrix}\sin t & 0\\ 0 & \cos t\end{bmatrix}, H_i=I,\ E_{1i}=E_{2i}=E_{3i}=\begin{bmatrix}0.02 & 0\\ 0 & 0.02\end{bmatrix},\ i=1,2.\ \mu=2.$$

The activation function is described by $f(x)=\dfrac{e^x-e^{-x}}{e^x+e^{-x}}$, the membership function is $\mu_1(z(t))=\sin^2 x_1$, $\mu_2(z(t))=\cos^2 x_1$, and we choose the parameter J with different values, the maximum allowable upper bound of h_2 with given h_1 is showed in Table 2.

Table 2: Maximum allowable upper bound of h_2 with given h_1

	h_1	**0.005**	**0.01**	**0.05**	**0.1**	**0.15**	**0.2**
J	0	0.2550	0.1662	0.1377	0.1291	0.1502	-
	$\sqrt{0.1}I$	0.1624	0.1667	0.1475	0.1389	0.1548	-
	$\sqrt{0.5}I$	0.0342	0.0392	0.0505	-	-	-

The time-varying delays are taken as $d(t)=0.01+0.15\sin^2 t$ and $J=\sqrt{0.1}I$, using MATLAB LMI Toolbox to solve the LMIs in Theorem 2, some positive definite feasible matrices are given as follows:

$$P=\begin{bmatrix}0.0356 & 0.0365\\ 0.0365 & 0.0376\end{bmatrix}, Q_1=\begin{bmatrix}0.0123 & 0.0118\\ 0.0118 & 0.0433\end{bmatrix}, Q_2=\begin{bmatrix}0.0123 & 0.0118\\ 0.0118 & 0.0433\end{bmatrix}, Q_3=\begin{bmatrix}0.0007 & 0.0011\\ 0.0011 & 0.0036\end{bmatrix},$$

$$R_1=\begin{bmatrix}0.0598 & 0.0061\\ 0.0061 & 0.0737\end{bmatrix}, R_2=\begin{bmatrix}2.0506 & 0.0543\\ 0.0543 & 2.2905\end{bmatrix},$$

and the state trajectories of the systems with different initial conditions are showed as follows(Figs. 4-6)

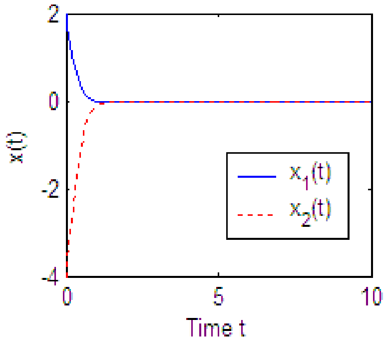

Fig. 4 The state trajectories with $x(0)=\begin{bmatrix}2 & -4\end{bmatrix}^T$

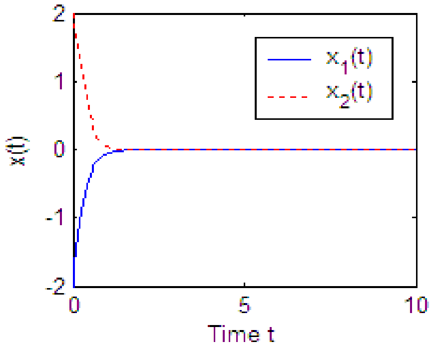

Fig. 5 The state trajectories with $x(0)=\begin{bmatrix}-2 & 2\end{bmatrix}^T$

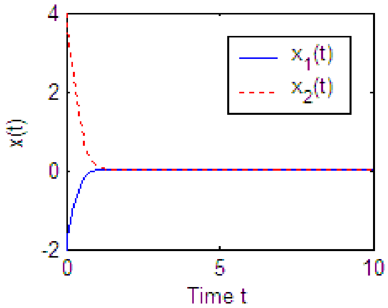

Fig. 6 The state trajectories with $x(0)=\begin{bmatrix}-2 & 4\end{bmatrix}^T$

Table 2 shows the maximum allowable upper bound of time-delay with given the allowable lower bound. From Figs.4-6, it can be seen that the state trajectories of the systems are converging to zero with different initial state, that is to say, system (9) is robust stable when Theorem 2 holds.

Example 4 In this example, we consider the DNNs (30) and corollary 2 with

$$A=\begin{bmatrix}2 & 0\\ 0 & 2\end{bmatrix}, B=\begin{bmatrix}0.7 & 0.8\\ -0.5 & 0.3\end{bmatrix}, C=\begin{bmatrix}0.2 & 0.2\\ 0.1 & -0.2\end{bmatrix}, E_1=E_2=E_3=\begin{bmatrix}0.02 & 0\\ 0 & 0.02\end{bmatrix}, F(t)=\begin{bmatrix}\sin t & 0\\ 0 & \cos t\end{bmatrix},$$

$H=I,\ \mu=2,$

the activation function is described by $f(x)=\dfrac{e^x-e^{-x}}{e^x+e^{-x}}$, and we choose the parameter J with different values, the maximum allowable upper bound of h_2 with given h_1 is showed in Table 3.

Tab. 3: Maximum allowable upper bound of h_2 with given h_1

h_1		**0.005**	**0.01**	**0.05**	**0.1**	**0.2**	**0.3**
J	0	0.3175	0.3224	0.2255	0.2168	0.2194	-
	$\sqrt{0.1I}$	0.1543	0.1593	0.1553	0.1350	-	-
	$\sqrt{0.5I}$	0.0401	0.0412	0.0504	-	-	-

The time-varying delays are taken as $d(t)=0.05+\ 0.1\sin^2 t$ and $J=\sqrt{0.1I}$, using MATLAB LMI Toolbox to solve the LMIs in corollary 2, some positive definite feasible matrices are given as follows:

$$P=\begin{bmatrix}0.0322 & 0.0012\\ 0.0012 & 0.0383\end{bmatrix}, Q_1=\begin{bmatrix}0.0463 & 0.0025\\ 0.0025 & 0.0542\end{bmatrix}, Q_2=\begin{bmatrix}0.0463 & 0.0025\\ 0.0025 & 0.0542\end{bmatrix}, Q_3=\begin{bmatrix}0.0050 & 0.0002\\ 0.0002 & 0.0057\end{bmatrix},$$

$$R_1 = \begin{bmatrix} 0.0629 & 0.0046 \\ 0.0046 & 0.0781 \end{bmatrix}, R_2 = \begin{bmatrix} 0.4790 & 0.0335 \\ 0.0335 & 0.4719 \end{bmatrix},$$

and the state trajectories of the systems with different initial conditions are showed as follows(Figs. 7-9)

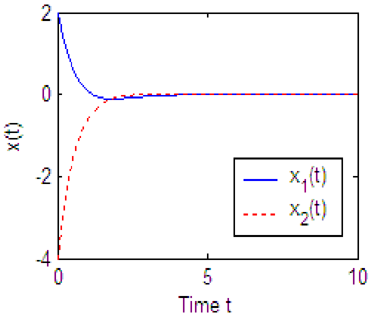

Fig. 7 The state trajectories with $x(0) = \begin{bmatrix} 2 & -4 \end{bmatrix}^T$

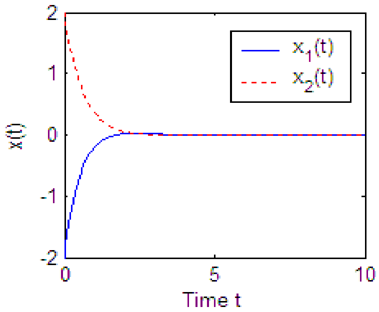

Fig. 8 The state trajectories with $x(0) = \begin{bmatrix} -2 & 2 \end{bmatrix}^T$

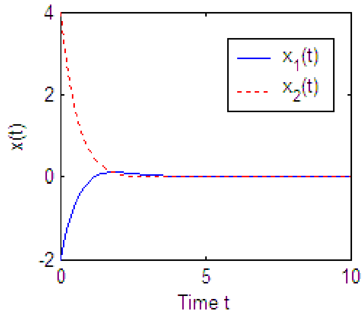

Fig. 9 The state trajectories with $x(0) = \begin{bmatrix} -2 & 4 \end{bmatrix}^T$

Table 3 shows the maximum allowable upper bound of time-delay with given the allowable lower bound. From Figs.7-9, it can be seen that the state trajectories of the systems are converging to zero with different initial state, that is to say, system (30) is robust stable when corollary 2 holds.

5. Conclusions

We present improved criteria of robust stability for HNNs with time-varying delay and uncertainties in this paper. The obtained stability conditions are expressed with LMIs. By comparing the experimental results from numerical examples, it is demonstrated the improvement of our proposed criteria over some existing ones.

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