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Research on the complex features about Stackelberg game model with retailers have dualidentities

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Abstract. This paper presents a 1-2 suppliers-retailers model with delayed bounded rationality. The retailers have their own products, and their products aren't manufacturer's products' substitutes, products are epiphytic relationship. The phenomenon of chaos and other complex phenomena are reported using stability region,bifu[1]rcation,attractors etc. We also introduce delayed decision into the model, study the influence of delayed decision on the stability of the model. The results show that the system's stability is mainly determined by the delay coefficient, appropriate delay coefficient can enhance the stability of the system, the inappropriate delay coefficient will reduce the stability of the system. In addition, we cannot simply think that more merchants adopt delayed decision can improve the stability of the system.

Keywords: bounded rationality, game theory, complex analysis,bifurcation.

1. Introduction

Puu[1] first discovered bifurcation and chaos exist in Duopoly Cournot model.Bischi[2] and others introduced the bounded rationality into Cournot duopoly game model with linear cost for the first time. The dynamic behavior of Bowley model under bounded rationality is studied by Agiza[3]. Since then, incomplete information has been introduced into the classic Cournot Oligopoly game model.The oligopoly enterprises are no longer the same type of decision makers, but also different type of decision makers. Literature [4-7] studied different types of heterogeneous duopoly game model, discussed the existence conditions and stability conditions of bounded equilibrium point and Nash equilibrium point.The complex dynamical behavior of the system is proved by numerical simulation of bifurcation, chaos, singular attractor and sensitivity, depending on initial conditions. The document [4] is the study of linear demand function and linear cost function, Cournot model with bounded rationality and complete rationality; literature [5] is the study of linear demand function and nonlinear cost function, double oligopoly game model of limited rationality and complete rationality; literature[6]is the study of nonlinear the demand function and linear cost function, Cournot model with bounded rationality and complete rationality; literature [7] is the study of linear demand function and asymmetric cost function, Cournot model with bounded rationality and complete rationality of the literature. Literature [8-9] studied a totally heterogeneous three oligopoly game model, analyzed the complex dynamic characteristics of the model. The difference is that the literature [8] is studied under linear cost functions, while the literature [9] is studied under nonlinear cost functions.Yao[10-11] improved the bounded rationality dynamic Cournot model, which were introduced into the advertising market and the financial field respectively, and analyzed the evolution process of the improved model.

Oligopoly competition between enterprises on the one hand is the production competition, on the other hand is the price competition. The earliest research on the price competition between the oligarch enterprises model is put forward by Bertrand in 1883[12].In document [13], a bounded rational duopoly Bert Rand model is proposed, and the dynamic characteristics of the model are analyzed.Ma[14] considered the macroeconomic model of money supply with time delays, discussed the effect of delay variation on system stability and Hopf bifurcation.literature[15] studied the Cournot-Bertrand duopoly model, analyzed the stability of the fixed points, and recognized the chaotic behavior of the system.The [16] manufacturers to use the delayed bounded rationality hypothesis establishes dynamic game model of the market based on the theoretical analysis. The application of the complicated systematic complexity of the state system. Our Fengshan [17] improved three oligopoly price game model, restricting the production cost function using the limited resource

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condition, study the influence on the time delay changes the dynamic characteristics of the system such as.Ahmed[18]on a dynamic product differentiation Bertrand duopoly game model based on the gradient adjustment mechanism take the limited rationality of the model in the enterprise update each cycle of price.

2. The one master-two slaves price game model

2.1 Assumptions

The following assumptions are made to develop our model in this paper.

(1) There are one manufacturer and two retailers(1 and 2) . Manufacturer is a master,has negative effects on Retailers.Retailers can’t affect manufacturer.

(2) Retailers can Product their products,which are substitutes,with manufacturer’s products are epiphytic relationship.

(3) Both the demand functions and the cost functions are linear, and the price is used as the decision objective.

(4) Three businessmen are bounded rationality.

2.2 symbol description

$p_i(t), i=1,2,3$ said the business i ‘s product price in the t period;

$Q_i(t)$ said in the t period,the demand of the market for the business i ’s products;

a_i said when the price is zero, the the biggest market demand for the product i ;

b_i is the own price sensitive coefficient of $Q_i(t)$;

d said the price sensitive factor of two ratailers’influence each other;

k said the adverse impact of manufacturer cause on retailers;

c_i said the marginal cost of business i ;

$\Pi_i(t)$ said the profit of business i .

2.3 The model

According to the actual situation, and refer to scholars’ previous research [19-20].Each business's demand function can be written :

$$\begin{aligned} Q_1(t) &= a_1 - b_1 p_1(t) \\ Q_2(t) &= a_2 - b_2 p_2(t) + dp_3(t) - kp_1(t) \\ Q_3(t) &= a_3 - b_3 p_3(t) + dp_2(t) - kp_1(t) \end{aligned} \tag{1}$$

where a_i, b_i, d, k are positive constant. $b_2, b_3 > d$,that is, the impact of the price of the product itself on the demand is greater than the impact of the substitute goods.

Each business's cost function can be written:

$$C_i(t) = c_i Q_i(t), i = 1,2,3 \tag{2}$$

The profit of business i can be written:

$$\Pi_i(t) = p_i(t)Q_i(t) - C_i(t) = Q_i(t)\left(p_i(t) - c_i\right), i = 1,2,3 \tag{3}$$

According to the hypothesis, the dynamic price adjustment mechanism for each merchant is as follows:

$$p_i(t+1) = p_i(t) + \alpha_i p_i(t) \frac{\partial \Pi_i(t)}{\partial p_i(t)}, i = 1,2,3. \tag{4}$$

where α_i is coefficient that capture the speed at which business i adjust its price according to the consequent marginal change in its profit, respectively.

Each business's marginal profit can be written:

$$\begin{aligned}
\frac{\partial \Pi_1(t)}{\partial p_1(t)} &= a_1 - 2b_1 p_1(t) + b_1 c_1 \\
\frac{\partial \Pi_2(t)}{\partial p_2(t)} &= a_2 - 2b_2 p_2(t) + d p_3(t) - k p_1(t) + b_2 c_2 \\
\frac{\partial \Pi_3(t)}{\partial p_3(t)} &= a_3 - 2b_3 p_3(t) + d p_2(t) - k p_1(t) + b_3 c_3
\end{aligned} \tag{5}$$

we can get the one master-two slaves price game model by substituting (5) into (4):

$$\begin{cases}
p_1(t+1) = p_1(t) + \alpha_1 p_1(t)\left(a_1 - 2b_1 p_1(t) + b_1 c_1\right) \\
p_2(t+1) = p_2(t) + \alpha_2 p_2(t)\left(a_2 - 2b_2 p_2(t) + d p_3(t) - k p_1(t) + b_2 c_2\right) \\
p_3(t+1) = p_3(t) + \alpha_3 p_3(t)\left(a_3 - 2b_3 p_3(t) + d p_2(t) - k p_1(t) + b_3 c_3\right)
\end{cases} \tag{6}$$

2.4 Equilibrium point and stability analysis of the model

In order to study the stability of dynamical systems (6), according to the definitions of fixed points, let $p_i(t+1) = p_i(t)$,We can get the eight fixed points as follows:

$$E_1 = (0,0,0), E_2 = \left(0, 0, \frac{a_3 + b_3 c_3}{2b_3}\right), E_3 = \left(0, \frac{a_2 + b_2 c_2}{2b_2}, 0\right), E_4 = \left(\frac{a_1 + b_1 c_1}{2b_1}, 0, 0\right),$$

$$E_5 = \left(0, \frac{2a_2 b_3 + 2b_2 b_3 c_2 + a_3 d + b_3 c_3 d}{4b_2 b_3 - d^2}, \frac{2a_3 b_2 + 2b_2 b_3 c_2 + a_2 d + b_2 c_2 d}{4b_2 b_3 - d^2}\right),$$

$$E_6 = \left(\frac{a_1 + b_1 c_1}{2b_1}, 0, \frac{2a_3 b_1 + 2b_1 b_3 c_3 - a_1 k - b_1 c_1 k}{4b_1 b_3}\right),$$

$$E_7 = \left(\frac{a_1 + b_1 c_1}{2b_1}, \frac{2a_2 b_1 + 2b_1 b_2 c_2 - a_1 k - b_1 c_1 k}{4b_1 b_2}, 0\right),$$

$$E^* = \left(p_1^*, p_2^*, p_3^*\right)$$

Where $p_1^* = \dfrac{a_1 + b_1 c_1}{2b_1}$

$$p_2^* = \frac{4a_2 b_1 b_3 + 4b_1 b_2 b_3 c_2 + 2a_3 b_1 d + 2b_1 b_3 c_3 d - k\left(2b_3 + d\right)\left(a_1 + b_1 c_1\right)}{8b_1 b_2 b_3 - 2b_1 d^2}$$

$$p_3^* = \frac{4a_3 b_1 b_2 + 4b_1 b_2 b_3 c_3 + 2a_2 b_1 d + 2b_1 b_2 c_2 d - k\left(2b_2 + d\right)\left(a_1 + b_1 c_1\right)}{8b_1 b_2 b_3 - 2b_1 d^2}$$

According to the literature of Li[21], $E_1, E_2, \ldots, E_7$ are the unstable bounded equilibrium point, only the Nash equilibrium point E^* has the economic meaning when

$$\begin{cases}
p_1^* > 0 \\
p_2^* > 0 \\
p_3^* > 0
\end{cases} \tag{7}$$

The local stability of each equilibrium point is studied below,the Jacobian matrix of System (6) at point $(p_{1,}, p_2, p_3)$ is needed:

$$J=\begin{bmatrix} 1+\alpha_1(a_1-4b_1p_1+b_1c_1) & 0 & 0 \\ -\alpha_2kp_2 & 1+\alpha_2(a_2-4b_2p_2+dp_3-kp_1+b_2c_2) & \alpha_2dp_2 \\ -\alpha_3kp_3 & \alpha_3dp_3 & 1+\alpha_3(a_3-4b_3p_3+dp_2-kp_1+b_3c_3) \end{bmatrix} \tag{8}$$

The stability of each equilibrium point can be investigated by the eigenvalues of the characteristic polynomial of its Jacobian matrix.

The Jacobian matrix of E_1 is

$$J(E_1)=J(0,0,0)=\begin{bmatrix} 1+\alpha_1(a_1+b_1c_1) & 0 & 0 \\ 0 & 1+\alpha_2(a_2+b_2c_2) & 0 \\ 0 & 0 & 1+\alpha_3(a_3+b_3c_3) \end{bmatrix}$$

Easy to get its eigenvalues are $\lambda_i=1+\alpha_i(a_i+b_ic_i)$, Obviously, $\lambda_i>1(i=1,2,3)$, so E_1 is an unstable equilibrium point. Similarly, the above seven bounded equilibria are unstable.

The local stability of Nash equilibrium point E^* is studied below.the Jacobian matrix of the E^* is

$$J(E^*)=\begin{bmatrix} 1-2\alpha_1b_1p_1^* & 0 & 0 \\ -\alpha_2kp_2^* & 1-2\alpha_2b_2p_2^* & \alpha_2dp_2^* \\ -\alpha_3kp_3^* & \alpha_3dp_3^* & 1-2\alpha_3b_3p_3^* \end{bmatrix}$$

By calculating, we know that its characteristic polynomial is $p(\lambda)=\lambda^3+A\lambda^2+B\lambda+C$

Where $A=2\alpha_1b_1p_1^*+2\alpha_2b_2p_2^*+2\alpha_3b_3p_3^*-3$

$$B=\left(2\alpha_1b_1p_1^*-1\right)\left(2\alpha_2b_2p_2^*+2\alpha_3b_3p_3^*-2\right)+\left(2\alpha_2b_2p_2^*-1\right)\left(2\alpha_3b_3p_3^*-1\right)-\alpha_2\alpha_3d^2p_2^*p_3^*$$

$$C=\left(2\alpha_1b_1p_1^*-1\right)\left(2\alpha_2b_2p_2^*-1\right)\left(2\alpha_3b_3p_3^*-1\right)-\alpha_2\alpha_3d^2p_2^*p_3^*\left(2\alpha_1b_1p_1^*-1\right)$$

According to the Jury conditions, the necessary and sufficient condition of the local stability of Nash equilibrium point should satisfy the following conditions:

$$\begin{cases} 1+A+B+C>0 \\ 1-A+B-C>0 \\ |C|<1 \\ |B-AC|<1-C^2 \end{cases} \tag{9}$$

The above (9) formula is the stability condition for Nash equilibrium. Because of its complexity, we give numerical simulation analysis.

2.5 numerical simulation analysis

For the sake of convenience, consider the actual market competition,we assign some values to parameter:

$$\begin{aligned} &a_1=6, a_2=5, a_3=4.5, b_1=0.5, b_2=0.4, b_3=0.5, c_1=0.3, c_2=0.35, \\ &c_3=0.25, d=0.2, k=0.5, p_1(0)=6, p_2(0)=2, p_3(0)=1.5 \end{aligned} \tag{10}$$

It is obvious that, the parameters satisfy (7), which is no business will exit the market.Right now, $E^* = (6.15, 3.125, 2.175)$.

According to (9), we can get 3-dimensional stability region of the Nash equilibrium point (see Fig.1).A two-dimensional stable domain can be obtained by projecting the 3D stable domain to each plane.

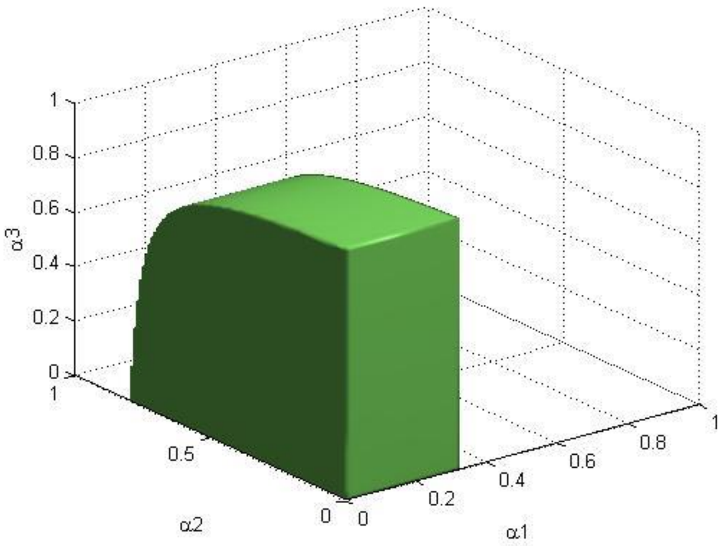

Fig. 1: 3-dimensional stability region of the Nash equilibrium point in the space $(\alpha_1, \alpha_2, \alpha_3)$.

When $\alpha_2 = 0.5$, $\alpha_3 = 0.5$,figure 2 shows the price bifurcation diagram of the system with α_1 .

Similarly, figures 3 and 4 show the price bifurcation diagram of the system with α_2 , α_3 respectively. We can see that p_2 with p_3 have similar evolutionary trajectory, but p_1 always remain the same. The reason is that we assume that retailer 1 and retailer 2 have no effect on the price setting of the manufacturer.

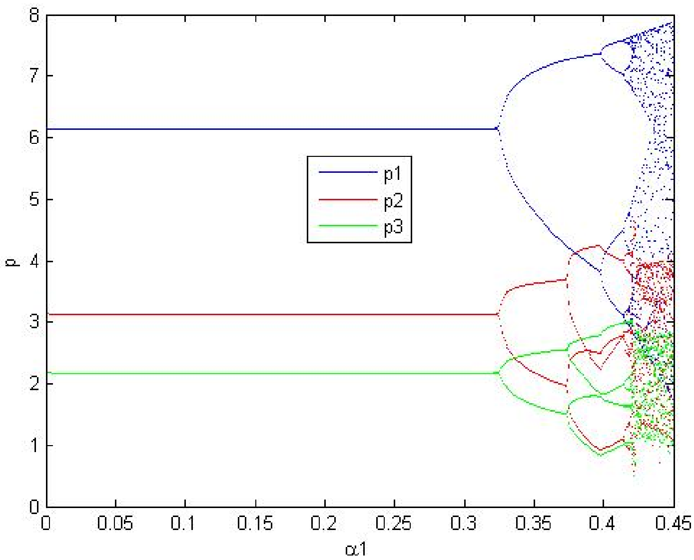

Fig. 2: Bifurcation diagram of price with respect to α_1 when $\alpha_2 = 0.5, \alpha_3 = 0.5$

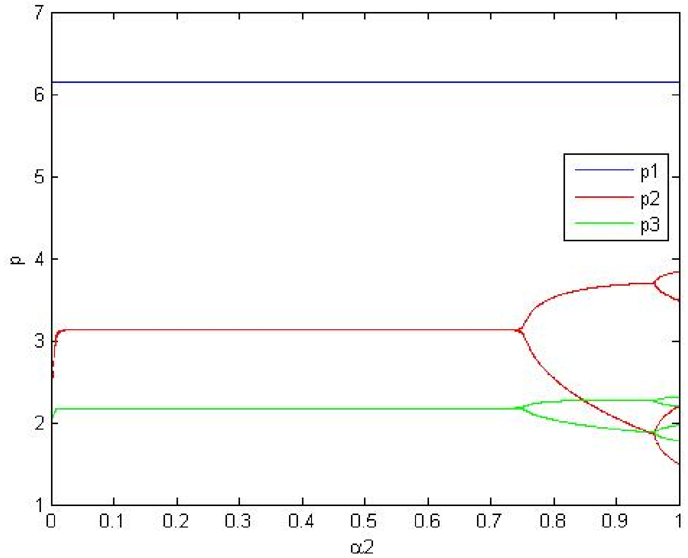

Fig. 3: Bifurcation diagram of price with respect to α_2 when $\alpha_1 = 0.3, \alpha_3 = 0.5$

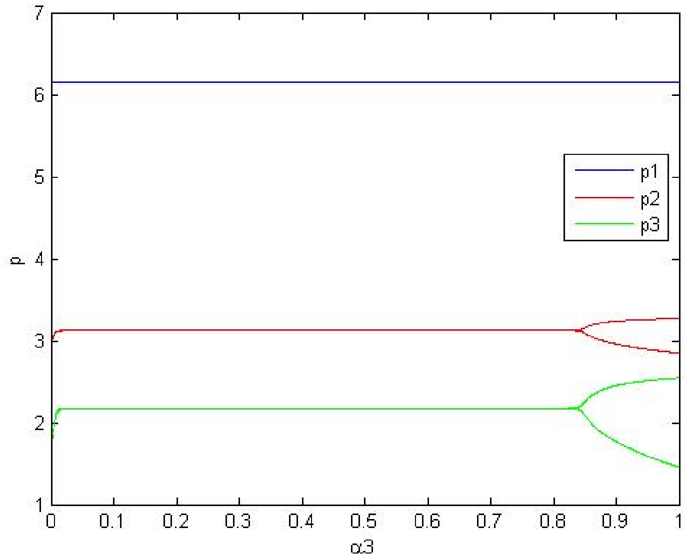

Fig. 4: Bifurcation diagram of price with respect to α_3 when $\alpha_1 = 0.3, \alpha_2 = 0.5$

Figure 5,6,7show that the singular attractors of the system when $\alpha_1 = 0.45, \alpha_2 = 0.4, \alpha_3 = 0.4$.

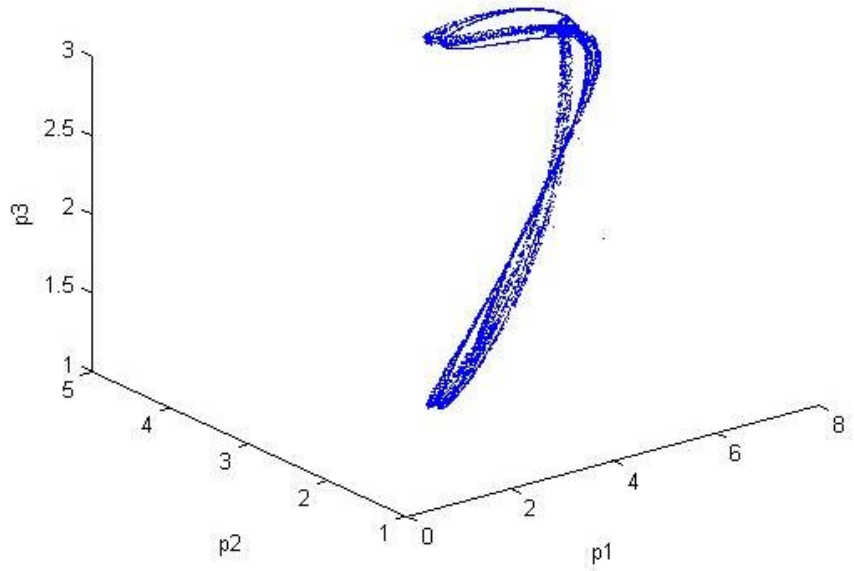

Fig. 5:Strange attractor in (p_1, p_2, p_3) space when $\alpha_1 = 0.45, \alpha_2 = 0.4, \alpha_3 = 0.4$

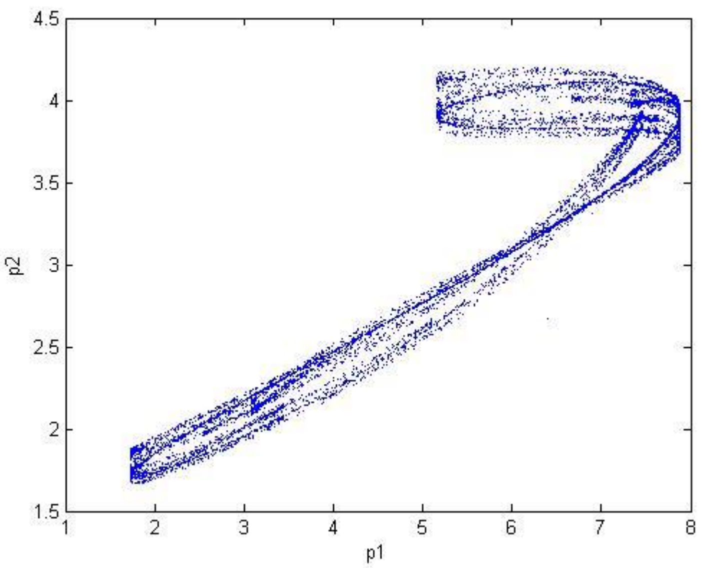

Fig. 6: Strange attractor in (p_1, p_2) plane when $\alpha_1 = 0.45, \alpha_2 = 0.4, \alpha_3 = 0.4$

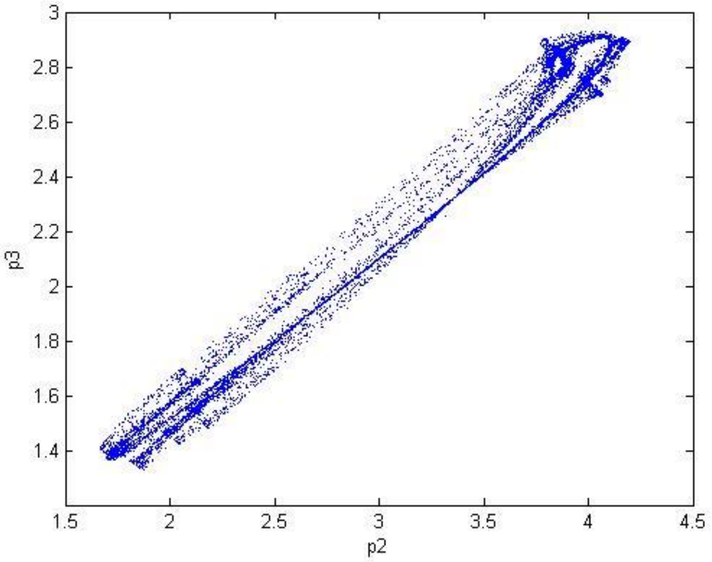

Fig. 7: Strange attractor in (p_2, p_3) plane when $\alpha_1 = 0.45, \alpha_2 = 0.4, \alpha_3 = 0.4$

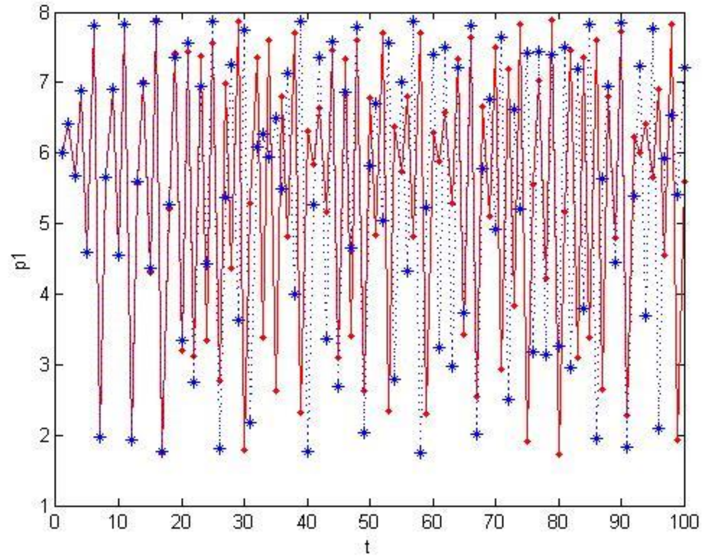

Fig. 8: Sensitive dependence on initial conditions for system (6)

Fig. 8 is to verify that the system (6) is sensitive to initial conditions.The initial value of the red curve in the diagram is $(p_1(0), p_2(0), p_3(0)) = (6.0, 2.0, 1.5)$,the blue curve in the diagram is $(p_1(0), p_2(0), p_3(0)) = (6.0001, 2.0, 1.5)$ .we can see that subtle changes in initial prices can have a major impact on the outcome.

3. delaying bounded rationality price game model

3.1 The model

the $p_i(t+1)$ in period $t+1$ is always adjusted based on the previous price $p_i(t)$ .Sometimes considered $p_i(t-1)$, $p_i(t-2)$, $p_i(t-3)$ and so on.The price adjustment strategy is stated as follows:

$$p_i(t+1) = p_i(t) + \alpha_i p_i(t) \frac{\partial \Pi_i(p^D)}{\partial p_i^D}, \ i = 1, 2, 3 \tag{11}$$

Where $p^D = \left(p_1^D, p_2^D, \cdots, p_T^D\right)$, $P_i^D = \sum_{l=0}^{T} w_i p_i(t-l)$, $w_l \geq 0, \sum_{l=0}^{T} w_l = 1$, $l = 0,1,\cdots,T$.

For simplicity, let $T = 1$,That is, when the merchant determines the product price of the $t+1$ period, it will consider the marginal profits of the t period and the $t-1$ period, The discrete systems as follows:

$$\begin{cases} p_1(t+1) = p_1(t) + \alpha_1 p_1(t)\left\{a_1 - 2b_1\left[(1-w_1)p_1(t) + w_1 p_1(t-1)\right] + b_1 c_1\right\} \\ p_2(t+1) = p_2(t) + \alpha_2 p_2(t)\left\{a_2 - 2b_2\left[(1-w_2)p_2(t) + w_2 p_2(t-1)\right] + \right. \\ \qquad \left. d\left[(1-w_2)p_3(t) + w_2 p_3(t-1)\right] - k\left[(1-w_2)p_1(t) + w_2 p_1(t-1)\right] + b_2 c_2\right\} \\ p_3(t+1) = p_3(t) + \alpha_3 p_3(t)\left\{a_3 - 2b_3\left[(1-w_3)p_3(t) + w_3 p_3(t-1)\right] + \right. \\ \qquad \left. d\left[(1-w_3)p_2(t) + w_3 p_2(t-1)\right] - k\left[(1-w_3)p_1(t) + w_3 p_1(t-1)\right] + b_3 c_3\right\} \end{cases} \tag{12}$$

When $w_i = 0$,the system (12) is the system (6).

In order to facilitate the study, the system (12) is changed into the following six-dimensional system:

$$\begin{cases} u_i(t+1) = p_i(t) \quad (i = 1,2,3) \\ p_1(t+1) = p_1(t) + \alpha_1 p_1(t)\left\{a_1 - 2b_1\left[(1-w_1)p_1(t) + w_1 u_1(t)\right] + b_1 c_1\right\} \\ p_2(t+1) = p_2(t) + \alpha_2 p_2(t)\left\{a_2 - 2b_2\left[(1-w_2)p_2(t) + w_2 u_2(t)\right] + \right. \\ \qquad \left. d\left[(1-w_2)p_3(t) + w_2 u_3(t)\right] - k\left[(1-w_2)p_1(t) + w_2 u_1(t)\right] + b_2 c_2\right\} \\ p_3(t+1) = p_3(t) + \alpha_3 p_3(t)\left\{a_3 - 2b_3\left[(1-w_3)p_3(t) + w_3 u_3(t)\right] + \right. \\ \qquad \left. d\left[(1-w_3)p_2(t) + w_3 u_2(t)\right] - k\left[(1-w_3)p_1(t) + w_3 u_1(t)\right] + b_3 c_3\right\} \end{cases} \tag{13}$$

3.2 numerical simulation analysis

Let $\alpha_2 = \alpha_3 = 0.5, w_2 = w_3 = 0$,Figure 9-Figure 12 show the price bifurcation diagram of the system with α_1 when $w_1 = 0.1, 0.2, 0.4, 0.7$ respectively. As compared with figure 2($w_1 = 0$), we can see that in figure 9- Figure 11, the range of α_1 that makes the system stable is increased, but in figure 12,the range of α_1 that makes the system stable is reduced. It's easy to get that when w_1 has other values, we will get a similar result. So, we can get the conclusion: an appropriate delay weighting factor can enhance the stability of the system; an inappropriate delay weight factor will reduce the stability of the system.

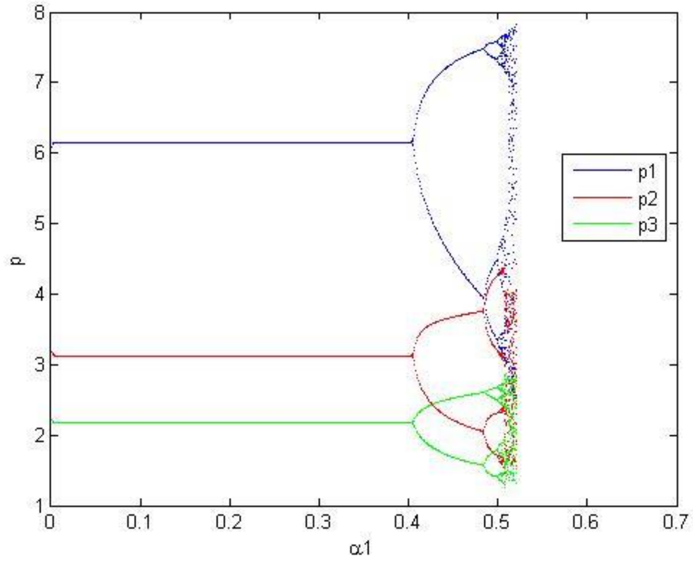

Fig. 9: Bifurcation diagram of price with respect to α_1 with delay $w_1 = 0.1, w_2 = w_3 = 0$

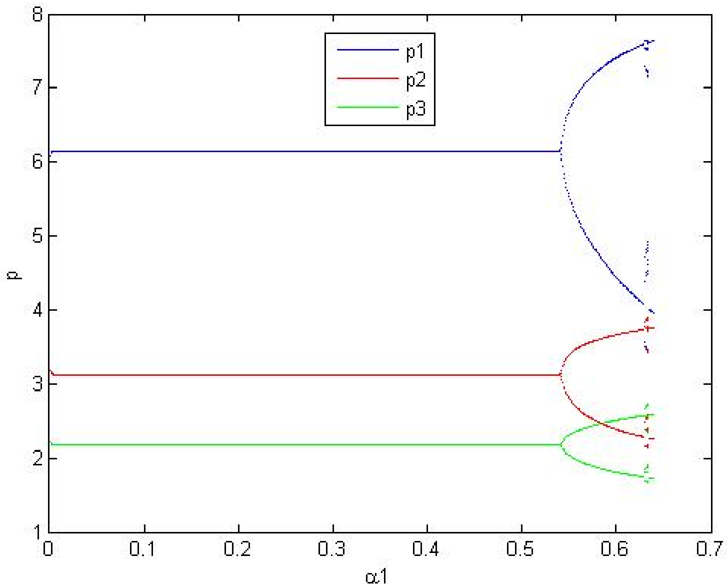

Fig. 10: Bifurcation diagram of price with respect to α_1 with delay $w_1 = 0.2, w_2 = w_3 = 0$

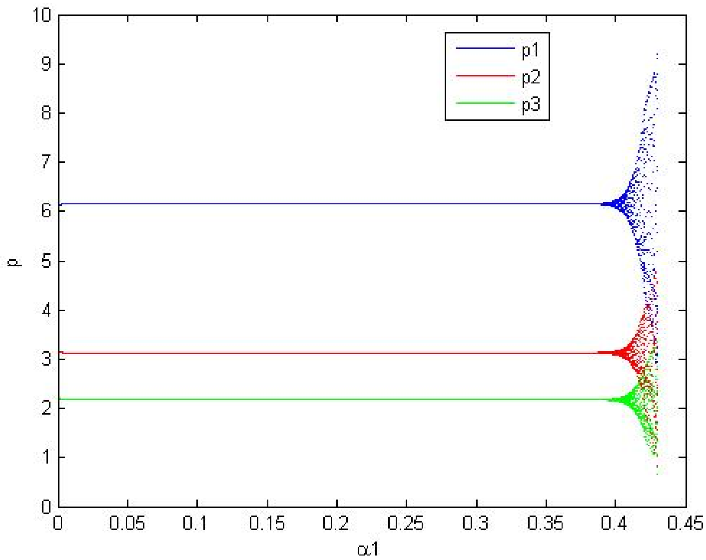

Fig. 11: Bifurcation diagram of price with respect to α_1 with delay $w_1 = 0.4, w_2 = w_3 = 0$

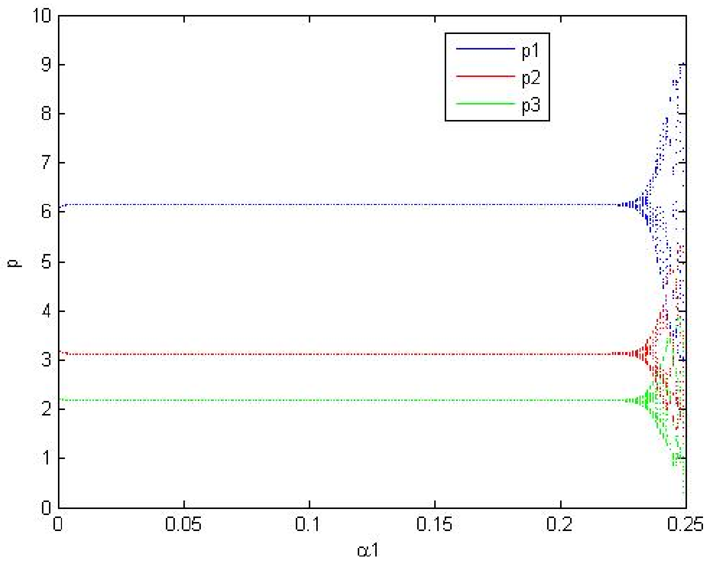

Fig. 12: Bifurcation diagram of price with respect to α_1 with delay $w_1 = 0.7, w_2 = w_3 = 0$

Let $\alpha_1 = 0.45, \alpha_2 = \alpha_3 = 0.5$,At this point the system is unstable,that's because α_1 beyonds the stability domain.figure 13 shows the price bifurcation diagram of the system with w_1 when $w_2 = w_3 = 0$ .As we can see from the diagram, an appropriate delay coefficient, w_1 , increases the system from chaos to stability and then to chaos.figure 14 shows the price bifurcation diagram of the system with w_i ,It has the same trajectory as figure 13.This means that even more businesses adopt delayed decision, and no further enhance stability. Figure 15 shows a three dimensional singular attractor of a system approaching its Nash equilibrium point when $w_1 = 0.25, w_2 = w_3 = 0$ .Compare Figure 15 (with delay) with figure 5 (without delay), again, it is proved that the proper delay coefficient will enhance the stability of the system.

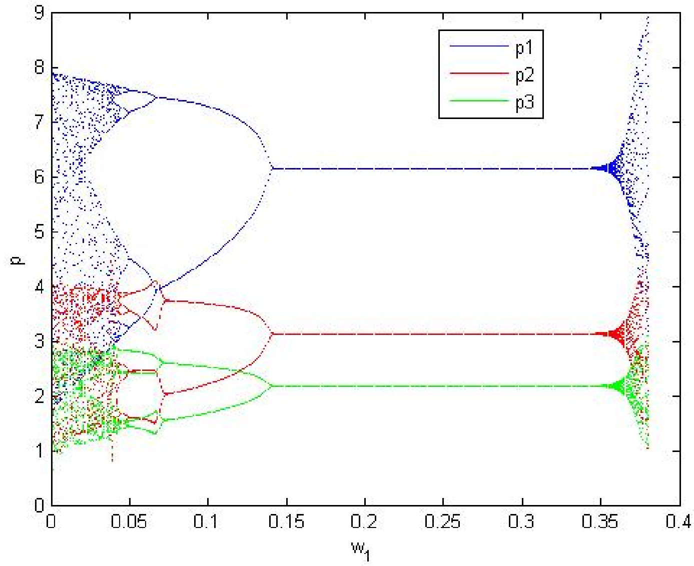

Fig. 13: Bifurcation diagram of price with respect to w_1 when $w_2 = w_3 = 0$

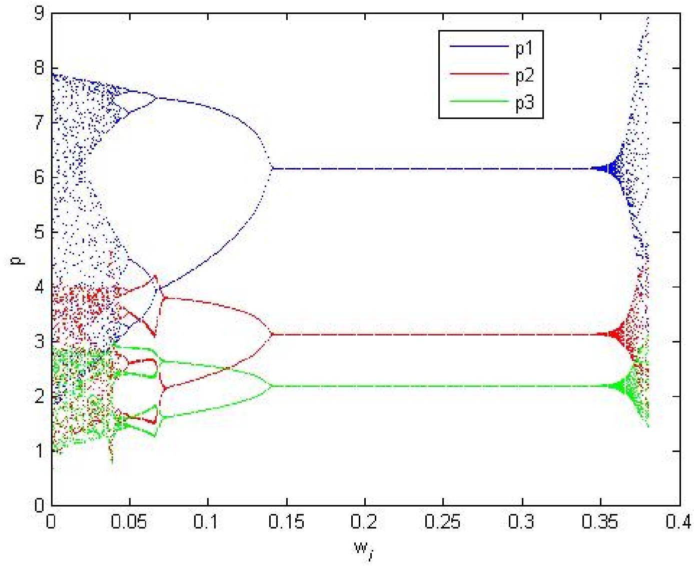

Fig. 14:Bifurcation diagram of price with respect to $w_i(i = 1, 2, 3)$

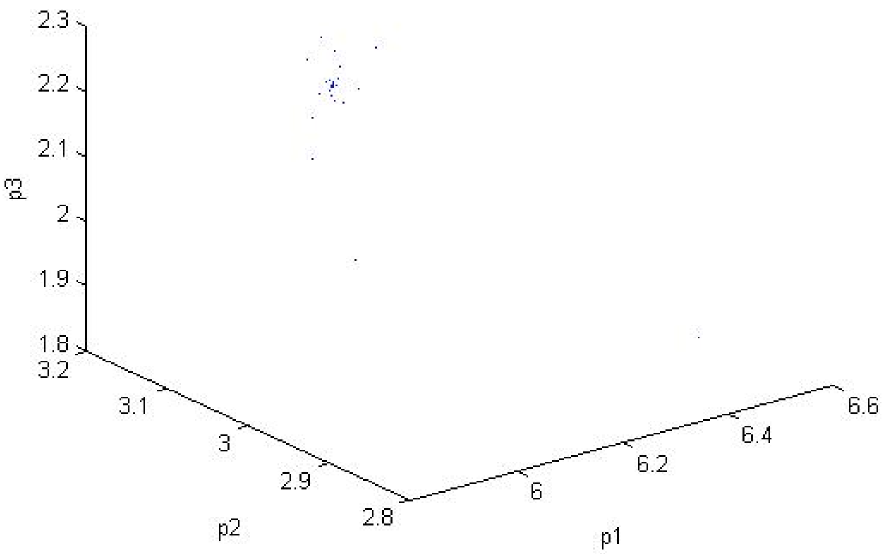

Fig. 15: Three-dimensional strange attractors when $w_1 = 0.25, w_2 = w_3 = 0$

Let $\alpha_1 = 0.3, \alpha_2 = \alpha_3 = 0.9$,at this time, α_1 is in the stable domain, α_2, α_3 beyond the stability domain. Figures 16 and 17 show only the price bifurcation diagram of retailer 1 or only retailer 2 use delayed decision. Figure 18 shows the price bifurcation diagram when two retailers all use delayed decision.At this point, more businesses adopt delayed decisions that will enhance system stability. Figure 19 shows the price bifurcation diagram of the system with $w_i (i = 1, 2, 3)$,it has the same trajectory as figure 18.At this point, the stability of the system has not been further enhanced. Therefore, it cannot be said that more businesses using delayed decision-making will make the stability of the system increase.

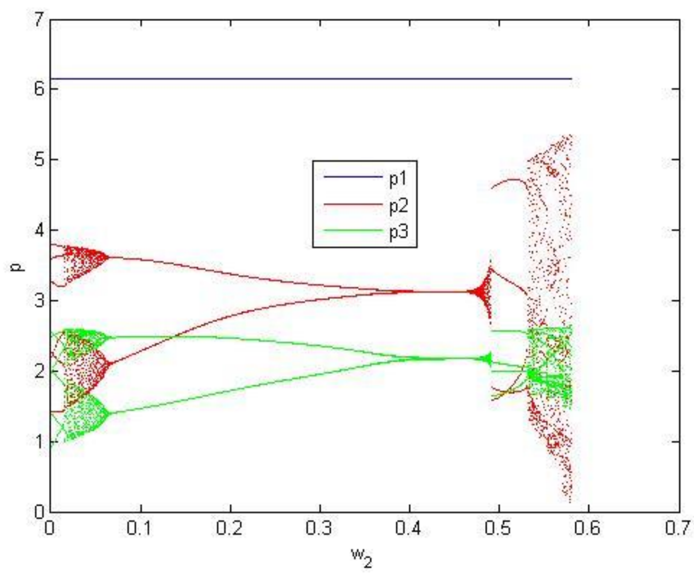

Fig. 16: Bifurcation diagram of price with respect to w_2 when $w_1 = w_3 = 0$

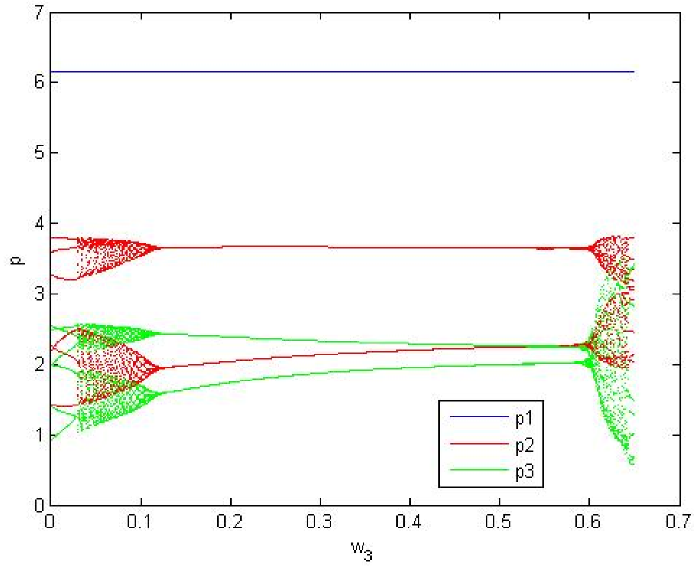

Fig. 17: Bifurcation diagram of price with respect to w_3 when $w_1 = w_2 = 0$

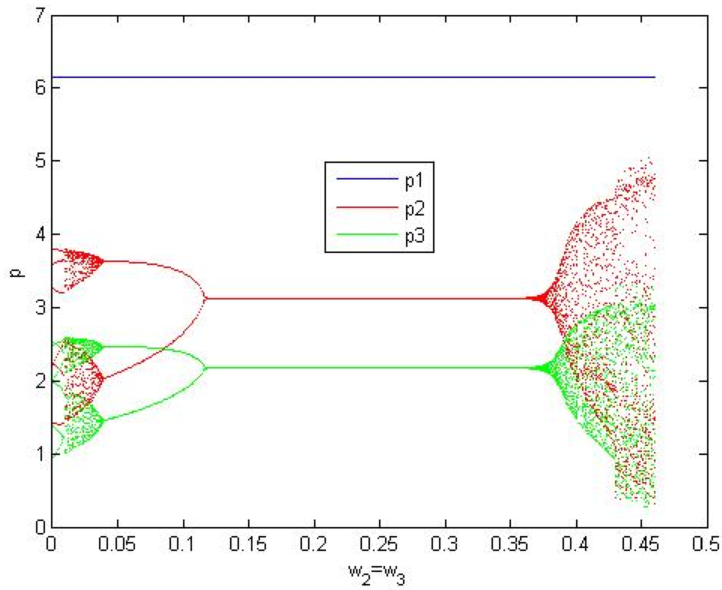

Fig. 18: Bifurcation diagram of price with respect to w_2 and w_3 when $w_1 = 0$

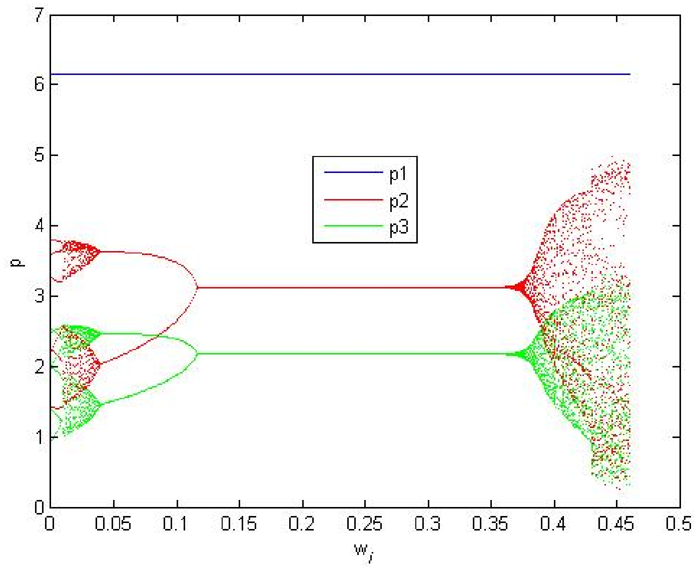

Fig. 19: Bifurcation diagram of price with respect to $w_i(i=1,2,3)$

Let $\alpha_1 = 0.3, \alpha_2 = \alpha_3 = 0.5$,the system is stable.figure 20 shows the price bifurcation diagram of the system with $w_i(i=1,2,3)$ .the system from stability to chaos with the bigger of w_i .Figure 21 shows three-dimensional strange attractors with $w_i = 0.56$ .This further proves that the delay decision does not necessarily increase the stability of the system, and the stability is mainly determined by the size of the delay coefficient.

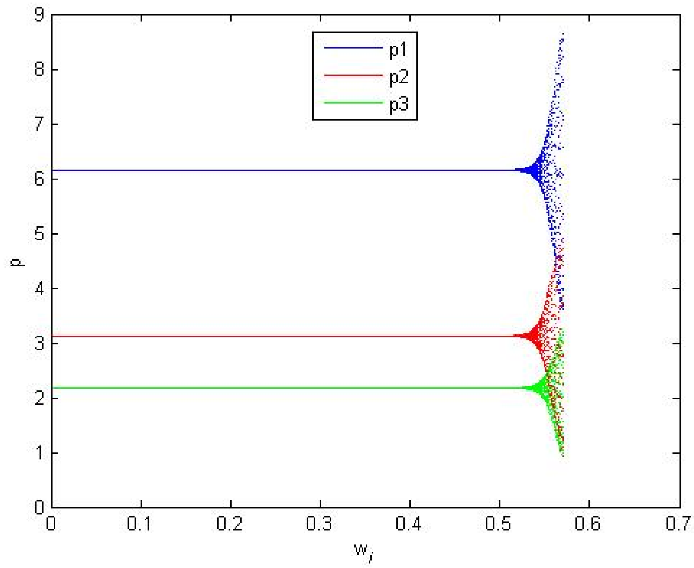

Fig. 20: Bifurcation diagram of price with respect to $w_i(i=1,2,3)$

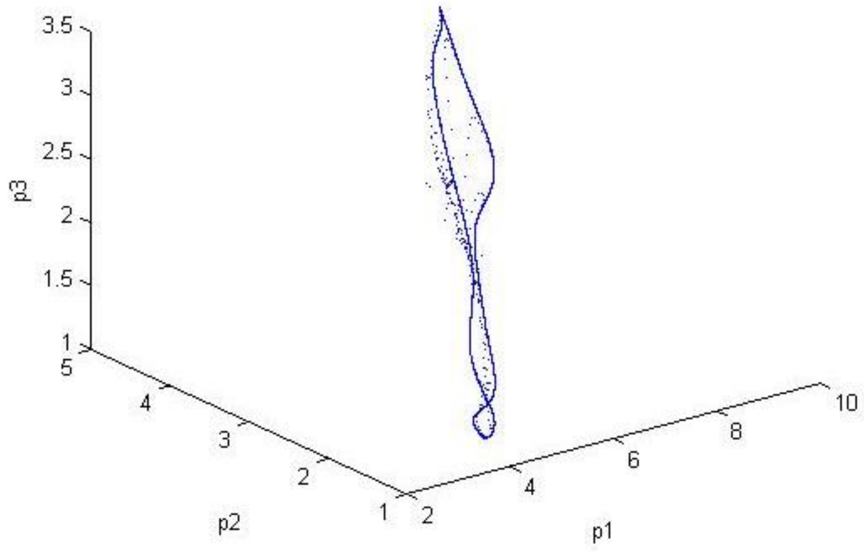

Fig. 21: Three-dimensional strange attractors with $w_i = 0.56$

4. Conclusions

This paper established a one master-two slaves price game model with bounded rationality, the stability of the model is analyzed. And the decision delay into the model, and analyses its influence on the stability of the model. The results showed that the appropriate delay coefficient can enhance the stability of the system; delay coefficient of inappropriate will reduce the stability of the system moreover, the system stability is mainly determined by the delay coefficient. In addition, not simply that more businesses with delayed decision can improve the stability of the system.

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