

Application of shadowing Filter in Weather Predictability

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Abstract: The physical meaning of the Ikeda model is analyzed. The influence of the parameters in the model on the state of the solution is discussed. The change process of the solution when the parameters are taken from different values is described, and the sensitivity of the solution to the initial value is studied. The tail filter is used to optimize the decision function through the gradient descent method, filtering the observation error, and obtaining the initial field which is closer to the true value.

Keywords: shadowing filter, weather forecast

1. Introduction

With the continuous development and perfection of the system model, numerical weather forecasting is increasingly attributed to the initial value problem. A clear analysis of the solution state of the model is helpful to understand the close relationship between the initial value and predictability and to highlight the importance of proper processing of the initial observation. Through the discussion of the predictability problem, the differences between the results obtained before and after filtering are compared, which can provide example support for the effective filtering error of the trailing filter and is helpful for the further popularization and application of the trailing filter in data optimization. At the same time, the filtered observation value can be regarded as a true value in a certain sense, and the accurate prediction results can be obtained by the initial observation within a certain error range. For example, the lower bound of the maximum predictable time instead of the maximum predictable time can be finally obtained, thus improving the prediction efficiency.

The numerical weather forecast uses a computer to calculate the development of the atmosphere in the next few days based on the weather observation data provided. Since the time integration of an atmospheric model belongs to the initial value problem, the ability to make accurate forecasts requires not only that the computer operating model can realistically represent the atmosphere, but also that the initial conditions of the atmosphere are considered accurate. With the continuous development and improvement of the model, the accuracy requirements for the initial conditions are also increasing day by day. Physicist Bjerknes once attributed the weather forecast to the initial value problem. Lorenz pointed out: after a possible transition period, stable systems with constant or periodic orbits will have trajectories starting from different points very close to each other, that is to say, they are completely predictable. As a kind of unstable dynamical system, even when the model is perfect and the initial conditions are almost completely correct, for example, only the rounding error of the initial values is considered, the two solutions obtained by the integration of the same model are very different in a few weeks, which shows that their predictability is limited. The research on predictability is concerned with the uncertainty of numerical weather forecast, which is mainly divided into problems related to initial error and problems related to model error. The former is the main research object. This project is also based on the assumption of perfect model. Furthermore, Mumu and others divided the predictability problem into three sub-problems: the lower bound of the maximum predictable time, the upper bound of the maximum prediction error, and the maximum allowable initial error and model error. The first two sub-problems consider the maximum prediction time that can be reached by a given prediction error and the maximum prediction error that can be reached by a given prediction time after the actual value is unknown but the error range between the given observation value and the actual value is taken as the initial field for numerical prediction. Since the true value is only included in the observation error range and the specific value is not known, the lower bound of the maximum predictable time can only be obtained by taking the minimum within the observation error range or the upper bound of the maximum predictive error can be obtained within the observation error range. As a method of filtering errors, Thomas Stemler and Kevin Judd [2] reveal that shadowing filter is processing the original observation data to improve the approximation degree between the original observation data and the true value, and finally achieve the effectiveness of improving the prediction effect.

2. Shadowing filters by gradient descent of indeterminism

Although the shadowing filter can be implemented in more general imperfect model situations, for the purposes of this discussion we will consider the perfect model scenario with isotropic Gaussian noise. Consider a discrete time dynamical system on \mathbb{R}^d with a dynamics given by the map $y_{t+1} = g(y_t)$. Assume $s_t = y_t + \xi$ is our observation of y_t , where ξ are independent Gaussian random variates with an isotropic variance σ^2 . Also assume to have a model f of the system that is identical to g and that f is differentiable. The task of a shadowing filter is to find a sequence of states $X = (x_1, x_2 \dots x_n)$ from a given sequence of observations $S = (s_1, s_2 \dots s_n)$. X should be a trajectory of the model f and should shadow S . For X to be a trajectory requires $x_{i+1} = f(x_i)$, $i = 1, \dots, n-1$. The trajectory will shadow S if the distances $\|s_i - x_i\|$ $i = 1, \dots, n$ are not large relative to σ .

One method of implementing a shadowing filter is gradient descent of indeterminism (GDI). For any sequence of states X define the indeterminism:

Define for any sequence of X its indeterminism:

$$I(X) = \frac{1}{n-1} \sum_{i=1}^{n-1} \|x_{i+1} - f(x_i)\|^2 \quad (1)$$

The sequence of states X can be considered as a point in \mathbb{R}^{nd} , and $I(X)$ is a scalar function on this a $(n \times d)$ -dimensional space. Clearly, X will be a trajectory of the model if and only if $I(X) = 0$. The indeterminism should be interpreted as a measure of how far a sequence of states X is from being a trajectory. The squared norm $\|\cdot\|^2$ can be replaced by any appropriate metric. Using a squared norm does not imply any assumption about a Gaussian distribution mismatches or otherwise, see elsewhere [1] for further discussion of this point. For the observations S the indeterminism $I(S)$ is almost surely non-zero. The gradient descent method takes the observations S as a starting sequence of states X , then follows the steepest descent of the gradient of $I(X)$ down to a minimum where $I(X) = 0$. This is equivalent to solving the differential equations on \mathbb{R}^{nd} : $\frac{dX}{d\tau} = -\nabla I(X(\tau))$, $X(0) = S$. The GDI shadowing filter is the result of taking the limiting sequence $X(\tau)$ as $\tau \rightarrow \infty$.

A more practical method is to solve the differential equation by a Euler iteration until suitable convergence is achieved, which provides an iterative GDI shadowing filter. Let $X_0 = s$ and $X_m = (x_{1,m} \dots x_{n,m})$, where

$$x_{i,m+1} = x_{i,m} - \frac{2\Delta}{n-1} \times \begin{cases} -A(x_{i,m})(x_{i+1,m} - f(x_{i,m})), & i = 1 \\ x_{i,m} - f(x_{i-1,m}) - A(x_{i,m})(x_{i+1,m} - f(x_{i,m})), & 1 < i < n \\ x_{i,m} - f(x_{i-1,m}), & i = n \end{cases} \quad (2)$$

Here $A(x)$ denotes the adjoint of f (transpose of the Jacobian matrix) evaluated at x , and Δ is an arbitrary step size. Later on we outline a method to find suitable values for the step size Δ , but typically the choice $2\Delta/(n-1) = 0.1$ will lead to a convergence of the iterative GDI shadowing filter. Details on the properties of such GDI shadowing filters are given elsewhere [5] Here we just want to mention that the GDI method always converges to a shadowing trajectory of the model and $I(X)$ converges monotonically to zero. Furthermore, given a long observation sequence with sufficiently small bounded measurement noise of a hyperbolic system it can be shown that for perfect models the GDI shadowing filter converges to the true trajectory. In practice we will iterate eq. (2) until X_m has converged sufficiently. The remaining magnitude I_m is one quantity that measures the quality of the estimated states. In addition we define below three other quantities that measure the quality: The magnitude of mismatch $I_{n,m}$, the root mean

$$I_{n,m} = \|x_{n,m} - f(x_{n-1,m})\| \quad (3)$$

$$E_m = \sqrt{\frac{1}{n} \sum_{i=1}^n \|x_{i,m} - y_i\|^2} \quad (4)$$

$$E_{n,m} = \|x_{n,m} - y_n\| \quad (5)$$

square error E_m and the last point error $E_{n,m}$:

In practice we will not know y_i and therefore are unable to evaluate E_m and $E_{n,m}$. Nevertheless, they can be used to assess successful implementation using artificial data. We are also interested in comparing the forecast $f_t(x_{n,m})$ with the future states y_{n+t} . To measure this forecast quality one can evaluate the separation time.

3. Application of the shadowing filter

Through the study of the state and stability of the Ikeda model, the important influence of the initial value on the change form of the solution is revealed, the cause of the predictability and the core technology to improve the prediction effect are clarified. It is proved that the trailing filter is an effective method of filtering error and through the tail of the lower bound of the maximum predictable time. With the true value of filtering, the concept of maximum prediction time and maximum prediction error is proposed.

The instability of the atmospheric system results in sensitivity to initial values. According to the stability theory of differential equations, the different values of the parameters in the control equation determine whether the chaotic phenomenon appears, and when the chaos appears, when the initial value is located, the state of the initial value will lead to a violent deviation with the change of the initial value. Therefore, in the study of predictability, first of all, we must make a clear analysis of the equation of the dynamic system and the stability and stability of the solution, so as to select the appropriate value and initial value, and do a good job of preparation for the further research. The stability theory of differential equations has been developed for hundreds of years, and has become a very important field in the study of differential equations. The number of papers and books related to it is huge. Therefore, the Ikeda model can be used for reference to the introduction of relevant knowledge.

$$\begin{aligned} u_{t+1} &= 1 + \mu(u_t \cos(\theta_t) - v_t \sin(\theta_t)) \\ v_{t+1} &= \mu(u_t \sin(\theta_t) + v_t \cos(\theta_t)) \\ \theta_t &= a - b/(1 + u_t^2 + v_t^2) \end{aligned}$$

Among them, the corresponding stability analysis is carried out. How the parameters affect the shape of the phase diagram and how the initial trajectories of different regions correspond to the development trend are important issues to be discussed.

According to the results of the stability study of the Ikeda model, the parameters and initial values of the chaotic phenomena can be determined. Because all the projects are ideal experiments, the solutions can be used as the real trajectories. In order to get the initial value of the observation with a certain error, the initial disturbance is obtained by adding the normal disturbance to each component of the solution. Because the solution is sensitive to the initial value, it is of great significance to how to use the appropriate method to effectively filter the error. It is of great significance to the degree of change of the flat remission and the accuracy of the prediction results. Thomas Stemler and Kevin Judd reveal that the shadowing filter (shadowing filter) can improve the degree of approximation between the original observation data and the true value, and finally improves the effectiveness of the prediction effect. This project follows the method of filtering initial observation error by trailing filter, through the decision function.

$$I(X) = \frac{1}{n} \sum_{i=0}^{n-1} \|x_{i+1} - f(x_i)\|^2$$

The gradient descent method is used to optimize the initial guess value. In the decision function, the state value of the I moment is expressed, which represents the one-step time integral of the state variables. In the optimization of the gradient descent method, the tangent linear model corresponding to the model is first required, then the adjoint model is obtained, and finally the gradient information of the decision function is obtained, and the fastest direction of the function value drops is determined, and each vector is optimized according to a certain step in the direction of the income, and the optimization process can be obtained. In order to continue, we know that the order of decision function is reduced to a threshold, and the optimization work of the initial observation of state variables can be concluded and the best initial guesses are obtained.

$$T_\varepsilon := \max_{\tau} \left\{ \tau \mid \|M_t(q_0^{obs}) - q'_t\| \leq \varepsilon, 0 \leq t \leq \tau \right\}$$

Since the true value of the time is impossible to obtain, according to the initial error information, a permissible perturbation is added to the initial observation, and the lower bounds of the maximum predictability time are obtained by replacing the true value and minimizing the perturbed set.

$$T_{el} = \min_{q_0 \in B(q_0^{obs}, \delta)} \left\{ T_{q_0} \mid T_{q_0} = \max_{\tau} \left\{ \tau \mid \|M_t(q_0^{obs}) - M_t(q_0)\| \leq \varepsilon, 0 \leq t \leq \tau \right\} \right\}$$

According to the definition, the deviation between the predicted value and the true value obtained by the propagator will not exceed the maximum permissible error from time to time.

The lower bound of the maximum forecast error is as follows: given the initial observation and forecast time, the prediction error of time is defined as:

$$E = \|M_T(q_0^{obs}) - q_T^t\|$$

Similar to the upper problem, the true value of the moment can not be obtained. The error of the upper definition can not be calculated, and the upper bound of the prediction error is obtained by replacing the true value and the maximum value of the permissible perturbation set.

$$E_u = \max_{q_0 \in B(q_0^{obs}, \delta)} \|M_T(q_0^{obs}) - M_T(q_0)\|$$

According to the definition, the deviation between the predicted value and the true value obtained by the propagator will not exceed the error at all times.

In the above definition, because the initial observations are not processed, the upper or lower bounds can only be obtained. After filtering the initial observation by tail filtering, the optimal result can be approximated as true value and it is used to replace the above predictability formula. So the original formula (1) (2) indicates the initial field of the observation value in the range of the true value of the observed error, and the given prediction error is given. The maximum forecast time and the maximum forecast error at a given time.

4. Conclusion

The physical meaning of the Ikeda model is analyzed. The influence of the parameters in the model on the state of the solution is discussed. The change process of the solution when the parameters are taken from different values is described, and the sensitivity of the solution to the initial value is studied. The tail filter is used to optimize the decision function through the gradient descent method, filtering the observation error, and obtaining the initial field that is closer to the true value.

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