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Daubechies wavelet based full approximation scheme for solving Burgers' equation arising in Fluid Dynamics

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Abstract. This paper presents, Daubechies wavelet based full approximation scheme (DWFAS) for the numerical solution of Burgers' equation, which is nonlinear partial differential equation (PDE) arising in fluid dynamics using Daubechies wavelet intergrid operartors. The numerical solutions obtained are compared with existing numerical methods and exact solution. Some of the test problems are presented to demonstrate that DWFAS has fast convergence in low computational time and is very effective, convenient and quite accurate to systems of PDEs.

Keywords: Daubechies wavelet; Multi-resolution analysis; Full approximation scheme; Burgers' equation.

1. Introduction

Burgers' equation has attracted much attention. Solving this equation has been an interesting task for mathematicians. This equation has been found to describe various kinds of phenomena such as a mathematical model of turbulence and approximate theory of flow through a shock wave traveling in a viscous fluid [1]. Consider one–dimensional non-linear PDE with the following initial and boundary conditions:

$$\frac{\partial u(x,t)}{\partial t} + u(x,t)\frac{\partial u(x,t)}{\partial x} = \upsilon \frac{\partial^2 (x,t)}{\partial x^2} \tag{1.1}$$

Initial condition:

$$u(x,0) = f(x), \quad 0 \le x \le 1 \tag{1.2}$$

Boundary conditions:

$$u(0,t) = g(t), \quad u(1,t) = h(t), \ t > 0 \tag{1.3}$$

is known as Burgers' equation. Burgers' model of turbulence is a very important fluid dynamics model and the study of this model and the theory of shock waves have been considered by many authors both for conceptual understanding of a class of physical flows and for testing various numerical methods. The distinctive feature of Eq. (1.1) is that it is the simplest mathematical formulation of the competition between non-linear advection and the viscous diffusion. It contains the simplest form of non-linear advection term and dissipation term where ' ' is the viscosity coefficient for formulating the physical phenomena of wave motion and thus determines the behavior of the solution. In 1915, such type of equation is introduced by Bateman [2] and proposed the steady-state solution of the problem. Burgers [3] introduced this equation in 1948, to capture some features of turbulent fluid in a channel caused by the interaction of the opposite effects of convection and diffusion, therefore it is referred as ''Burgers' equation''. The structure of Burgers' equation is roughly similar to that of Navier–Stokes equations due to the presence of the non-linear convection term and the occurrence of the diffusion term with viscosity coefficient. So, this equation can be considered as a simplified form of the Navier–Stokes equations and also it is the simplest model of nonlinear partial differential equation for diffusive waves in fluid dynamics. The study of the general properties of Burgers' equation has attracted attention of scientific community due to its applications in many physical

problems including one-dimensional sound/shock waves in a viscous medium, waves in fluid filled viscous elastic tubes, magneto hydrodynamic waves in a medium with finite electrical conductivity, mathematical modeling of turbulent fluid, and in continuous stochastic processes.

Analytical methods for solving Burger's equation are very restricted and can be used in very special cases; so they cannot be used to solve equations of numerous realistic scenarios. Numerical methods which are commonly used such as finite difference, finite element methods etc. are need a large amount of computation and usually the effect of round-off error causes the loss of accuracy.

So far many authors are applied various numerical methods to solve Burgers equations, some of them are finite element method [4], Least-squares quadratic B-spline finite element method [5], Cubic B-splines collocation method [6] etc. For large systems, these methods are inefficient in terms of both computer storage and computational cost.

The multigrd approach is an alternative scheme to overcome these drawbacks. In 1964 Fedorenko [7] formulated a multigrid scheme to solve the Poisson equation in a recatangular domain. Bachvalov [8] generalized the technique for general elliptic PDEs in 1966. Up to this time, the approch was not yet practical. In 1973 the first practical results were published in a pioneering paper by Brandt [9]. He outlined the purpose and practical utility of multigrid methods. Hackbush [10] independently discovered multigrid methods and provided some theoretical foundation in 1976. The multigrid method is largely applicable in increasing the efficiency of iterative methods used to solve large system of algebraic equations. Since their early application to elliptic partial differential equations, multigrid methods have been applied successfully to a large and growing class of problems. Classical multigrid begins with a two-grid process. First, iterative relaxation is applied, whose effect is to smooth the error. Then a coarse-grid correction is applied, in which the smooth error is determined on a coarser grid. This error is interpolated to the fine grid and used to correct the fine-grid approximation. Applying this method recursively to solve the coarse-grid problem leads to multigrid.

Bastian et al. [11] was investigated in series of experiments to solve parabolic PDEs using multigrid methods. However, when meet by certain problems, for example parabolic type of problems with discontinuous or highly oscillatory coefficients, as well as advection-dominated problems, the standard multigrid procedure converges slowly with larger computational time or may break down. For this reason we go for wavelet multigrid method in which by choosing the filter operators obtained from wavelets to define the prolongation and restriction operators.

"Wavelets" have been very popular topic of conversations in many scientific and engineering gatherings these days. Some of the researchers have decided that, wavelets as a new basis for representing functions, as a technique for time-frequency analysis, and as a new mathematical subject. Of course, "wavelets" is a versatile tool with very rich mathematical content and great potential for applications. However wavelet analysis is a numerical concept which allows one to represent a function in terms of a set of basis functions, called wavelets, which are localized both in location and scale. In wavelet applications to the solution of partial differential equations the most frequently used wavelets are those with compact support introduced by Daubechies [12]. Recently, many authors De Leon [13], Bujurke et al. [14] and Shiralashetti et al. [15] have developed wavelet multigrid methods.

This paper gives an alternative method i.e. Daubechies Wavelet based full approximation scheme for the numerical solution of Burgers equation using Daubechies filter coefficients. Daubechies FAS is formulated in this paper have the following characteristics:

- Provide approximations which are continuous and continuously differentiable throughout the domain of the problems, and have piecewise continuous second derivatives.
- The methods possess super convergence properties.
- The methods incorporate IC and BCs in a systematic fashion.

The organization of the paper is as follows. Preliminaries of Daubechies wavelets are given in section 2. Section 3 deals with Wavelet multigrid operators. Method of solution is discussed in section 4. Numerical findings and error analysis are presented in section 5. Finally, conclusions of the proposed work are discussed in section 6.

2. Preliminaries of Daubechies wavelets

A major problem in the development of wavelets during the 1980s was the search for a multiresolution analysis where the scaling function was compactly supported and continuous. As we know, the Haar multiresolution analysis is generated by a compactly supported scaling function that is not continuous. The

B-spines are continuous and compactly supported but fail to form an orthonormal basis. In 1988 Ingrid Daubechies constructed a family of multiresolution analyses generated by scaling functions which are both compactly supported and continuous. That created great excitement among mathematicians and scientists performing research area of wavelets. Daubechies' construction leads to a family of scaling functions that are compactly supported and smooth.

Before the explanation of Daubechies wavelets based numerical methods, we are interested to explain some definitions

2.1. Compactly supported wavelets

The class of compactly supported wavelet bases was introduced by Daubechies [16]. They are an orthonormal bases for functions in $L^2(R)$. The construction of wavelet functions starts from building the scaling or dilation function, $\phi(x)$ and set of coefficients $b_k, k \in Z$, satisfies the two-scale refinement relation,

$$\phi(x) = \sum_{k=0}^{N-1} a_k \, \phi(2x-k),$$

The wavelet function is

$$\psi(x) = \sum_{k=0}^{N-1} b_k \, \phi(2x-k),$$

where $b_k = \left(-1\right)^k a_{N-1-k}$ and $\int_{-\infty}^{\infty} \phi(x)dx = 1$

2.2. Vanishing moment

The wavelet is said to have $M\left(M \in N\right)$ vanishing movement if it verifies the following condition

$$\int_{-\infty}^{\infty} x^m \psi(x)dx = 0, \;\; m = 0,1,2....M-1$$

where $N = 2M$ for the Daubechies wavelets.

2.3. Vanishing moment

The wavelet bases induces a multiresolution analysis on $L^2(R)$ i.e. the decomposition of Hilbert space $L^2(R)$ into a chain of closed subspaces

$$\ldots \subset V_{-2} \subset V_{-1} \subset V_0 \subset V_1 \subset V_2 \ldots$$

Such that

$$\cup_j W_j = L^2(R)$$

and

$$\cap_j V_j = \{0\}$$

By defining the W_j as an orthogonal complement of V_j and V_{j+1},

$$\text{i.e. } V_{j+1} = V_j \oplus W_j$$

The space $L^2(R)$ is represented as the direct sum of W_j' s as

$$L^2(R) = \oplus W_j$$

On each fixed scale j, the wavelets $\left\{\psi_{j,k}(x) = 2^{j/2}\phi(2^j x - k) : k \in Z\right\}$ form an orthonormal basis of W_j and the functions $\left\{\phi_{j,k}(x) = 2^{j/2}\phi(2^j x - k) : k \in Z\right\}$ form an orthonormal basis of V_j .The coefficients

$H = \{a_k\}$ and $G = \{b_k\}$ are quadrature mirror filters. Once the filter H has been chosen, it completely determines the function ϕ and ψ.

In Daubechies wavelet system the coefficients $\{a_k\}$ and $\{b_k\}$ that define the refinement relation and also decide shape of the scaling function and wavelet function. This shape in turn decides the application where we can use the particular wavelet. The coefficients $\{a_k\}$ and $\{b_k\}$ acts as signal filters. Wavelets must satisfy certain necessary conditions like orthogonality and certain desirable properties for specific kind of applications. These conditions, in turn, put restrictions on scaling and wavelet function coefficients. Some of the conditions are mentioned in above and remaining are as follows.

- A wavelet basis is orthonormal if any two translated or dilated wavelets satisfy the condition

$$\int_{-\infty}^{\infty} \psi_{n,k}(x)\psi_{m,l}(x)\,dx = \delta_{n,m}\,\delta_{k,l}$$

Where δ is the Kronecker Delta function.

- The necessary and sufficient condition for this to hold is that integer translates of the scaling function ϕ exactly interpolate polynomials of degree up to *K*. That is, for each *k*, there exists constants c_l such that

$$x^k = \sum_l c_l^k \phi_l(x)$$

Daubechies introduced scaling functions satisfying this property and distinguished by having the shortest possible support. The scaling function ϕ_N has support $[0,\ N-1]$, while the corresponding wavelet ψ_{2^J} has support in the interval $[1-N/2,\ N/2]$ and has $(N/2-1)$ vanishing wavelet moments. Thus, according to Daubechies scaling functions of order N can exactly represent any polynomial of order up to, but not greater than $N/2-1$.

For example, Daubechies family of wavelets when $N = 4$, we have filter coefficients,

$$a_0 = \frac{1+\sqrt{3}}{4\sqrt{2}},\quad a_1 = \frac{3+\sqrt{3}}{4\sqrt{2}},\quad a_2 = \frac{3-\sqrt{3}}{4\sqrt{2}},\quad a_3 = \frac{1-\sqrt{3}}{4\sqrt{2}}\quad;$$

$$b_0 = \frac{1-\sqrt{3}}{4\sqrt{2}},\quad b_1 = -\frac{3-\sqrt{3}}{4\sqrt{2}},\quad b_2 = \frac{3+\sqrt{3}}{4\sqrt{2}},\quad b_3 = -\frac{1+\sqrt{3}}{4\sqrt{2}}$$

2.4. Discrete wavelet transform

The discrete wavelet transform (DWT) takes a series of *N* observations and produces *N* new values called wavelet coefficients. The DWT is structured around a set of filtering operations that determine coefficients. The wavelet coefficients contain information about the magnitude and location of different scales or, in filtering parlance, different pass-bands. The decomposition is performed sequentially, starting with the smallest scales and progressing to the larger scales, with the scale doubling in size with each iteration. The matrix formulation of the discrete wavelet transforms (DWT), which play an important part in the wavelet method. This is highly expedient and informative, particularly for the numerical computations. Then the discrete Daubechies wavelet transformation matrix DWM for the D_4 filters, whose matrix is given by

$$DWM = \begin{pmatrix} a_0 & a_1 & a_2 & a_3 & 0 & 0 & \dots & 0 & 0 \\ b_0 & b_1 & b_2 & b_3 & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & a_0 & a_1 & a_2 & a_3 & \dots & 0 & 0 \\ 0 & 0 & a_0 & a_1 & a_2 & a_3 & \dots & 0 & 0 \\ \vdots & & \ddots & & \dots & & \dots & 0 & 0 \\ a_2 & a_3 & 0 & 0 & \dots & & \dots & 0 & a_0 & a_1 \\ b_2 & b_3 & 0 & 0 & \dots & & \dots & 0 & b_0 & b_1 \end{pmatrix}_{N\times N}$$

3. Wavelet multigrid (WMG) operators

Multigrid method is well known among the fastest solution in the numerical method. Particularly for elliptic problems, they have been proved to be highly accurate. Multigrid method have earned a reputation as an efficient and versatile approach for other types of computational problems as well, including other types of partial differential equations and systems and some integral equations. In multigrid method, the residual is passed from the fine grids to the coarser grids. Vectors from fine grids are transferred to coarser grids with Restriction operator (R_o), while vectors are transferred from coarse grids to the finer grids with a Prolongation operator (P_o).

In the wavelet-multigrid, the scaling functions, which are both compactly supported and continuous, were first constructed by Daubechies that created great excitement among mathematicians and scientists performing research in the area of wavelets. Daubechies high pass and low pass filter coefficients are used in the wavelet intergrid operators (restriction and prolongation) by the DWT matrix as given in section 2.4, authors [13] used restriction and prolongation operators as

$$DW_R = \begin{pmatrix} a_0 & a_1 & a_2 & a_3 & 0 & 0 & \dots & 0 & 0 \\ b_0 & b_1 & b_2 & b_3 & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & a_0 & a_1 & a_2 & a_3 & \dots & 0 & 0 \\ 0 & 0 & b_0 & b_1 & b_2 & b_3 & \dots & 0 & 0 \\ \vdots & & \ddots & & \dots & & \dots & 0 & 0 \\ 0 & 0 & & \dots & b_0 & b_1 & b_2 & b_3 & \dots & 0 & 0 \end{pmatrix}_{N/2\times N} \quad \text{and } DW_P = \left(DW_R\right)^T \text{ respectively}$$

4. Daubechies wavelet based full approximation Method of solution

In this section, we applied the wavelet based full approximation scheme for the numerical solution of Burgers equation as follows:

Nonlinear multigrid initiate by Brandt [9], which seeks to use concepts from the linear multigrid iteration and apply them directly in the nonlinear setting. Many problems of physical interest are nonlinear in character and for these problems the multigrid strategy provides new powerful algorithms. But this is not the case for nonlinear problems, and different strategies must be employed. In this paper, we describe how to apply multigrid to nonlinear problems. Applying multigrid method directly to the nonlinear problems by employing the method so-called Full Approximation Scheme (FAS). FAS is suitable for nonlinear problems [17, 18] which treats directly the nonlinear equations on finer and coarser grids. In FAS, Gauss-Seidel method is applied as nonlinear iteration to smoothen the error. Differently from linear multigrid, the full-scale equation is solved on the coarse grid instead of the residual equation, because of the nonlinearity.

Consider the Burgers equation,

$$\frac{\partial u(x,t)}{\partial t} + u(x,t)\frac{\partial u(x,t)}{\partial x} = \upsilon \frac{\partial^2 (x,t)}{\partial x^2} \tag{4.1}$$

subject to initial condition (IC) and boundary conditions (BCs).

Where u the real valued function and that is is assumed to be in $L^2(R)$ in the interval $0 \le x,\ t \le 1$. We assume that Eq. (4.1) has a unique solution i.e. u to be determined.

Now discretizing the Eq. (4.1) by using finite difference scheme, we get the system of nonlinear equations of the form

$$F\left(u_{i,j}\right) = b_{i,j} \tag{4.2}$$

where $i, j = 1,2,.....N$, which have $N\times N$ equations with $N\times N$ unknowns.

Solving Eq. (4.2) through the Gauss Seidel iterative method, we get approximate solution v.

Approximate solution contains some errors, and therefore required solution equals to sum of approximate solution and error. There are many methods to minimize such error to get the accurate solution. Some of them are FAS, WFAS etc. Now we are discussing the method of solution of the above mentioned methods as below.

4.1. Full-Approximation Scheme (FAS)

Now, we are deliberating about the Full-Approximation Scheme (FAS) of solutions the algortith given by Briggs et. al [17] is as follows,

From the system Eq. (4.2), we get the approximate solution v for u. Now we find the residual as

$$[r]_{N\times N} = [b]_{N\times N} - A([v]_{N\times N}). \tag{4.3}$$

Reduce the matrices in the finer level to coarsest level using Restriction operator and then construct the matrices back to finer level from the coarsest level using Prolongation operator.

$$R_O = \frac{1}{4}\begin{pmatrix} 1 & 2 & 1 & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & 1 & 2 & 1 & \dots & 0 & 0 \\ \vdots & & \ddots & & & \dots & 0 & 0 \\ 0 & 0 & & \dots & & \dots & 1 & 2 \end{pmatrix}_{N/2\times N}$$

and then construct the matrices back to finer level from the coarsest level using Prolongation operator as.

$$P_O = \frac{1}{2}\begin{pmatrix} 1 & 0 & 0 & \dots & 0 \\ 2 & 0 & 0 & \dots & 0 \\ 1 & 1 & 0 & \dots & \vdots \\ 0 & 2 & \vdots & \dots & \vdots \\ 0 & 1 & & & \\ \vdots & \vdots & \ddots & & \vdots \\ 0 & 0 & 0 & \dots & 1 \\ 0 & 0 & 0 & \dots & 2 \end{pmatrix}_{N\times N/2}$$

From Eq. (4.3),

$$[r]_{\frac{N}{2}\times\frac{N}{2}} = [R_O]_{\frac{N}{2}\times N}[\mathrm{r}]_{N\times N}[P_O]_{N\times\frac{N}{2}}.$$

Similarly,

$$[v]_{\frac{N}{2}\times\frac{N}{2}} = [R_O]_{\frac{N}{2}\times N}[v]_{N\times N}[P_O]_{N\times\frac{N}{2}}$$

and

$$A([v]_{\frac{N}{2}\times\frac{N}{2}} + [e]_{\frac{N}{2}\times\frac{N}{2}}) - A([v]_{\frac{N}{2}\times\frac{N}{2}}) = [r]_{\frac{N}{2}\times\frac{N}{2}}. \tag{4.4}$$

Solve Eq. (4.4) with initial guess '0', we get $[e]_{\frac{N}{2}\times\frac{N}{2}}$.

From Eq. (4.4),

$$[r]_{\frac{N}{4}\times\frac{N}{4}} = [R_O]_{\frac{N}{4}\times\frac{N}{2}}[r]_{\frac{N}{2}\times\frac{N}{2}}[P_O]_{\frac{N}{2}\times\frac{N}{4}}$$

Similarly,

$$[v]_{\frac{N}{4}\times\frac{N}{4}} = [R_O]_{\frac{N}{4}\times\frac{N}{2}}[v]_{\frac{N}{2}\times\frac{N}{2}}[P_O]_{\frac{N}{2}\times\frac{N}{4}}$$

and

$$A([v]_{\frac{N}{4}\times\frac{N}{4}} + [e]_{\frac{N}{4}\times\frac{N}{4}}) - A([v]_{\frac{N}{4}\times\frac{N}{4}}) = [r]_{\frac{N}{4}\times\frac{N}{4}}. \tag{4.5}$$

Solve Eq. (4.5) with initial guess '0', we get $[e]_{\frac{N}{4}\times\frac{N}{4}}$.

Then the procedure is continue up to the coarsest level, we have,

$$[r]_{1\times1} = [R_O]_{1\times2}[r]_{2\times2}[P_O]_{2\times1}.$$

Similarly, $$[v]_{1\times1} = [R_O]_{1\times2}[v]_{2\times2}[P_O]_{2\times1}$$

and

$$A([v]_{1\times1} + [e]_{1\times1}) - A([v]_{1\times1}) = [r]_{1\times1}. \tag{4.6}$$

Solve Eq. (4.6) we get, $[e]_{1\times1}$.

Now correct the solution to the finer level, i.e.

$$[e]_{2\times 2} = [P_O]_{2\times 1}[e]_{1\times 1}[R_O]_{1\times 2}$$
$$[e]_{4\times 4} = [P_O]_{4\times 2}[e]_{2\times 2}[R_O]_{2\times 4}$$

and so on we have,

$$[e]_{N\times N} = [P_O]_{N\times \frac{N}{2}}[e]_{\frac{N}{2}\times\frac{N}{2}}[R_O]_{\frac{N}{2}\times N} \ .$$

Correct the solution with error. $[u]_{N\times N} = [v]_{N\times N} + [e]_{N\times N}$.

This is the required solution of the given Eq. (4.1).

4.2. Wavelet Full Approximation Scheme (WFAS)

The same procedure is applied as explained the FAS (Section 4.1) in which replacing operators DW_R and DW_P in place of R_O and P_O respectively.

5. Numerical examples

In this section, we applied FAS, WFAS for the numerical solution of Burgers equation and subsequently presented the efficiency of the methods in the form of tables and figures. The error analysis is considered as $E_{max} = \max|u_e - u_a|$, where u_e and u_a are exact and approximate solutions respectively.

Test problem. Consider the Burgers equation (4.1) with initial and boundary conditions [19], subject to the I.C.:

$$u(x,0) = \frac{2\upsilon\pi \sin(\pi x)}{\sigma + \cos(\pi x)}, \ 0 \le x \le 1 \tag{5.1}$$

and B.C.s:

$$u(0\ ,t) = 0 = u(1\ ,t)\ , \quad 0 \le t \le 1 \tag{5.2}$$

which has the exact solution

$$u(x,t) = \frac{2\upsilon\pi e^{-\pi^2\nu t}\sin(\pi x)}{\sigma + e^{-\pi^2\nu t}\cos(\pi x)}, \ 0 \le x, t \le 1 \tag{5.3}$$

where $\sigma > 1$ is a parameter.

By applying the method explained in the section 4.1, for different values of ν i.e.

$$\upsilon = \ 0.1,\ 0.05,\ 0.025,\ 0.005 \ (\text{with } t = 0.01, \sigma = 2),$$

we obtain the numerical solutions of the problem are compared with exact solution is presented in figures 1 and Physical behavior of numerical solutions of problem in 3D are presented in figures 2-5. The maximum absolute errors with CPU time of the methods for $\nu = 0.001$ and $\sigma = 2$ are presented in table 1 and Error analysis of the method for different values υ and σ is presented in table 2.

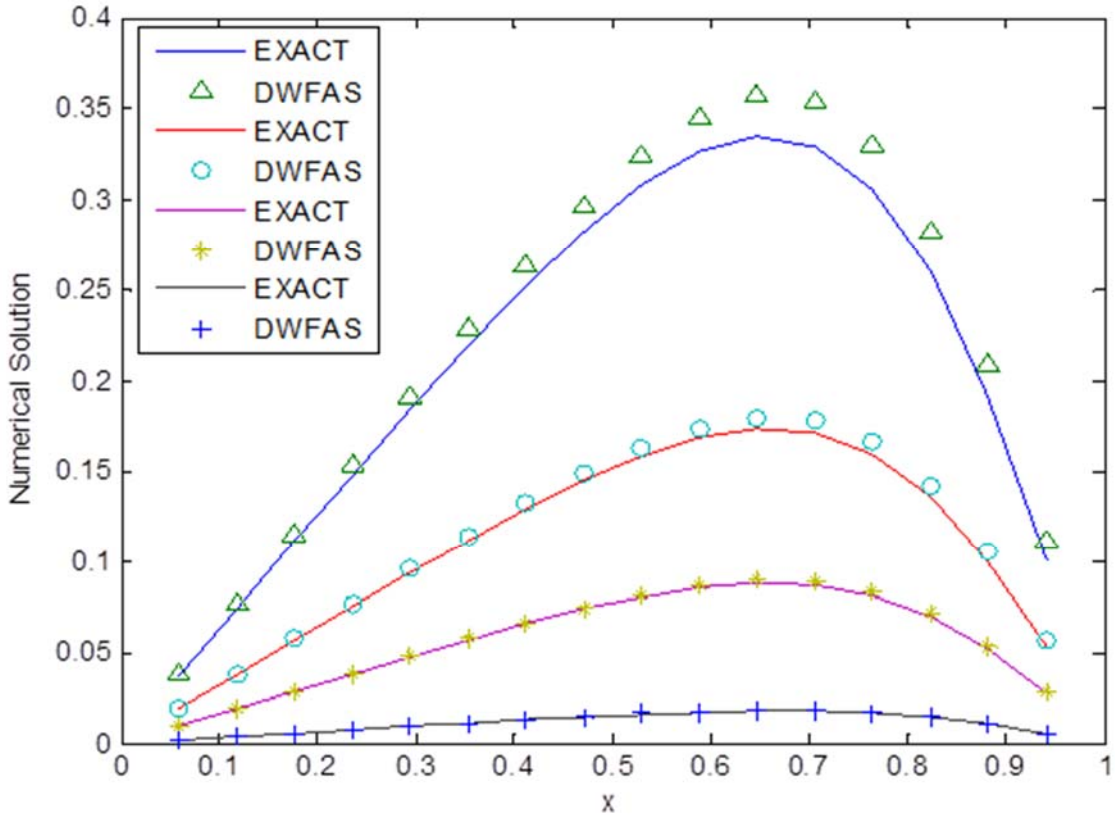
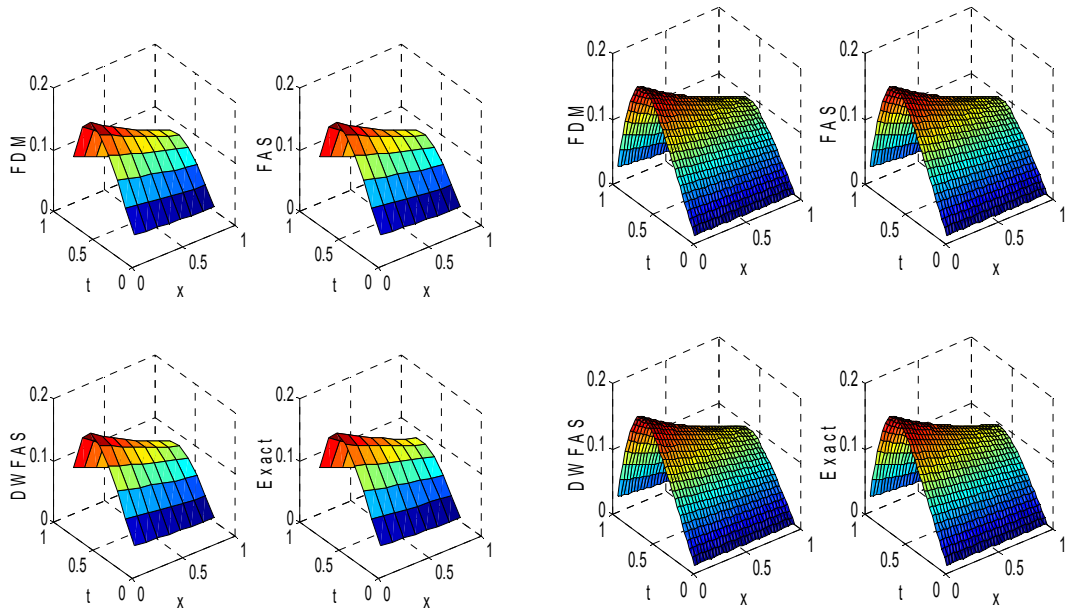


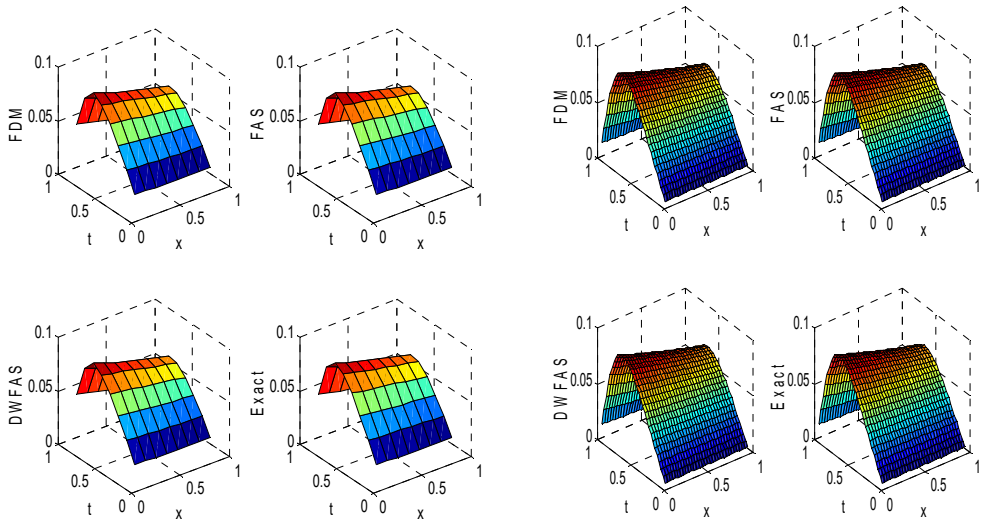
Fig. 1. Physical behavior of numerical solutions of test problem with $t = 0.01$, $\sigma = 2$.



N = 64 *N = 1024*

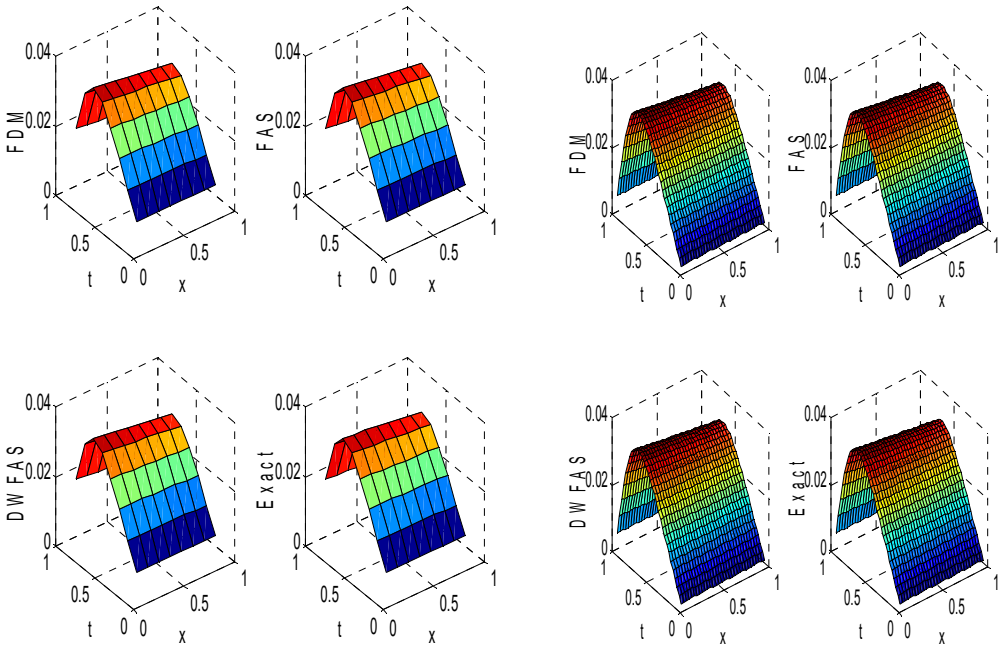
Fig. 2. Comparison of numerical solutions with exact solution of test problem

when $\upsilon = 0.05$ and $\sigma = 2$ for N=64 & 1024.



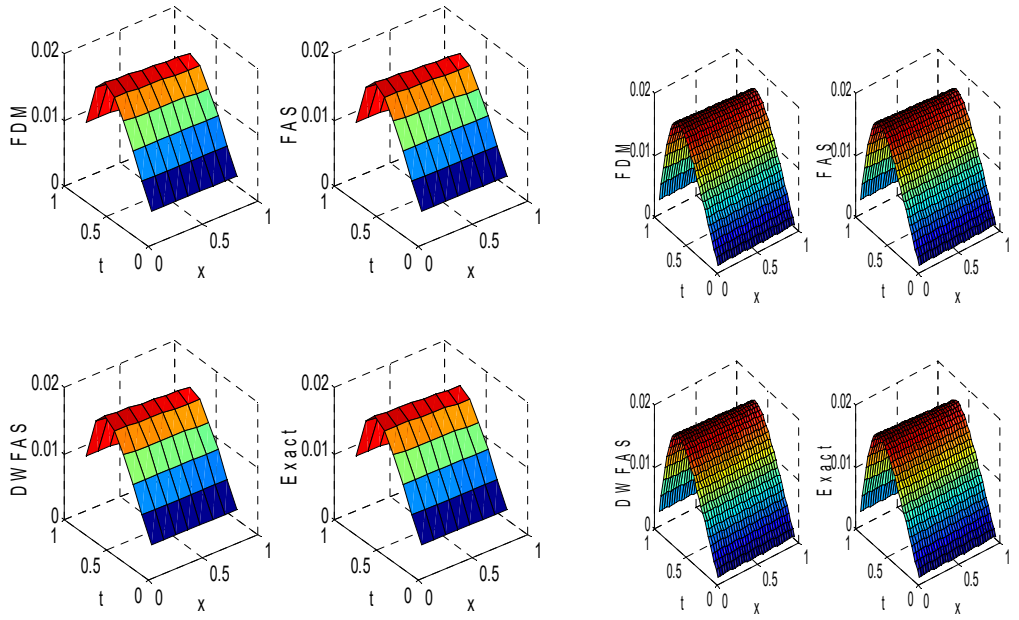
N = 64 *N = 1024*

Fig. 3. Comparison of numerical solutions with exact solution of test problem when $\upsilon = 0.025$ and $\sigma = 2$ for N=64 & 1024.



N = 64 *N = 1024*

Fig. 4. Comparison of numerical solutions with exact solution of test problem when $\upsilon = 0.01$ and $\sigma = 2$ for N=64 & 1024.



N = 64 *N = 1024*

Fig. 5. Comparison of numerical solutions with exact solution of test problem when $\upsilon = 0.005$ and $\sigma = 2$ for N=64 & 1024.

Table 1. Maximum error and CPU time (in seconds) of the methods of test problem for $\upsilon = 0.001$ and $\sigma = 2$.

N	Method	$E_{\max}$	Setup time	Running time	Total time
64	FDM	7.8968e-06	7.8204	0.0012	7.8216
	FAS	7.8968e-06	0.0192	0.0003	0.0195
	DWFAS	7.8968e-06	0.0164	0.0001	0.0165
256	FDM	5.4390e-06	6.2829	0.0012	6.2841
	FAS	5.4390e-06	0.0204	0.0003	0.0207
	DWFAS	5.4390e-06	0.0165	0.0001	0.0166
1024	FDM	3.1252e-06	8.2394	0.0016	8.2410
	FAS	3.1252e-06	0.0277	0.0003	0.0280
	DWFAS	3.1252e-06	0.0166	0.0001	0.0167
4096	FDM	1.6882e-06	9.5276	0.0028	9.5304
	FAS	1.6882e-06	0.0208	0.0003	0.0211
	DWFAS	1.6882e-06	0.0167	0.0002	0.0169

Table 2. Error analysis of the method for different values υ and σ .

N	$E_{max} = \max\lvert u_e - u_a \rvert$					
	For different values of σ with $\upsilon = 0.01$			For different values of υ with $\sigma = 2$		
	$\sigma = 2$	$\sigma = 5$	$\sigma = 50$	$\upsilon = 0.1$	$\upsilon = 0.001$	$\upsilon = 0.0001$
64	5.0678e-04	4.7588e-05	9.5429e-07	2.1572e-03	7.8968e-06	8.3262e-08
256	3.5537e-04	3.1105e-05	2.7523e-07	2.0833e-03	5.4390e-06	5.7210e-08
1024	2.0786e-04	1.7883e-05	7.8304e-08	1.4278e-03	3.1252e-06	3.2769e-08
4096	1.1210e-04	9.5983e-06	2.4747e-08	8.3131e-04	1.6882e-06	1.7708e-08

6. Conclusions

In this paper, a Daubechies wavelet based full approximation scheme (DWFAS) for the numerical solutions of Burgers' equation with Dirichlet's boundary conditions has been presented. The presented scheme is tested on one test problem and the obtained numerical results are reasonably agreeable in comparison to the existing numerical methods (i.e. FDM, FAS) and exact solution. From this the following conclusions about presented scheme are:

i) The main advantages of the DWFAS is that the scheme is able to capture the behavior of numerical solutions for small coefficient of kinematic viscosity υ.

ii) From the error analysis, the convergence of the presented method is observed i.e. the error decreases when the level of resolution *N* increases and also for smaller values of υ & larger values of σ.

iii) DWFAS is capable of reducing the volume of the computational work as compared to the classical methods and is still maintaining the high accuracy of numerical result.

The presented scheme seems to be easily extended to solve model equations including more mechanical, physical or biophysical effects, such as nonlinear convection, reaction, linear diffusion and dispersion.

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