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# Complete synchronization of 4D Chua system via linear controllers

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**Abstract.** In this paper, the complete synchronization of 4D Chua systems is investigated via linear controllers. Especially, the synchronization can be realized using only one or two linear controllers. By analyzing simulation results, it is known that when realizing the synchronization, the choice of linear controller has greater flexibility and the controlled variables can also be randomly selected.

**Keywords:** Complete synchronization, hyperchaotic Chua system, linear controller

## 1. Introduction

Since being proposed by Pecora and Carroll [1], synchronization of chaotic systems has been paid much attention by many researchers[2-6]. Many kinds of methods have been discussed to study synchronization of chaotic systems, such as a stable-manifold-based method[7], adaptive method[8], back stepping scheme[9], sliding mode control[10], nonlinear control[11], and so on. Correspondingly, different types of synchronization have been proposed, for example, complete synchronization[12], anti-synchronization[13], phase synchronization[14], hybrid synchronization [15], etc.

With further research on the synchronization, many chaotic systems have been explored, for instance, Lorenz system, Chen system, Rössler system, Chua system, etc. In these systems, at least one nonlinear term is involved in addition to linear terms. Chua system is a simple non-linear electronic circuits with chaotic behaviors. Initially, Leon. O. Chua addressed a 3D Chua's circuit, which becomes a typical model for studying chaos due to its simplicity and representation of the nonlinear circuit. Chua system attracted researchers' attention and has been used widely. Later, some researchers found that 3D Chua system can not satisfied the requirements of studying circuit system and a 4D Chua system was put forward[16]. Afterwards, the dynamics of the 4D Chua system has been extensively studied.

Based on above, complete synchronization of a 4D Chua system is investigated via linear controllers in this paper. Other parts of this paper are arranged as follows. Section 2 depicts the 4D Chua system and the attractors of it. In Section 3, the complete synchronization of the 4D Chua system is discussed via linear controllers using numerical simulations. Conclusions are drawn in Section 4.

## 2. Model description

In this section, hyperchaotic Chua system is considered as:

$$\begin{aligned} \dot{x}_1 &= a(x_2 - px_1 - qx_1^3), \\ \dot{x}_2 &= x_1 - x_2 + x_3 + x_4, \\ \dot{x}_3 &= -bx_2 + x_4, \\ \dot{x}_4 &= -\gamma x_1 + \rho x_2 + \omega x_4 . \end{aligned} \tag{1}$$

If the parameters are chosen as $b$= 16, $\gamma$= 0.1, $\rho$= 0.6, $\omega$= -0.03, $p$= −1/7 and $q$= 6/7, the dynamical behaviors of system (1) shows diversity for different values of $a$ , which can be verified by simulation results. In our numerical calculations, the fourth order Runge-kutta algorithm is used, the time step is $h$=0.01, and the initial values for the variables of model (1) are selected as (0.01, 0.02, 0.03, 0.04), (-0.6, -0.5, -0.9, -0.8), (0.6, 0.5, 0.9, 0.8), (-0.01, -0.02, -0.03, -0.04), respectively.

The phase portraits of system (1) are plotted in Figs.1-4. When $a$=7.5, system (1) has only one periodical attractor for different initial values (Fig.1). But for $a$=7.745 and $a$=8.6, system (1) performs coexistence of two periodical attractors (Fig.2) and three periodical attractors (Fig.3), respectively. When $a$=8.9, two periodical attractors and one chaotic attractor can coexist in system (1) (Fig.4).

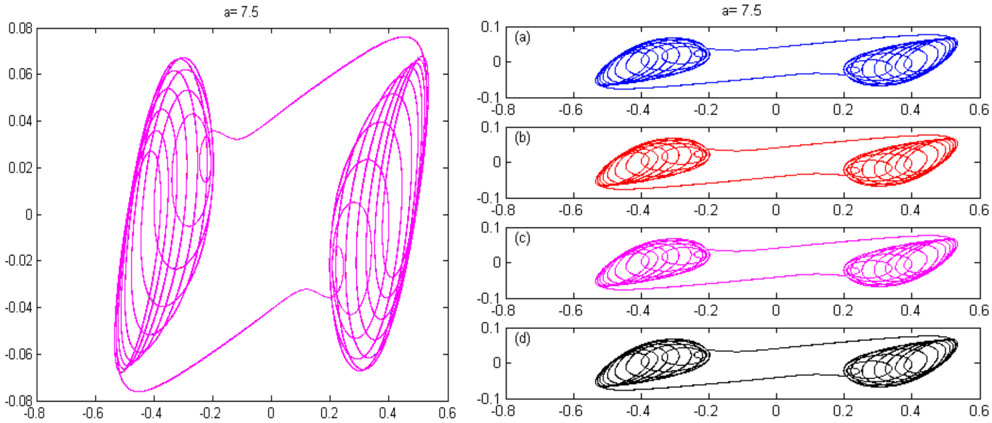


Fig.1 Periodical attractor on the platform of $(x, y)$ of system (1) when a=7.5,
the right part is the cases for different initial values of system (1),
(a) (0.01,0.02,0.03,0.04), (b) (-0.6,-0.5,-0.9,-0.8), (c) (0.6,0.5,0.9,0.8), (d) (-0.01,-0.02,-0.03,-0.04).

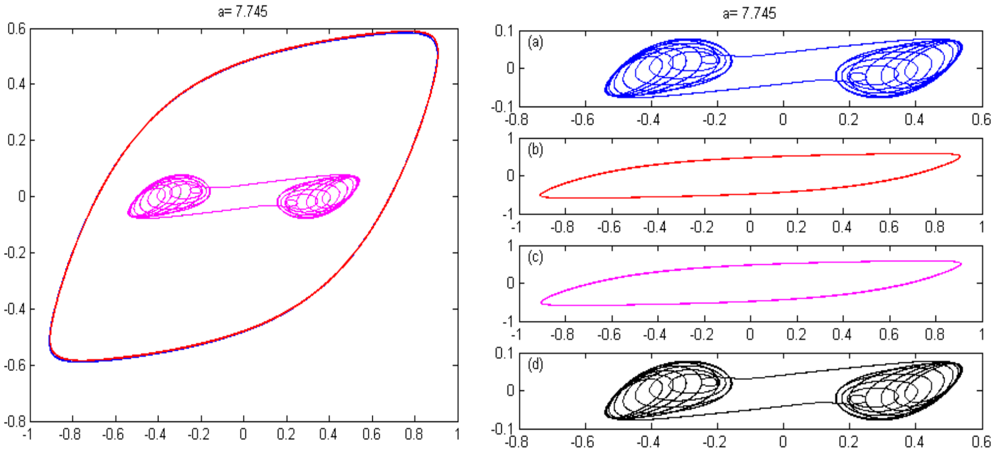


Fig.2 Coexistence of two periodical attractors on the platform of $(x, y)$ of system (1) when a=7.745,
the right part is the cases for different initial values of system (1),
(a) (0.01,0.02,0.03,0.04), (b) (-0.6,-0.5,-0.9,-0.8), (c) (0.6,0.5,0.9,0.8), (d) (-0.01,-0.02,-0.03,-0.04) .

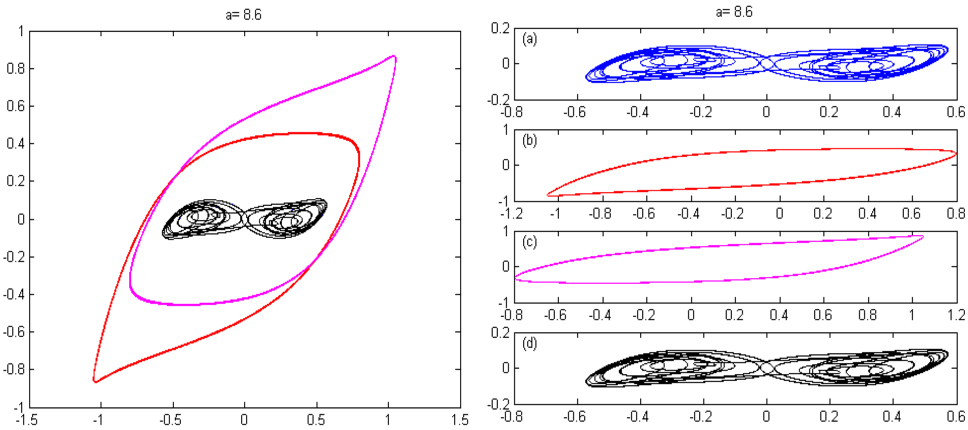


Fig.3 Coexistence of three periodical attractors on the platform of $(x, y)$ of system (1) when a=8.6,
the right part is the cases for different initial values of system (1),
(a) (0.01,0.02,0.03,0.04), (b) (-0.6,-0.5,-0.9,-0.8), (c) (0.6,0.5,0.9,0.8),(d) (-0.01,-0.02,-0.03,-0.04) .

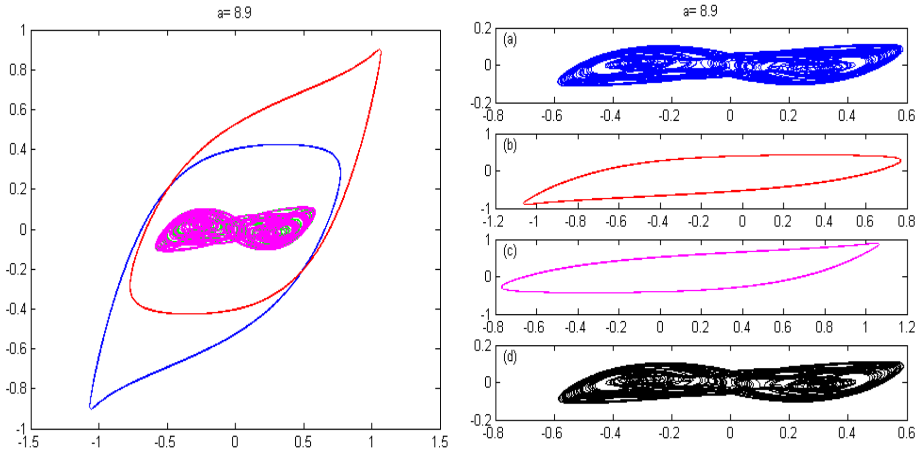


Fig.4 Coexistence of two periodical attractors and one chaotic attractor on the platform of $(x, y)$ of system (1) when a=8.9,the right part is the cases for different initial values of system (1), (a) (0.01,0.02,0.03,0.04), (b) (-0.6,-0.5,-0.9,-0.8), (c) (0.6,0.5,0.9,0.8),(d) (-0.01,-0.02,-0.03,-0.04).

From Figs.1-4, it is easy to know that system (1) can be provided with multiple dynamical behaviors with the change of the system parameter *a* while other parameters are fixed. Figs.1-4 suggest that, when the parameter *a* is changing, attractors of system (1) can show a variety of patterns. This phenomenon is helpful for further expanding the application of Chua circuit system.

## 3. Complete synchronization of hyperchaotic Chua system

In this section, the complete synchronization of hyperchaotic Chua system is investigated via numerical simulations. In the simulations, the fixed system parameters are taken as $b$= 16, γ= 0.1, ρ= 0.6, ω= -0.03, $p$= −1/7, $q$= 6/7, while $a$ is regarded as a changeable parameter. System (1) is chosen as the drive system, and the response system with controllers is written as

$$\begin{aligned} \dot{y}_1 &= a(y_2 - \mathrm{p}y_1 - \mathrm{q}y_1^3) + u_1, \\ \dot{y}_2 &= y_1 - y_2 + y_3 + y_4 + u_2, \\ \dot{y}_3 &= -\beta y_2 + y_4 + u_3, \\ \dot{y}_4 &= -\gamma y_1 + \rho y_2 + \omega y_4 + u_4, \end{aligned} \tag{2}$$

where $u_1, u_2, u_3, u_4$ are controllers to be constructed.

To realize the complete synchronization of systems (1) and (2), let the errors be

$$e_1 = x_1 - y_1,\ e_2 = x_2 - y_2,\ e_3 = x_3 - y_3,\ e_4 = x_4 - y_4. \tag{3}$$

If the variables of systems (1) and (2) satisfying the conditions $\lim_{t\to\infty} e_i = 0$ $(i = 1,2,3,4)$, then it can be said that the two systems gain the complete synchronization.

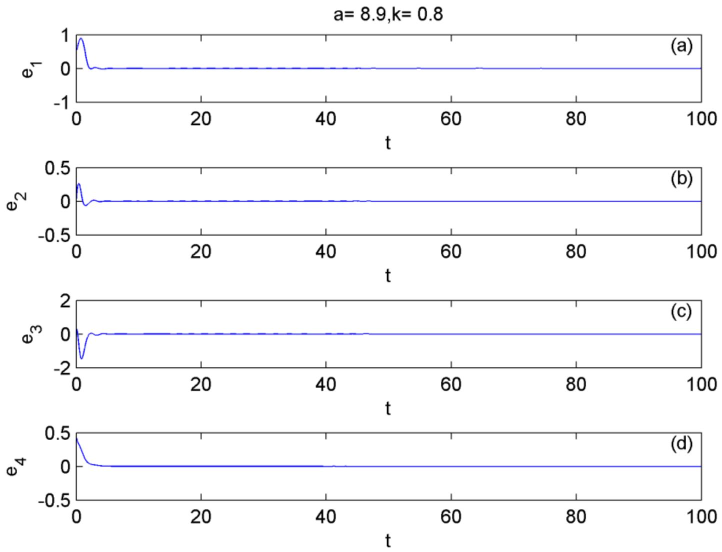


Fig.5 The time series of error variables in (3), where $u_i = ke_i$ $(i = 1,2,3,4)$ and k=0.8.

It is easy to prove that the complete synchronization between systems (1) and (2) can be achieved if the controllers are chosen as $u_i = ke_i$ $(i = 1,2,3,4)$ with enough large value of positive $k$. This result can be seen from the numerical simulation (Fig.5).

In specific application of circuit system, the fewer the controllers are, the more convenient the system is to be used. For this reason, the complete synchronization between systems (1) and (2) are to be discussed using one or two controllers via numerical simulation.

Case 1. Using only one controller

In this case, numerical simulations are used to verified the synchronization results that, if one controller of $u_i = ke_i$ $(i = 1,2,3,4)$ is chosen and used to control the response system, the complete synchronization between systems (1) and (2) can also be obtained. The main results are to be given as follows.

If $u_2 = k(x_1 - y_1)$ is chosen and the controlled response system can be written as

$$\begin{aligned}\dot{y}_1 &= a(y_2 - py_1 - qy_1^3),\\ \dot{y}_2 &= y_1 - y_2 + y_3 + y_4 + k(x_1 - y_1),\\ \dot{y}_3 &= -\beta y_2 + y_4,\\ \dot{y}_4 &= -\gamma y_1 + \rho y_2 + \omega y_4,\end{aligned} \tag{4}$$

the time series of errors between systems (1) and (4) are depicted in Fig.6, from which it is obvious to see that the complete synchronization between systems (1) and (4) can be arrived for enough large value of $k$. Fig.7 gives the bifurcation diagram of $e_1$ with the change of parameter $k$, from which we know that the critical value of $k$ to realize complete synchronization is about 0.425.

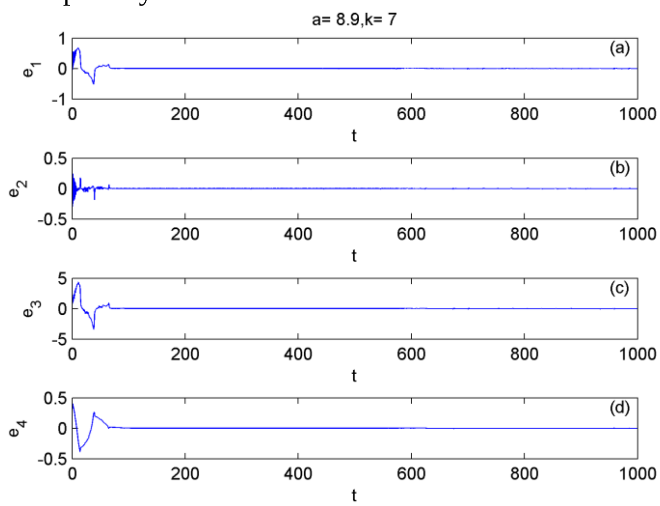


Fig.6 The time series of errors between systems (1) and (4).

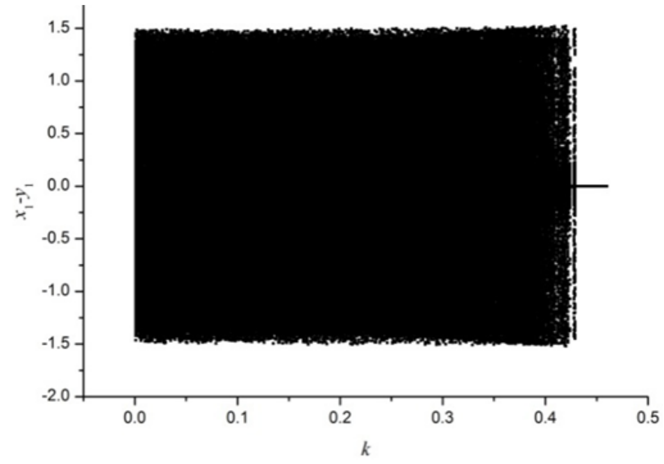


Fig.7 The bifurcation diagram of $e_1$ with the change of parameter k for $u_2 = k(x_1 - y_1)$.

If the controller is taken as $u_2 = k(x_2 - y_2)$, and the controlled response system is written as

$$\begin{aligned} \dot{y}_1 &= a(y_2 - py_1 - qy_1^3), \\ \dot{y}_2 &= y_1 - y_2 + y_3 + y_4 + k(x_2 - y_2), \\ \dot{y}_3 &= -\beta y_2 + y_4, \\ \dot{y}_4 &= -\gamma y_1 + \rho y_2 + \omega y_4, \end{aligned} \tag{5}$$

the error dynamics of systems (1) and (5) is drawn in Fig.8, from which we know that system (5) can synchronize system (1) for enough large value of $k$. Fig.9 gives the bifurcation diagram of $e_1$ with the change of parameter $k$, from which it is known that the critical value of $k$ to obtain the complete synchronization is about 2.96.

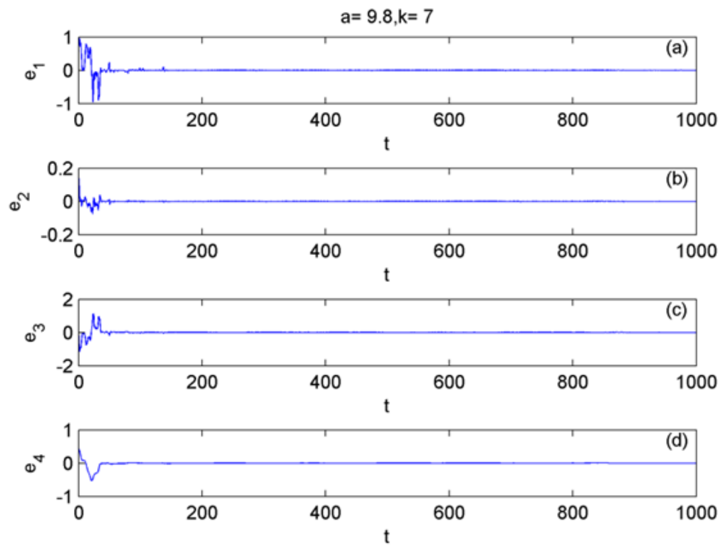


Fig.8 Error dynamics of systems (1) and (5).

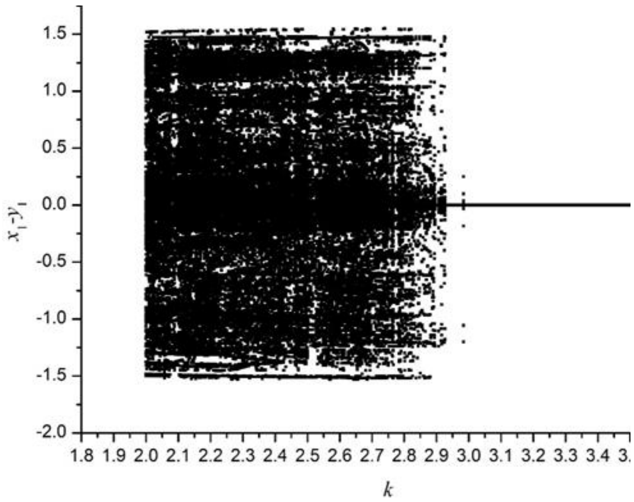


Fig.9 The bifurcation diagram of $e_1$ with the change of parameter k for $u_2 = k(x_2 - y_2)$.

When the controller is chosen as $u_3 = k(x_2 - y_2)$, and the controlled response system is

$$\begin{aligned} \dot{y}_1 &= a(y_2 - py_1 - qy_1^3), \\ \dot{y}_2 &= y_1 - y_2 + y_3 + y_4, \\ \dot{y}_3 &= -\beta y_2 + y_4 + k(x_2 - y_2), \\ \dot{y}_4 &= -\gamma y_1 + \rho y_2 + \omega y_4, \end{aligned} \tag{6}$$

the curve evolutions of the errors between systems (1) and (6) is given in Fig.10, from which we know that the complete synchronization of systems (6) and (1) can be obtained for enough large value of $k$.

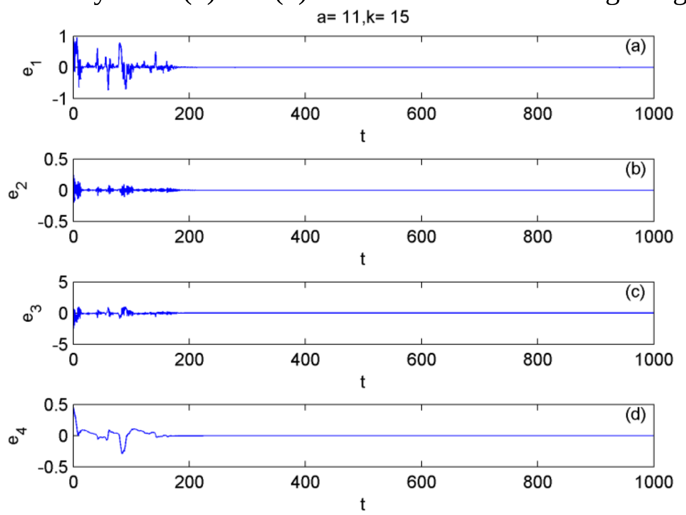


Fig.10 The curve evolution of the errors between systems (1) and (6).

In this case, only one linear controller is used to achieve the complete synchronization of hyperchaotic Chua systems. Simulation results suggest that the selection of the controller and controlled variable are all flexible. Whichever one controller of $u_i = ke_i$ (i = 1,2,3,4) is chosen and whichever variable is selected to be controlled, the complete synchronization of hyperchaotic Chua systems can be realized.

Case 2. Using two controllers

To further investigate the effect of controller selection on the complete synchronization, two linear controllers are used to discuss the complete synchronization of hyperchaotic Chua systems.

If the controllers are taken as $u_1 = k(x_1 - y_1)$, $u_4 = k(x_4 - y_4)$, and the controlled response system is

$$\begin{aligned} \dot{y}_1 &= a(y_2 - \mathrm{p}y_1 - \mathrm{q}y_1^3) + k(x_1 - y_1), \\ \dot{y}_2 &= y_1 - y_2 + y_3 + y_4, \\ \dot{y}_3 &= -\beta y_2 + y_4, \\ \dot{y}_4 &= -\gamma y_1 + \rho y_2 + \omega y_4 + k(x_4 - y_4), \end{aligned} \tag{7}$$

the time series of the errors between systems (1) and (7) is plotted in Fig.11, from which we know that the complete synchronization of systems (1) and (7) can be realized when $a$=11 and $k$=3.

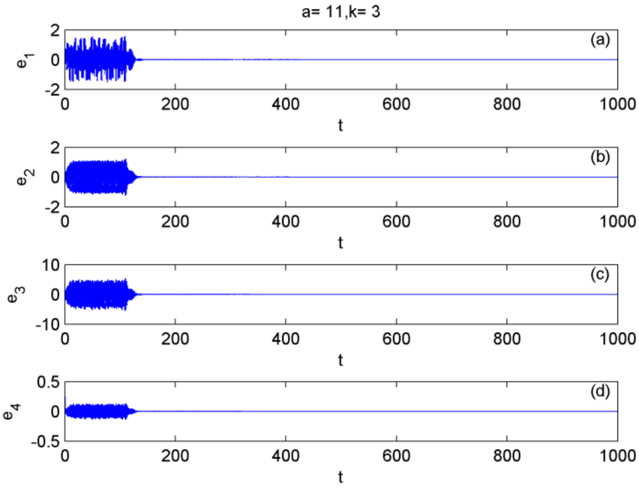


Fig.11 Time series of the errors between systems (1) and (7).

If the controllers are chosen as $u_1 = k(x_2 - y_2), u_4 = k(x_4 - y_4)$, the controlled response system is written as

$$\begin{aligned}\dot{y}_1 &= a(y_2 - \mathrm{p}y_1 - \mathrm{q}y_1^3) + k(x_2 - y_2),\\ \dot{y}_2 &= y_1 - y_2 + y_3 + y_4,\\ \dot{y}_3 &= -\beta y_2 + y_4,\\ \dot{y}_4 &= -\gamma y_1 + \rho y_2 + \omega y_4 + k(x_4 - y_4),\end{aligned} \tag{8}$$

the dynamical behaviors of the errors between systems (1) and (8) is drawn in Fig.12, which indicates that the complete synchronization between systems (1) and (8) can be gotten when $a$=11 and $k$=3.

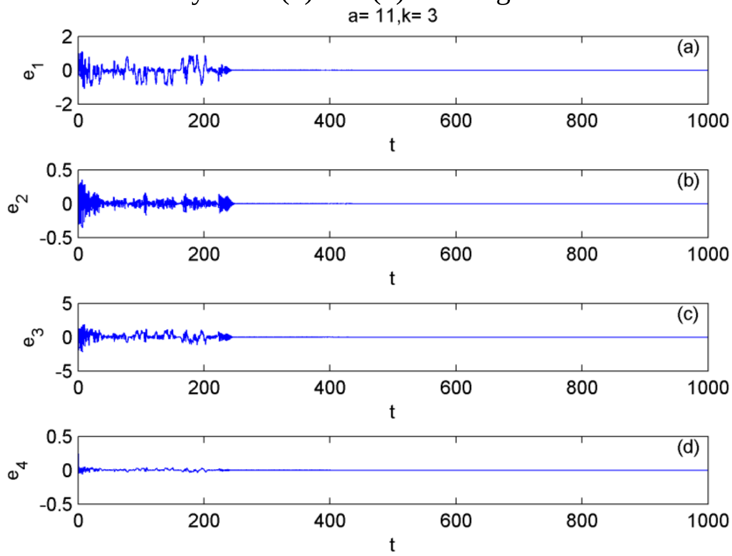


Fig.12 The dynamical behaviors of the errors between systems (1) and (8).

In this case, simulation results illustrate that, if we select two from the four controllers $u_i = ke_i$ (i = 1,2,3,4), the controlled response system can complete synchronize the drive system. Further investigation can describe that, whichever two controllers are selected and no matter which two variables are under control, the complete synchronization between the hyperchaotic Chua systems can be achieved.

## 4. Conclusions

In this paper, the complete synchronization of hyperchaotic Chua systems is investigated via linear controllers. Firstly, the synchronization between the drive system and the controlled hyperchaotic Chua system with four controllers is considered. Numerical simulation suggests that the synchronization can be easily realized via four controllers. To further simplify the controlled system, the synchronization between the drive system and the controlled hyperchaotic Chua system with one or two controllers is discussed. Simulation resultions indicate that fewer controllers can also make hyperchaotic Chua systems achieve synchronization. Simultaneously, it is found that, when we select one or two of the four linear controllers randomly and make any one or two variables be controlled, the complete synchronization of 4D Chua system can all be obtained. Therefore, it can be said that linear controller has certain superiority in achieving synchronization of hyperchaotic systems.

## 5. Acknowledgement

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