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Finite-time burst synchronization of time-delay neural system with parameters disturbed by periodic signal

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Abstract. In this manuscript, finite-time burst synchronization of time-delay neuron system is investigated for two cases. In one case, parameters are known. In another case, parameters are unknown and some parameters are disturbed by periodic signal. The time to gain burst synchronization is derived and a factor affecting the synchronization time is given via theoretical analysis. The relationship between the time and the factor is described. Finally, simulation results are given to verify the effectiveness of the proposed methods.

Keywords: Finite-time burst synchronization; Hindmarsh-Rose system; Periodic perturbation; Time-delay

1. Introduction

In the past decades, synchronization of chaotic system has attracted more and more attention of researchers due to its powerful applications in many areas [1-3], such as secure communication, chemical reactions, biological systems and mechanical systems. Many kinds of synchronization have been investigated, such as complete synchronization [4], lag synchronization [5], generalized synchronization [6], phase synchronization [7], anti-synchronization [8], cluster synchronization [9], etc. For this, various effective methods have been proposed to synchronize chaotic systems, e.g., sliding mode control [10], back stepping method [11], adaptive control [12], observer-based control [13], nonlinear control [14], control Lyapunov function method [15], and so on.

With further research on synchronization, more and more people have realized that the time to achieve synchronization is very important in real applications. For this reason, many methods have been introduced to get faster convergence speed, among which finite-time control is an effective technique. Since finite-time synchronization means the optimality in convergence time, many contributions have been made to it [16-21]. Meanwhile, in real systems, time-delay is often inevitable. So it is necessary to investigate the finite-time synchronization of time delay system.

Motivated by potential applications of synchronization, many researchers have been engaged in the synchronization of neuron system. Studies [22-26] have shown that synchronization is an important phenomenon in information processing of neurons and neurons engage in various activities, among which, burst is one pattern consisting of the active phase and silent phase. Burst synchronization [27] naturally refers to the introduction of a temporal relationship between the bursts produced by two or more neurons. It is typically used to refer to a temporal relationship between active phase onset or offset times across neurons. This form of synchronization can be observed, when either excitatory synaptic coupling or diffusive coupling is introduced between a pair of model respiratory neurons [28, 29]. Clinical evidences suggest that burst synchronization plays an important role in some pathology, such as Parkinson's disease, essential tremor, and epilepsies [30]. Therefore, controlling this synchronization has a practical importance for undesirable neuronal rhythms [31, 32]. In addition, in real neuron system, time-delay always exists when signals are communicated among neurons, even in the same neuron. So it is necessary to investigate the finite-time burst synchronization of the neuron system with time-delay.

Based on above, the main contribution of this paper is to investigate the finite-time burst synchronization of time-delay neuron system with various parameters. The rest parts of this paper are arranged as follows. Some preliminaries are given in Section 2. In Section 3, the finite-time burst synchronization of time-delay neuron system is discussed for two cases. One is with known parameters and another is with periodic perturbation parameters. Section 4 gives numerical simulations to verify the effectiveness of the proposed method. Some conclusions are reached in Section 5.

2. Preliminaries

Definition 1. Considering two chaotic systems as follows:

$$\dot{x} = f(x), \dot{y} = g(y) \tag{1}$$

Where x,y are two n-dimensional state vectors. $f, g: R^n \to R^n$ are vector-valued functions. If there exists a positive constant T such that

$$\lim_{t \to T} \|x - y\| = 0,$$

and $\|x - y\| \equiv 0$ when $t \geq T$, then it is said that the two systems of (1) can achieve finite-time synchronization.

Lemma 1[33]. Assume that a continuous, positive-definite function $V(t)$ satisfies the following differential inequality

$$\dot{V}(t) \leq -cV^{\eta}(t), \forall t \geq t_0, V(t_0) \geq 0, \tag{2}$$

where $c > 0$, $0 < \eta < 1$ are all constants. Then for any given t_0, $V(t)$ satisfies following inequality:

$$V^{1-\eta}(t) \leq V^{1-\eta}(t_0) - c(1-\eta)(t - t_0), t_0 \leq t \leq t_1, \tag{3}$$

and

$$V(t) \equiv 0, \ \forall t \geq t_1 \tag{4}$$

with t_1 given by

$$t_1 = t_0 + \frac{V^{1-\eta}(t_0)}{c(1-\eta)}. \tag{5}$$

Lemma 2 [34]. Suppose $0 < r \leq 1$, a, b are all positive numbers, then the inequality

$$(a + b)^r \leq a^r + b^r$$

is quite straightforward .

3. Finite-time burst synchronization of neuron system with time-delay

3.1 System description

In this section, the neuron system with time-delay is considered as following Hindmarsh -Rose (HR) system:

$$\dot{x} = ax^2 - bx^3 + y - z(t - \tau) + I_{ext},$$

$$\dot{y} = c - dx^2 - y,$$

$$\dot{z} = r(S(x + k) - z), \tag{6}$$

where $\tau > 0$ is the time-delay. x, y and z represent the membrane potential of the neuron, the recovery variable, and the adaptation current, respectively. a, b, c, d, r, S, k are real constants. I_{ext} is an external influence on the system. When τ =0, model (6) is a mathematical representation of the firing behavior of neurons proposed by Hindmarsh and Rose [35]. In Eq. (6), time delay emerges in the third variable z, which is thought as adaptation current. The electric synaptic exists in many neurons, and its effect is often described by an additional current with time delay [36, 37]. With various I_{ext}, model (6) can show different dynamical behaviors. For example, if $\tau = 1$ and other parameters are chosen as a =3.0, b =1.0, c =1.0, d =5.0, r =0.006, S =4.0, k =1.6, system (6) can show regular bursting for $I_{ext} = 2.6$ (Fig.1) and chaotic bursting for $I_{ext} = 3.1$ (Fig.2), respectively.

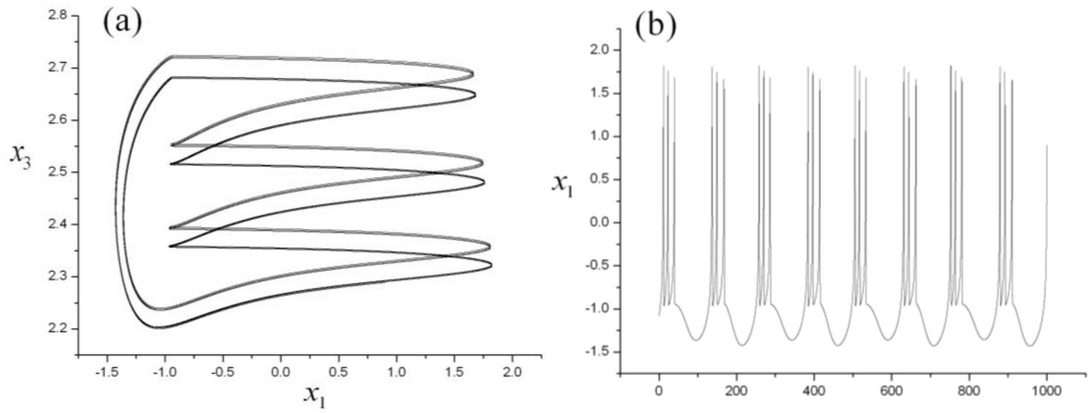

Fig.1 Regular bursting of system (6) for I_{ext} = 2.6. (a) Phase portrait, (b) Time series

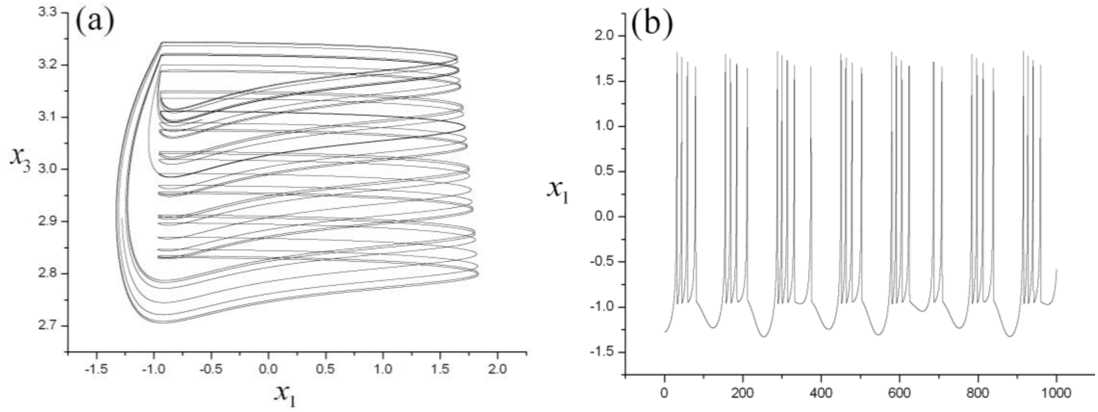

Fig.2 Chaotic bursting of system (6) for I_{ext} = 3.1. (a) Phase portrait, (b) Time series

For simplicity, let $rS = p$, $rSk = q$, system (6) can be rewritten as

$$\dot{x} = ax^2 - bx^3 + y - z(t-\tau) + I_{ext},$$
$$\dot{y} = c - dx^2 - y,$$
$$\dot{z} = px + q - rz. \tag{7}$$

To realize the finite-time burst synchronization of HR system, the drive system is considered as

$$\dot{x}_1 = ax_1^2 - bx_1^3 + y_1 - z_1(t-\tau) + I_{ext},$$
$$\dot{y}_1 = c - dx_1^2 - y_1,$$
$$\dot{z}_1 = px_1 + q - rz_1, \tag{8}$$

and the controlled response system is taken as

$$\dot{x}_2 = ax_2^2 - bx_2^3 + y_2 - z_2(t-\tau) + I_{ext} + u_1,$$
$$\dot{y}_2 = c - dx_2^2 - y_2 + u_2,$$
$$\dot{z}_2 = px_2 + q - rz_2 + u_3. \tag{9}$$

To obtain the main results, following **Assumption 1** is put forward.

Assumption 1(**A1**). Due to the bounded trajectories of chaotic system, there is a positive constant M satisfying $|x_i| < M$, $|y_i| < M$, $|z_i| < M$ $(i = 1, 2)$.

Let $e_1 = x_2 - x_1$, $e_2 = y_2 - y_1$, $e_3 = z_2 - z_1$, the error system of (9) and (8) can be obtained as

$$\dot{e}_1 = ae_1(x_2 + x_1) - be_1(x_2^2 + x_2x_1 + x_1^2) + e_2 - e_3(t-\tau) + u_1,$$
$$\dot{e}_2 = -de_1(x_2 + x_1) - e_2 + u_2,$$

$$\dot{e}_3 = pe_1 - re_3 + u_3 . \tag{10}$$

Next, controllers u_i (i=1, 2, 3) are to be constructed to make the error system (10) achieve finite-time stability, that is, the finite-time burst synchronization of systems (9) and (8) can be realized. For this, two cases are to be discussed as follows.

3.1 Finite-time burst synchronization of time-delay HR system with known parameters

In this section, the parameters of HR system are considered to be known. For complete synchronization, you may only need one controller to realize it. Generally speaking, with enhanced synchronization, it is stricter in the number and expression of controller. It means that the condition of controller required for finite-time burst synchronization is harsher than for complete synchronization. Therefore, in this section, we take three controllers and the main result is given in **Theorem 1**.

Theorem 1 The finite-time burst synchronization of systems (8) and (9) can be achieved if the controllers are chosen as $u_1 = -M_1 e_1 - e_1^{\beta} + e_3(t-\tau) - e_2$, $u_2 = e_2 - e_2^{\beta}$, $u_3 = re_3 - e_3^{\beta}$, where $M_1 > 2aM + 3bM^2$, $\beta = n/m$ is a proper rational number. M is the constant in A1. m, n are positive odd integers satisfying $m > n$.

Proof. Firstly, substitute u_1 into the first equation of (10), and we can get

$$\begin{aligned}\dot{e}_1 &= ae_1(x_1 + x_2) - be_1(x_1^2 + x_2^2 + x_1x_2) + e_2 - e_3(t-\tau) - M_1e_1 - e_1^{\beta} + e_3(t-\tau) - e_2 \\ &\le (2Ma + 3M^2 b - M_1)e_1 - e_1^{\beta} .\end{aligned} \tag{11}$$

Let $V_1 = \frac{1}{2} e_1^2$, then we have

$$\begin{aligned}\dot{V}_1 &= e_1\dot{e}_1 \le -e_1^2(M_1 - 2Ma - 3M^2 b) - e_1^{\beta+1} \\ &\le -e_1^{\beta+1} = -2^{\frac{\beta+1}{2}} (\frac{1}{2}e_1^2)^{\frac{\beta+1}{2}} = -2^{\frac{\beta+1}{2}} V_1^{\frac{\beta+1}{2}} .\end{aligned}$$

According to **Lemma 1**, system (11) is finite-time stable. It means that there is a constant $T_1 = t_0 + 2V_1^{\frac{1-\beta}{2}}(t_0)/(2^{\frac{1+\beta}{2}}(1-\beta)) > 0$, such that $e_1 \equiv 0$ provided that $t \ge T_1$. Then, the last two equations of (10) become

$$\begin{aligned}\dot{e}_2 &= -e_2 + u_2 , \\ \dot{e}_3 &= -re_3 + u_3 .\end{aligned} \tag{12}$$

Secondly, substitute u_2, u_3 into (12), and it can be obtained that

$$\begin{aligned}\dot{e}_2 &= -e_2^{\beta} , \\ \dot{e}_3 &= -e_3^{\beta} .\end{aligned} \tag{13}$$

Let $V_2 = \frac{1}{2}(e_2^2 + e_3^2)$, make use of Lemma 2, and it can be reached that

$$\dot{V}_2 = e_2\dot{e}_2 + e_3\dot{e}_3 = -e_2^{\beta+1} - e_3^{\beta+1} = -2^{\frac{\beta+1}{2}}(\frac{1}{2}e_2^2)^{\frac{\beta+1}{2}} - 2^{\frac{\beta+1}{2}}(\frac{1}{2}e_3^2)^{\frac{\beta+1}{2}}$$

$$= -2^{\frac{\beta+1}{2}}((\frac{1}{2}e_2^2)^{\frac{\beta+1}{2}} + (\frac{1}{2}e_3^2)^{\frac{\beta+1}{2}}) \le -2^{\frac{\beta+1}{2}}(\frac{1}{2}e_2^2 + \frac{1}{2}e_3^2)^{\frac{\beta+1}{2}} = -2^{\frac{\beta+1}{2}}(V_2)^{\frac{\beta+1}{2}} .$$

From **Lemma 1**, system (13) is finite-time stable. That is to say, there exists a constant

$$T_2 = t_0 + 2V_2^{\frac{1-\beta}{2}}(t_0)/(2^{\frac{1+\beta}{2}}(1-\beta)) \geq T_1$$

satisfying $e_2 \equiv 0$, $e_3 \equiv 0$ after a finite- time T_2.

According to **Definition1**, the finite-time burst synchronization of systems (8) and (9) can be realized if the controllers are chosen as mentioned in **Theorem 1**.

Remark1. For given t_0, derivate $T_1 = t_0 + 2V_1^{1-\beta/2}(t_0)/(2^{1+\beta/2}(1-\beta))$ of β, it can be gotten that $\dot{T}_1 = -V_1^{1-\beta/2}(t_0)[\ln(V_1(t_0)) + 2^{1+\beta/2}((1-\beta)\ln 2 - 1)] \big/ (2^{1+\beta/2}(1-\beta))^2$. Analyzing the nature of $\dot{T}_1$ by using Matlab Program, it's known that $\dot{T}_1 > 0$ for given t_0 and $0<\beta<1$. It means that the time T_1 required to go stable become longer with β increasing. For $T_2 = t_0 + 2V_2^{1-\beta/2}(t_0)/(2^{1+\beta/2}(1-\beta))$, the same result can be gained. This result suggests that smaller value of β $(0<\beta<1)$ is helpful for achieving finite-time burst synchronization of time-delay neuron system with known parameters.

3.2 Finite-time burst synchronization of time-delay HR system for parameters with periodic perturbation

In this section, some parameters are with periodic perturbations. Without loss of generality, it is thought that two parameters are provided with periodic perturbations. System (8) is also considered as the drive system. And the time-delay HR system with periodic perturbation parameters is supposed as

$$\begin{aligned} \dot{x}_2 &= a(1+\theta_1)x_2^2 - b(1+\theta_2)x_2^3 + y_2 - z_2(t-\tau) + I_{ext}, \\ \dot{y}_2 &= c - dx_2^2 - y_2, \\ \dot{z}_2 &= px_2 + q - rz_2. \end{aligned} \tag{14}$$

The corresponding controlled response system is

$$\begin{aligned} \dot{x}_2 &= a(1+\theta_1)x_2^2 - b(1+\theta_2)x_2^3 + y_2 - z_2(t-\tau) + I_{ext} + v_1, \\ \dot{y}_2 &= c - dx_2^2 - y_2 + v_2, \\ \dot{z}_2 &= px_2 + q - rz_2 + v_3, \end{aligned} \tag{15}$$

where v_1, v_2, v_3 are controllers to be determined, θ_1, θ_2 are bounded periodic perturbations satisfying $|\theta_1| \leq L_1, |\theta_2| \leq L_2$ and L_1, L_2 are positive constants .

For systems (8) and (15), let $e_1 = x_2 - x_1$, $e_2 = y_2 - y_1$, $e_3 = z_2 - z_1$, and the main result can be given in following **Theorem 2**.

Theorem 2 The finite-time burst synchronization of the drive system (8) and the response system (15) can be achieved if the controllers are taken as

$$\begin{aligned} v_1 &= -M_1 e_1 - L_1 x_2^2 - L_2 x_2^3 - e_2 + e_3(t-\tau) - e_1^{\beta}, \\ v_2 &= e_2 - e_2^{\beta}, \\ v_3 &= re_3 - e_3^{\beta}, \end{aligned} \tag{16}$$

where M_1 and β are defined as the same as in **Theorem 1,** L_1, L_2 are positive constants satisfying $|\theta_1| \leq L_1, |\theta_2| \leq L_2$.

Proof. Subtract (8) from (15), and the error system can be arrived as following:

$$\begin{aligned} \dot{e}_1 &= a(x_2 + x_1)e_1 + \theta_1 x_2^2 - b(x_2^2 + x_2 x_1 + x_1^2)e_1 - \theta_2 x_2^3 + e_2 - e_3(t-\tau) + v_1, \\ \dot{e}_2 &= d(x_2 + x_1)e_1 - e_2 + v_2, \\ \dot{e}_3 &= pe_1 - re_3 + v_3. \end{aligned} \tag{17}$$

At first, let $W_1 = \frac{1}{2}e_1^2$, and we have

$$\begin{aligned}\dot{W}_1 = e_1\dot{e}_1 &= e_1[a(x_2+x_1)e_1 + \theta_1 x_2^2 - b(x_2^2 + x_2x_1 + x_1^2)e_1 - \theta_2 x_2^3 + e_2 - e_3(t-\tau) + v_1] \\ &\le e_1[(2aM + 3bM^2)e_1 + L_1x_2^2 + L_2x_2^3 + e_2 - e_3(t-\tau) + v_1] \\ &= e_1[(2aM + 3bM^2 - M_1)e_1 - e_1^\beta] \\ &\le -e_1^{\beta+1} = -2^{\frac{\beta+1}{2}}(\frac{1}{2}e_1^2)^{\frac{\beta+1}{2}} = -2^{\frac{\beta+1}{2}}W_1^{\frac{\beta+1}{2}}.\end{aligned}$$

According to **Lemma 1**, the first Equation of system (17) is finite-time stable with v_1 in (16). In another word, there exists a constant $T_3 = t_0 + 2W_1^{1-\beta/2}(t_0)/(2^{1+\beta/2}(1-\beta)) > 0$ satisfying $e_1 \equiv 0$ if $t \ge T_3$. Under this condition, the last two equations of (17) become

$$\begin{aligned}\dot{e}_2 &= -e_2 + v_2, \\ \dot{e}_3 &= -re_3 + v_3. \end{aligned} \tag{18}$$

Next, let $W_2 = \frac{1}{2}(e_2^2 + e_3^2)$, applying **Lemma 2**, and it can be obtained that

$$\begin{aligned}\dot{W}_2 &= e_2\dot{e}_2 + e_3\dot{e}_3 = e_2[-e_2+v_2] + e_3[-re_3+v_3] = -e_2^{\beta+1} - e_3^{\beta+1} = -2^{\frac{\beta+1}{2}}((\frac{1}{2}e_2^2)^{\frac{\beta+1}{2}} + (\frac{1}{2}e_3^2)^{\frac{\beta+1}{2}}) \\ &\le -2^{\frac{\beta+1}{2}}(\frac{1}{2}e_2^2 + \frac{1}{2}e_3^2)^{\frac{\beta+1}{2}} = -2^{\frac{\beta+1}{2}}(W_2)^{\frac{\beta+1}{2}}.\end{aligned}$$

According to **Lemma 1**, there exists a constant $T_4 = t_0 + 2W_2^{1-\beta/2}(t_0)/(2^{1+\beta/2}(1-\beta)) \ge T_3$, such that $e_2 \equiv 0$, $e_3 \equiv 0$ if $t \ge T_4$.

In the light of **Definition1**, system (17) is finite-time stable, that is to say the finite-time burst synchronization of systems (8) and (15) can be realized if controllers v_1, v_2 and v_3 are chosen as in (16).

Remark2. By the similar analysis as T_1, T_2, it's known that the time T_3, T_4 also become larger with β increasing. And smaller value of β is benefit for achieving finite-time burst synchronization of systems (8) and (15).

4. Numerical simulations

In this section, some numerical simulations are given to demonstrate the effectiveness of the theoretical results. In the simulations, the time delay is assumed as $\tau = 1$. The parameters of HR system are chosen as a =3.0, b =1.0, c =1.0, d =5.0, r =0.006, S =4.0, k =1.6 and I_{ext} =3.1, with which the HR system is chaotic bursting. The initial values of the drive system and the response system are set as $(x_1(0), y_1(0), z_1(0)) =$ (0.3, 0.2, 0.1) and $(x_2(0), y_2(0), z_2(0)) =$ (0.6, 0.7, 0.3), respectively. Fig.3 shows the dynamical behavior of system (10) without controllers, from which it is known that error system (10) is be unstable, e.g., the burst synchronization of systems (9) and (8) can't be achieved.

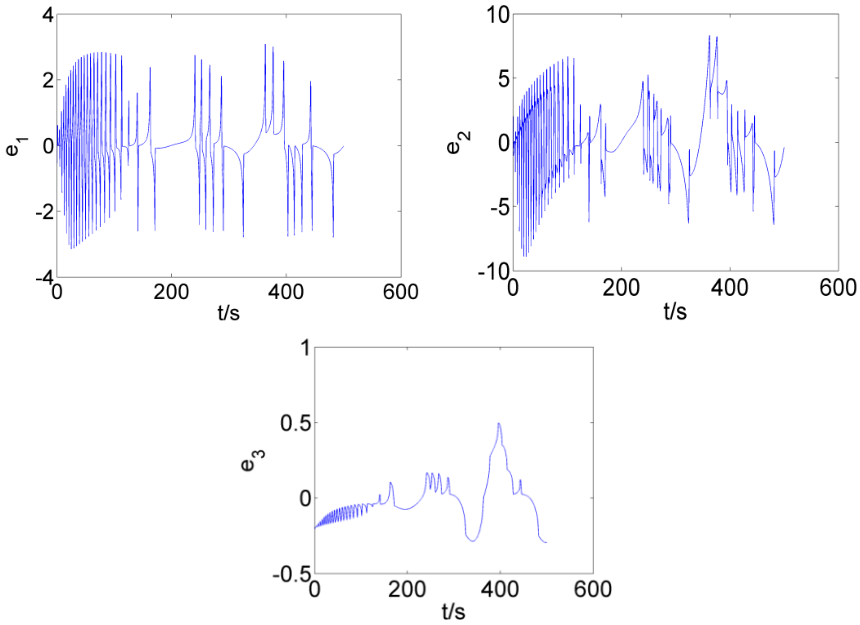

Fig.3 Dynamical behavior of error system (10) without controller

To verify the results in Theorem 1 and Theorem 2, simulations are given by constructing appropriate controllers and the finite-time burst synchronizations of HR systems in two cases are realized, which can be shown in Fig.4-Fig.9. In order to investigate the impact of parameter β on the burst synchronization time, β is taken as 0.5, 0.75, 0.90, respectively.

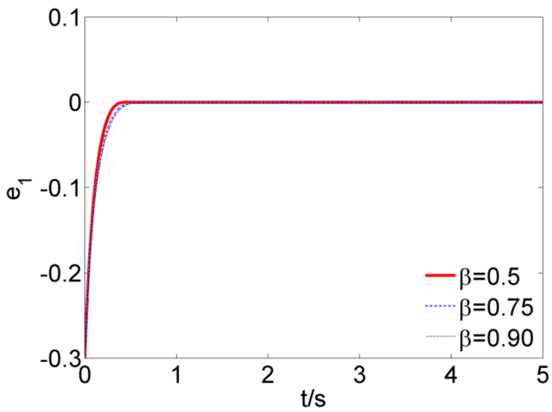

Fig.4 Dynamics of e_1 in system (10) with controller in Theorem 1 for different β

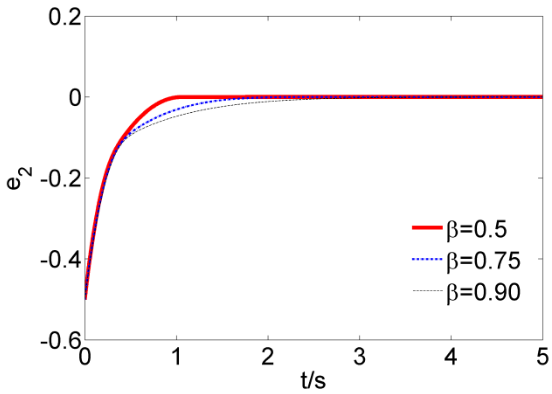

Fig.5 Dynamics of e_2 in system (10) with controller in Theorem 1 for different β

Firstly, when parameters are known and the controllers are chosen as in **Theorem 1**, the dynamics of error system (10) are pictured in Figs.4-6 with different values of β, from which, it can be seen that the finite-time burst synchronization of systems (9) and (8) can be obtained. Furthermore, the time to realize burst synchronization becomes larger with β increasing.

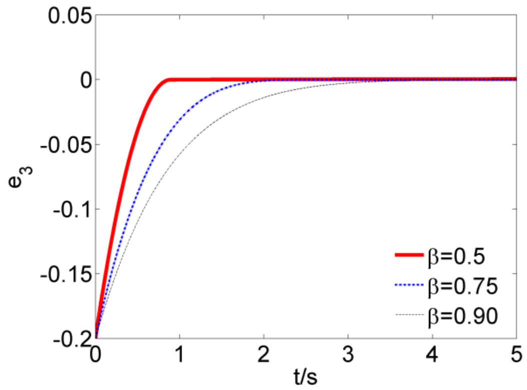

Fig.6 Dynamics of e_3 in system (10) with controller in Theorem 1 for different β

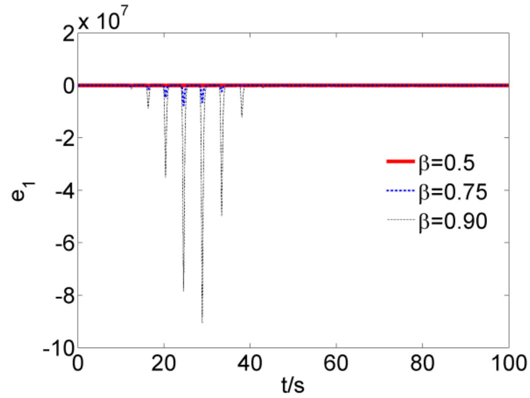

Fig.7 Dynamics of e_1 in system (17) with controller in (16) for different β

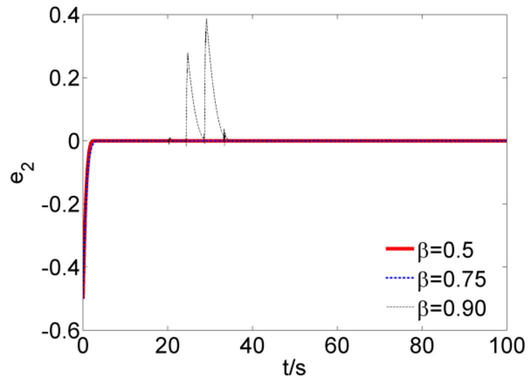

Fig.8 Dynamics of e_2 in system (17) with controller in (16) for different β

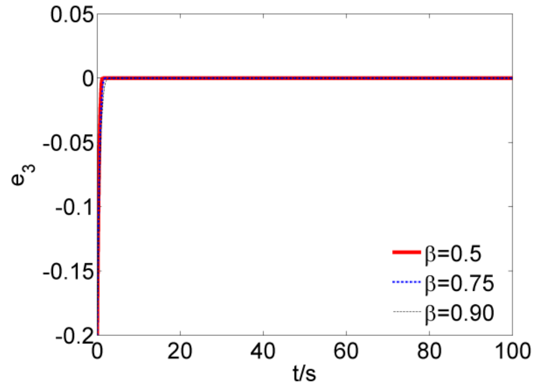

Fig.9 Dynamics of e_3 in system (17) with controller in (16) for different β

Secondly, when two parameters are with periodic perturbations, e.g., the periodic perturbations are taken as θ_1=0.1cos(t), $\theta_2 = 0.1\sin(t)$ and the controllers are taken as in (16). The dynamics of error system (17) are drawn in Figs.7-9, from which, it is known that when two parameters are with periodic perturbations, the finite-time burst synchronization can also be achieved. In addition, we can see that more time is required to realize the finite-time burst synchronization between system (8) and (15) with larger β .

5. Conclusion

In this paper, based on Lyapunov stability theory, schemes are proposed to realize the finite-time burst synchronization of time-delay neural system both for known parameters and for parameters with periodic perturbations. At the same time, a factor affecting the finite-time burst synchronization is obtained and the relationship between them is analyzed. Because time-delay and periodic perturbation parameters are taken into account in the considered HR system, the proposed scheme is attractive and practical in controlling the dynamic behavior of neural network. The research result gives a piece of advice to shorten the synchronization time of time-delay neuron network with known or unknown parameters under certain conditions. It may be helpful for treating pathologies.

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