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An approach for solving fuzzy matrix games using signed distance method

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Abstract. The goal of this paper is to solve a matrix game with fuzzy payoffs. In this paper, a fuzzy matrix game has been considered and its solution methodology has been proposed. In this paper, fuzzy payoff values are assumed to be trapezoidal fuzzy numbers. Then the corresponding matrix game has been converted into crisp game using defuzzification of fuzzy number. Here, widely known signed distance method has been used for defuzzification of fuzzy number. The value of the matrix game and strategy for each player is obtained by solving corresponding crisp game problems using the traditional method. Finally, a numerical example has been presented and solved.

Keywords: matrix games; muzzy payoff,fuzzy number; signed distance method.

1. Introduction

In every competitive situation, it is often required to take the decision where there are two or more opposite parties with conflicting of interests and the action of one depends upon the action which is taken by the opponent. A variety of competitive situation is seen in real life society like, in political campaign, elections, advertisement, marketing, etc. Game theory is a mathematical way out for describing the strategic interactions among multiple players who select several strategies from the set of admissible strategies. In 1944, Von Neumann and Oscar Morgenstern [1] introduced game theory in their most pioneer work "Theory of Games and Economic Behavior". Since then many diverse kinds of mathematical games have been defined and different types of solution methodologies have been proposed. The participants in the game are called the players. During the past, it is assumed that all the information about game is known precisely by players. But in traditional game theory, the precise information about the game is more difficult to collect due to the lack of information about the exact values of certain parameters and uncertain measuring of several situations by players. To overcome these types of situation, the problem can be formulated using the concept of uncertainty theory and the domain of payoffs are considered from uncertain environment like fuzzy, interval, stochastic, fuzzy-stochastic environment etc. In such cases fuzzy set theory is a vital tool to handle such situation. Fuzzy set theory, introduced by Zadeh [2], has been receiving considerable attention amongst researchers in game theory. Several researchers have applied the fuzzy set concepts to deal with the game problems. Fuzziness in game problem has been well discussed by Campos [3]. Compos introduced fuzzy linear programming model to solve fuzzy matrix game. Sakawa and Nishizaki [4] solved multi-objective fuzzy games by introducing Max-Min solution procedure. Based on fuzzy duality theory, Bector et al. [5, 6, 7] and Vijay et al. [8] proved that a two person zero-sum matrix game with fuzzy goals and fuzzy payoffs is equivalent to a pair of linear programming problems. Nayak and Pal [9,10] well discussed about interval games as well as fuzzy matrix games. Çevikel and Ahlatçıoglu [11] described new concepts of solutions for multi-objective two person zero-sum games with fuzzy goals and fuzzy payoffs using linear membership functions. Li and Hong [12] gave an approach for solving constrained matrix games with payoffs considering the triangular fuzzy numbers. Bandyopadhyay et al. [13] well studied a matrix game with payoff as triangular intuitionistic fuzzy number. Mijanur et al. [14] introduced an alternative approach for solving fuzzy matrix games. Effect of defuzzification methods in solving fuzzy matrix games has been discussed by Sahoo [15]. Very recently, Sahoo [16] introduced a new technique based on parametric representation of interval number for solving fuzzy matrix game. In this paper, we have treated imprecise parameters considering fuzzy numbers. Therefore, the concept of fuzzy game theory provides an efficient framework which solves real-life conflict problems with fuzzy information. In this paper, a matrix game has taken into consideration. The element of payoff matrix is considered to be trapezoidal fuzzy number. Then the corresponding problem has been converted into crisp equivalent matrix game using defuzzification of trapezoidal fuzzy numbers. Here, well known signed distance method [17] has been used for defuzzification of fuzzy number. The value of the matrix game and strategy for each player is obtained by solving corresponding crisp game problems using linear programming problem method. Finally, to illustrate the methodology, a numerical example has been solved and the computed results have been presented.

The paper is arranged in six sections as follows. The preliminaries are presented in Section 2. Mathematical model of matrix game is presented in Section 3. Transformation of game problem into a linear programming problem is discussed in Section 4. Numerical example and computational results are reported in Section 5. Finally, conclusions have been made in Section 6.

2. Preliminaries

The fuzzy set theory was developed to deal with the problems in which fuzzy phenomena exist. Fuzzy set theory handles imprecise data as probability distributions in terms of membership function. Let X be a non empty set. A fuzzy set $\tilde{A}$ is defined by a membership function $\mu_{\tilde{A}}(x)$, which maps each element x in X to a real number in the interval $[0,1]$. The function value $\mu_{\tilde{A}}(x)$ represents the grade of membership of x in $\tilde{A}$.

Definition 1: (*α-level set or α-cut*): The α-cut of a fuzzy set $\tilde{A}$ is a crisp subset of X and is denoted by $\tilde{A}_\alpha = \{x \in X : \mu_{\tilde{A}}(x) \geq \alpha\}$, where $\mu_{\tilde{A}}(x)$ is the membership function of $\tilde{A}$ and $\alpha \in [0,1]$. The lower and upper points of $\tilde{A}_\alpha$, are represented by $A_L(\alpha) = \inf \tilde{A}_\alpha$ and $A_U(\alpha) = \sup \tilde{A}_\alpha$.

Definition 2: (*Normal fuzzy set*): A fuzzy set $\tilde{A}$ is called a normal fuzzy set if there exists at least one $x \in X$ such that $\sup_x \mu_{\tilde{A}}(x) = 1$.

Definition 3: (*Convex fuzzy set*): A fuzzy set $\tilde{A}$ is called convex iff for every pair of $x_1, x_2 \in X$, the membership function of $\tilde{A}$ satisfies the inequality $\mu_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \geq \min\{\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)\}$, where $\lambda \in [0,1]$. Alternatively, a fuzzy set is convex if all α-level sets are convex.

Definition 4: (*Fuzzy point*): Let $\tilde{a}$ be a fuzzy set on $R = (-\infty, \infty)$. It is called a fuzzy point if its membership function is $\mu_{\tilde{a}}(x) = \begin{cases} 1, & x = a \\ 0, & \text{otherwise} \end{cases}$

Definition 5: (*Fuzzy number*): A fuzzy number $\tilde{A}$ is a fuzzy set on the real line R, which is both normal and convex.

Definition 6: (*Trapezoidal fuzzy number*): The trapezoidal fuzzy number (TrFN) is a normal fuzzy number denoted as $\tilde{A} = (a_1, a_2, a_3, a_4)$ where $a_1 < a_2 < a_3 < a_4$ and its membership function $\mu_{\tilde{A}}(x) : X \to [0,1]$ is defined by

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1} & \text{if } a_1 \leq x \leq a_2 \\ 1 & \text{if } a_2 \leq x \leq a_3 \\ \dfrac{a_4 - x}{a_4 - a_3} & \text{if } a_2 \leq x \leq a_3 \\ 0 & \text{otherwise} \end{cases}$$

Definition 7: (*α-level interval of Trapezoidal fuzzy number*):

Let $\tilde{A} = (a_1, a_2, a_3, a_4)$ is a Trapezoidal fuzzy number. The α-level interval of $\tilde{A}$ is defined as $(\tilde{A})_\alpha = [A_L(\alpha), A_R(\alpha); \alpha]$ where $A_L(\alpha) = a_1 + (a_2 - a_1)\alpha$ and $A_R(\alpha) = a_4 - (a_4 - a_3)\alpha$. We can represent $\tilde{A} = (a_1, a_2, a_3)$ as $\tilde{A} = \bigcup_{0 \leq \alpha \leq 1} [A_L(\alpha), A_R(\alpha); \alpha]$.

2.1 Signed distance method

The signed distance between the real numbers x and 0, denoted by $D(x,0)$ and is defined by $D(x,0) = x$. Therefore, $D(A_L(\alpha),0) = A_L(\alpha)$ and $D(A_R(\alpha),0) = A_R(\alpha)$. The signed distance of the α-level interval

$(\tilde{A})_\alpha = [A_L(\alpha), A_R(\alpha);\alpha]$ measured from the origin 0 is $D((\tilde{A})_\alpha, 0) = D([A_L(\alpha), A_R(\alpha);\alpha], 0)$ and given by $D([A_L(\alpha), A_R(\alpha);\alpha], 0) = \frac{1}{2}[D(A_L(\alpha), 0) + D(A_R(\alpha), 0)] = \frac{1}{2}(A_L(\alpha) + A_R(\alpha))$ provided $A_L(\alpha)$ and $A_R(\alpha)$ exists and are integrable for $0 \le \alpha \le 1$.

The signed distance of $\tilde{A}$ measured from $\tilde{0}$ defined as $D(\tilde{A},\tilde{0}) = \frac{1}{2}\int_0^1 (A_L(\alpha) + A_R(\alpha))d\alpha$.

Lemma 1: If $\tilde{A} = (a_1, a_2, a_3, a_4)$ is a Trapezoidal fuzzy number then $A_L(\alpha) = a_1 + (a_2 - a_1)\alpha$ and $A_R(\alpha) = a_4 - (a_4 - a_3)\alpha$, respectively, where $0 \le \alpha \le 1$. Here, $A_L(\alpha)$ and $A_R(\alpha)$ exists and are integrable for $0 \le \alpha \le 1$. Then we have $D(\tilde{A},\tilde{0}) = \frac{1}{4}(a_1 + a_2 + a_3 + a_4)$.

2.2 Ranking of fuzzy numbers

Let $D(\tilde{A},\tilde{0})$ and $D(\tilde{B},\tilde{0})$ are the signed distance measured from $\tilde{0}$ of two fuzzy numbers $\tilde{A}$ and $\tilde{B}$ respectively. Now, we have defined the ranking of fuzzy numbers as follows: (i) $\tilde{A} > \tilde{B} \Leftrightarrow D(\tilde{A},\tilde{0}) > D(\tilde{B},\tilde{0})$,

(ii) $\tilde{A} < \tilde{B} \Leftrightarrow D(\tilde{A},\tilde{0}) < D(\tilde{B},\tilde{0})$

and

(iii) $\tilde{A} = \tilde{B} \Leftrightarrow D(\tilde{A},\tilde{0}) = D(\tilde{B},\tilde{0})$.

Lemma 2: Let $\tilde{a}_i, i = 1, 2, ..., n$ be n fuzzy numbers and $x_i, i = 1, 2, ..., n$ be n crisp constants. If $\sum_{i=1}^{n} \tilde{a}_i x_i \le \tilde{b}$ holds then $D\left(\sum_{i=1}^{n} \tilde{a}_i x_i, \tilde{0}\right) \le D(\tilde{b},\tilde{0}) \Leftrightarrow \sum_{i=1}^{n} D(\tilde{a}_i,\tilde{0}) x_i \le D(\tilde{b},\tilde{0})$.

3. Mathematical model of a matrix game

Let $\{A_1, A_2, ..., A_m\}$ be a pure strategy available for player A and $\{B_1, B_2, ..., B_n\}$ be a pure strategy available for player B. When player A chooses a pure strategy A_i and the player B chooses a pure strategy B_j, then g_{ij} is the payoff for player A and $-g_{ij}$ be a payoff for player B. The two-person zero-sum matrix game G can be represented as a pay-off matrix $G = (g_{ij})_{m\times n}$ and G is represented in matrix form as follows:

$$G = (g_{ij})_{m\times n} = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{array} \begin{array}{c} \begin{array}{cccc} B_1 & B_2 & \dots & B_n \end{array} \\ \begin{pmatrix} g_{11} & g_{12} & \cdots & g_{1n} \\ g_{21} & g_{22} & \cdots & g_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ g_{m1} & g_{m2} & \cdots & g_{mn} \end{pmatrix} \end{array}$$

3.1. Fuzzy payoff matrix:

Let $\tilde{g}_{ij}$ be the fuzzy payoff which is the gain of player A from player B if player A chooses strategy A_i where as player B chooses B_j. Then the fuzzy payoff matrix of player A and B is $\tilde{G} = (\tilde{g}_{ij})_{m\times n}$. Where $A_i \in \{A_1, A_2, ..., A_m\}$, $B_j \in \{B_1, B_2, ..., B_n\}$ and it is assumed that each player has his/her choices from amongst the pure strategies and $\tilde{G}$ is represented in matrix form as follows:

$$\tilde{G} = (\tilde{g}_{ij})_{m\times n} = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{array} \begin{array}{c} \begin{array}{cccc} B_1 & B_2 & \dots & B_n \end{array} \\ \begin{pmatrix} \tilde{g}_{11} & \tilde{g}_{12} & \cdots & \tilde{g}_{1n} \\ \tilde{g}_{21} & \tilde{g}_{22} & \cdots & \tilde{g}_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ \tilde{g}_{m1} & \tilde{g}_{m2} & \cdots & \tilde{g}_{mn} \end{pmatrix} \end{array}$$

3.2 Mixed strategy:

Let us consider the fuzzy matrix game $\tilde{G}$ without saddle point whose payoff matrix is $\left(\tilde{g}_{ij}\right)_{m\times n}$. The mixed strategy for the player A and B are denoted by S_A and S_B respectively and defined as follows:

$$S_A = \left\{ x = \left(x_1, x_2, \cdots, x_m\right), x_i \geq 0 \text{ and} \sum_{i=1}^{m} x_i = 1 \right\}$$

$$S_B = \left\{ y = \left(y_1, y_2, \cdots, y_n\right), y_j \geq 0 \text{ and} \sum_{j=1}^{n} y_j = 1 \right\}$$

Definition 8: A pair (x, y) of mixed strategies for the players in a matrix game is called a situation in mixed strategies. In a situation (x, y) of mixed strategies each usual situation (i, j) in pure strategies becomes a random event occurring with probabilities x_i and y_j. Since in the situation (i, j), player A receives a payoff $D(\tilde{g}_{ij}, \tilde{0})$, the mathematical expectation of his payoff under (x, y) is equal to

$$E\left(x, y\right) = \sum_{i=1}^{m}\sum_{j=1}^{n} D\left(\tilde{g}_{ij}, \tilde{0}\right) x_i\, y_j$$

Definition 9: (Saddle *point of a function*): Let $E\left(x, y\right)$ be a function of two variables (vectors) x and y. The point $(x_\circ, y_\circ)$ is said to be the saddle point of the function $E(x, y)$ if $E(x, y_\circ) \leq E(x_\circ, y_\circ) \leq E(x_\circ, y)$

Theorem 1: Let $E(x, y)$ be a function of two variables such that $\max_x \min_y E(x, y)$ and $\min_y \max_x E(x, y)$ exist. Then the necessary and sufficient condition for the existence of a saddle point (x_0, y_0) of $E(x, y)$ is that $E(x_\circ, y_\circ) = \max_y \min_x E(x, y) = \min_x \max_y E(x, y)$

Definition 10: (*Value of a Matrix Game*): The common value of $\max_x \left\{ \min_y E\left(x, y\right) \right\}$ and $\min_y \left\{ \max_x E\left(x, y\right) \right\}$ is called the value of the matrix game with payoff matrix $G = D\left(\tilde{g}_{ij}, \tilde{0}\right)_{m\times n}$ and denoted by $v(G)$ or simply v.

4. Transformation of game problem into a linear programming problem (LPP)

Let us consider a game with the payoff $G = D\left(\tilde{g}_{ij}, \tilde{0}\right)_{m\times n}$. The maximizing player A has m strategies and

$$S_A = \left\{ x = \left(x_1, x_2, \cdots, x_m\right), x_i \geq 0 \text{ and} \sum_{i=1}^{m} x_i = 1 \right\}.$$

And similarly the minimizing player B has n strategies and

$$S_B = \left\{ y = \left(y_1, y_2, \cdots, y_n\right), y_j \geq 0 \text{ and} \sum_{j=1}^{n} y_j = 1 \right\}$$

The expected gain to the maximizing player A when the player B chooses his/her j-th course of action out of his/her courses is $\sum_{i=1}^{m} D(\tilde{g}_{ij}, \tilde{0}) x_i$. If v is the value of the game, then Player A expects to get at least v and $v = \min_j \left(\sum_{i=1}^{m} D(\tilde{g}_{ij}, \tilde{0}) x_i \right)$ and $\sum_{i=1}^{m} D(\tilde{g}_{ij}, \tilde{0}) x_i \geq D(v, 0), \forall j$. As v is the minimum of entire expected gains and the value of game is the maximum value of v, if such exists. Thus $x_1, x_2, ..., x_m$ are to be determined as to satisfies $\sum_{i=1}^{m} x_i = 1$ and $x_i \geq 0$.

Set $X_i = v^{-1} x_i$ then $\sum_{i=1}^{m} D(\tilde{g}_{ij}, \tilde{0}) X_i \geq D(\tilde{1}, \tilde{0}), \forall j$ and $\sum_{i=1}^{m} X_i = v^{-1} \sum_{i=1}^{m} x_i = v^{-1}$.

Hence the LPP is as follows:

Minimize $v^{-1} = \sum_{i=1}^{m} X_i$, (1)

subject to $\sum_{i=1}^{m} D(\tilde{g}_{ij}, \tilde{0}) X_i \geq D(\tilde{1}, \tilde{0}), \forall j$, and $X_i \geq 0$.

Problem (1) is a linear programming problem with respect to the player A and x_i is given by $x_i = vX_i, i = 1, 2, ..., m$.

The dual problem of the problem (1) is as follows:

Maximize $v^{-1} = \sum_{j=1}^{n} Y_j$ (2)

subject to $\sum_{j=1}^{n} D(\tilde{g}_{ij}, \tilde{0}) Y_j \leq D(\tilde{1}, \tilde{0}), \forall i$, and $Y_j = v^{-1} y_j \geq 0$.

Problem (2) is a linear programming problem with respect to the player B and y_j is given by $y_j = vY_j, j = 1, 2, ..., n$.

Algorithm I (Solution Procedure)

The procedural steps of this algorithm are as follows:

Step-1: *Solve Minimize* $v^{-1} = \sum_{i=1}^{m} X_i$ *, subject to* $\sum_{i=1}^{m} D(\tilde{g}_{ij}, \tilde{0}) X_i \geq D(\tilde{1}, \tilde{0}), \forall j$ *, and* $X_i \geq 0$

Step-2: *Solve Maximize* $v^{-1} = \sum_{j=1}^{n} Y_j$ *, subject to* $\sum_{j=1}^{n} D(\tilde{g}_{ij}, \tilde{0}) Y_j \leq D(\tilde{1}, \tilde{0}), \forall i$ *, and* $Y_j \geq 0$

Step-3: *Set* $x_i = vX_i, i = 1, 2, ..., m$ and $y_j = vY_j, j = 1, 2, ..., n$

Step-4: *For player* A*, print* v, $x_1, x_2, ..., x_m$

Step-5: *For player* B*, print* v, $y_1, y_2, ..., y_n$

Step-6: *Stop*

5. Numerical example and computational results

In this section, a numerical example has been considered to illustrate the proposed method. In this example, the elements of payoff matrix are trapezoidal fuzzy numbers. The payoff is given as follows:

$$\tilde{G} = \left(\tilde{g}_{ij}\right)_{3\times4} = \begin{array}{cc} & \textit{Player } B \\ \textit{Player } A & \begin{pmatrix} (1,4,5,6) & (1,2,4,5) & (3,4,5,8) & (4,5,7,8) \\ (5,10,12,17) & (8,9,13,18) & (5,7,10,14) & (7,10,11,12) \\ (-1,0,2,3) & (-1,2,3,4) & (8,17,21,30) & (5,6,7,10) \end{pmatrix} \end{array}$$

Now, we obtain values of $D(\tilde{g}_{ij}, \tilde{0})$ for each $\tilde{g}_{ij}$ of the fuzzy game $\tilde{G}$ .and we have the following reduced crisp payoff matrix as

$$G = \left(D(\tilde{g}_{ij}, \tilde{0})\right)_{3\times4} = \begin{array}{cc} & \textit{Player } B \\ \textit{Player } A & \begin{pmatrix} 4 & 3 & 5 & 6 \\ 11 & 12 & 9 & 10 \\ 1 & 2 & 19 & 7 \end{pmatrix} \end{array}$$

Using Step-1 and Step-2 of the proposed Algorithm I, we have formulated linear programming problems (LPP) for players A and players B which are as follows:

LPP 1: (*For players* A*)*

Minimize $v^{-1} = X_1 + X_2 + X_3$

subject to $4X_1 + 11X_2 + X_3 \geq 1$; $3X_1 + 12X_2 + 2X_3 \geq 1$; $5X_1 + 9X_2 + 19X_3 \geq 1$; $6X_1 + 10X_2 + 7X_3 \geq 1$ and $X_1, X_2, X_3 \geq 0$

LPP2: (*For players* B*)*

Maximize $v^{-1} = Y_1 + Y_2 + Y_3 + Y_4$

subject to $4Y_1 + 3Y_2 + 5Y_3 + 6Y_4 \leq 1$; $11Y_1 + 12Y_2 + 9Y_3 + 10Y_4 \leq 1$; $Y_1 + 2Y_2 + 19Y_3 + 7Y_4 \leq 1$ and $Y_1, Y_2, Y_3, Y_4 \geq 0$.

Finally, we have solved the LPP 1 and LPP 2 with the help of LINGO software. Now we obtained the optimal strategies for players A and players B. The optimal strategy for player A is (0, 0.923, 0.077) and the optimal strategy for players B is (0, 0, 0.231, 0.769). Also the value of the game is 9.769.

6. Concluding remarks

In most of the cases, setting precisely the exact value of the payoff elements is more difficult and that, their values are considered as imprecise data, therefore, in this paper the elements of payoff matrix are considered as fuzzy numbers. The fuzziness in payoff elements is represented by the Trapezoidal Fuzzy Number (TrFN). In this paper, a method of solving fuzzy game problem using defuzzification of fuzzy numbers has been considered. Several methods for defuzzification of fuzzy numbers have been proposed in the existing literature. But, in this paper, we have used signed distance method for defuzzification as this method has wider applicability in several existing studies. A numerical example has been considered and solved to illustrate the proposed method. The fuzzy game problem has been converted into crisp game problem after defuzzification of fuzzy number in which the converted payoff values are crisp valued. The crisp game problem can be solved by any existing traditional methods like dominance rule, graphical rule etc. Here, it is solved by using linear programming method. Finally, the games with their strategies and value of the game have been presented and it can be summarized that the entire method attempted in this work well to solve real-life decision making problems in the areas such as economics, operations research and management science.

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