

Enhancement of Performance of TCP Using Normalised Throughput Gradient in Wireless Networks

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Abstract. Transmission control protocol (TCP) is a dominant transport layer protocol for reliable data delivery in the internet. When packet loss occurs, TCP makes an implicit assumption that all packet loss is due to congestion. This results in unnecessary degradation in the TCP performance while traversing over a wireless link. In this paper, to improve the performance of TCP, we use NTG loss-predictor to distinguish congestion loss from transmission loss. Based on the prediction of type of loss, an appropriate algorithm is invoked by using NTG loss-prediction parameter β . Frequency of congestion loss and wireless loss predictions are analysed. Performance of TCP is improved further by discussing stability analysis over the system. A time-delay control theory is used and by constructing Hermit matrix, analysis is made for asymptotic stability of the system. Explicit conditions are derived for P_{max} (RED controller) and β (NTG loss controller) in terms of wireless network parameters. Using the characteristic equation of the matrix, convergence of the queue length at the bottleneck router is discussed. Analysis of convergence of queue length to a given target value is analysed. This establishes stability in the router performance. An approximate solution of queue length is derived. Our results provide better solutions for global stability and convergence conditions of the system.

Keywords: congestion-loss, convergence, immediate-recovery, loss-predictor, stability, transmission-loss, wireless-networks.

1. Introduction

Realistically, the existing Internet service uses the combination of wired and wireless media for communication, and is called as heterogeneous network. In wireless networks, packet losses are due to congestion and high bit error rate transmission over the wireless link. TCP, a widely used transport protocol, performs satisfactorily over wired networks and fails to perform well in the wireless networks because it is not be able to differentiate the loss caused by congestion or transmission. When a packet loss occurs during wireless transmission, TCP diagnoses it as a congestion event and reduce the flow rate. This policy will affect the performance of TCP and results in considerable degrade over wireless link [1], [2], [3], [20]. To improve the performance of TCP over wireless network, many techniques are proposed by the researchers. Some of them are, (i) I-TCP (Indirect TCP) - use two different links in the base station. One link is from base station to the mobile station and the other one is from wired host to the base station. Using wired link the base station receives data from the source and transmits the data to the mobile station using wireless link which feels like sending the data by using wired link [4]. (ii) M-TCP (Mobile TCP)-has the facility of keeping a retransmission timer in the base station. This timer is used by the sender TCP for checking the time-out period. For each receive of the data packet, base station sends an acknowledgement packet to the source before the timer period expires [5]. (iii) SNOOP is data link layer assisted protocol used to improve the performance of TCP in wireless networks (environment). The base station keeps track of all the packets transmitted from a source TCP. Base station keeps a copy of the packet in its buffer till it receives the acknowledgement. It removes the packet from its buffer after acknowledgement is received. When a packet loss occurs because of wireless transmission error, base station retransmits the lost packet. SOONP protocol does not answer for time-out event occurrence in mobile wireless transmission or the duplicate acknowledgements [6]. (iv) ELN (Explicit Loss Notification)-keeps the wireless loss information in the base station, and the base station sets the ELN bit to the duplicate acknowledgement. For the ELN-set duplicate acknowledgement, source TCP assumes that the packet loss is due to wireless transmission error and does not decrease its window size which results in congestion [7]. (v) W-TCP (Wireless-TCP)-is a rate-based wireless congestion control protocol. W-TCP assumes that the packet losses due to burst traffic as router congestion and random losses as the wireless transmission error. In each case, the receiver measures the time interval of the received packets and communicates the source to control the sending rate by using this time interval estimate. [8]. (vi) TCP Westwood- is a sender-side modification for wireless TCP. From the acknowledgement packet received, the sender estimates the current sending rate. For the congestion notification, the sender decides the congestion window size from the estimated sending rate. TCP Westwood has problem in time estimation and needs a fine-grained timer. In a dynamically changing network, the estimation cannot be accurate [9]. (vii) The ACK Pacing algorithm - can be used for black-out and hand-off delays. By sending ACK Pacing packets, burst data delivery is prevented from old path and the route update for new path [10]. (viii) JTCP (Jitter-based)-JTCP method is for heterogeneous wireless networks to adopt sending rates to the packet losses and jitter ratios [11]. (ix) ACK-Splitting-improves the TCP throughput over wired and wireless heterogeneous networks [12]. However, these schemes may not have good performance or fairness or it is too complex to deploy in wireless systems. To satisfy these challenges and for applying TCP over wireless link, we propose a model based scheme that can dynamically change the sending rate to the packet loss due to congestion and transmission. The model has integrated with the capability of distinguishing the packet losses due to congestion or transmission. If the loss is due to congestion, congestion control algorithm is invoked to reduce the network congestion and when the loss is due to transmission, immediate-recovery algorithm is invoked to recover from the decreased flow rate. One of the popular congestion avoidance schemes called RED (random early detection) and an accurate losspredictor function called NTG loss-predictor which is designed based on CAT are used in the system model. Congestion avoidance technique (CAT) monitors the level of congestion in the network, and instructs the source that the sender window should be increased or decreased and vice versa. A TCP source understands that a particular packet has lost due to congestion or due to wireless transmission error and take appropriate action [18]. To enhance performance of TCP for further, we apply stability [13], [14], [22]. A time-delay control theory is used and hermit matrix method is applied to analyse the asymptotic stability. A relationship between RED parameter p_{max} and NTG loss-predictor parameter β is established. The stability boundaries are established in terms of wireless network parameters. The work is further enhanced by analysing the queue convergence at the ingress point of the bottleneck link. Using characteristic equation of the Hermite matrix, an approximate solution for the instantaneous queue length, q(t) is derived. The convergence boundaries of q_0 , p_{max} and β are presented. Using these boundary values, convergence analysis of the queue length for a given target value is discussed. The helps in maintaining RED router stable. Using Matlab numerical results are given to validate the analytical results. The above illustrated characteristics collectively make TCP robust by minimising the packet losses and maximising throughput. Our results provide global stability and convergence conditions of the system.



Fig.1: Network model.

2. System model for wireless networks

The The extended fluid model [15], [16], [17], [21] that describe the dynamics of the TCP congestion window size in wireless networks is,

$$dW(t) = \frac{dt}{RTT} - \alpha W(t)L_a(t) + \beta L_t(t)$$
(1)

TCP operates on AIMD congestion avoidance strategy. The factor α is decrease rate of source window which is normally 0.5, L_a (t) is the rate of arrival of packet losses due to congestion at time t and $L_a(t) =$

 $\frac{w(t-R(t))}{R(t-R(t))} p(t-R(t-R(t)))$. This loss is proportional to the throughput at the source. First term on the right hand side of equation (1) refers to exponential increase of the sender window size until congestion occurs at the destination. The second term refers to congestion avoidance scheme based on RED. The third term $L_t(t)$ refers to immediate-recovery due to transmission loss. The transmission loss is proportional to the sending rate of the source. Therefore, $L_t(t)$ is proportional to $\frac{w(t-R(t))}{R(t-R(t))}$

The equation (1) can be modified to,

$$\frac{\partial w}{\partial t} = \frac{1}{R(t)} - \frac{w(t)w(t-R(t))}{2R(t-R(t))} p(t-R(t)) + \beta \frac{w(t-R(t))}{R(t-R(t))}$$
(2)

 β is called NTG loss-predictor parameter. It has some constant rate of transmission-loss and by choice β ϵ [0,1]. The differential version of Lindley's equation for capturing the dynamic behaviour of instantaneous queue length is given by,

$$\frac{\partial q(t)}{\partial t} = \frac{N w(t)}{R(t)} - C_d \tag{3}$$

Where, w(t)/R(t) is increase in queue length due to arrival of the packets from N- TCP flows. The down-link capacity, $C_d = q(t)/R(t)$ is the decreasing factor in queue length due to servicing of the packets and delay of the packet departure from the router. The service time is variable. One of the most important advantages of using instantaneous queue length over average queue length is faster detection of congestion. The mathematical version of RED scheme for dropping packets with probability is given by,

$$P(t) = \begin{cases} 0, & q(t) \in [0, t_{min}] \\ \frac{q(t) - t_{min}}{W_{max} - t_{min}} P_{max}, q(t) \in [t_{min}, W_{max}] \\ 1, & q(t) \ge W_{max} \end{cases}$$
(4)

Congestion loss is assumed to take place when queue buffer of the router reaches a value of W_{max} packets. The maximum buffer size, W_{max} of the router is given by, $W_{max} = \frac{C_d}{S} R(t) + M$, Where C_d/S is the bandwidth delay product, M (in packets) is the buffer size of the router, $\tilde{R(t)}$ is the round trip time. The buffer overflow takes place when the congestion window size becomes larger than W_{max} value. C_d is the down-link bandwidth. The model describing round trip time (RTT) in wireless networks is given by, (t) = $\frac{q(t)}{C_{\rm c}} + T_p$, Tp is the propagation delay in the wireless media, q(t)/C_d models the queuing delay.

2.1. Normalised throughput gradient (NTG)

NTG is a metric used to deduce the traffic load from the acknowledgements. When a TCP connection has no data to transmit, then other TCP sources detect the change using NTG metric and absorb the released resources. When a source initiates a connection, the window is set to one packet, after receiving an acknowledgement, the scheme checks the NTG. (i) If the NTG is lesser than the NTG threshold value, sender window enters the increased mode. (ii) If the NTG is greater than or equal to the NTG threshold value, increase the cwnd (sender window size) by one packet [18]. Wang and Crowcroft [19] proposed a congestion avoidance method based on normalised throughput gradient (f_{NTG}). Let P_i be the ith monitored packet. The throughput gradient of P_i is given by $TG_i = \frac{T_i - T_{i-1}}{W_i - W_{i-1}}$

The normalised throughput gradient,

$$f_{NTG} = \frac{TG_i}{TG_1}$$

But,
$$TG_1 = \frac{T_1 - T_0}{W_1 - W_0} = \frac{1}{RTT_i}$$

Chose,

$$w_0 = 0, w_1 = 1, T_0 = 0, and T_1 = \frac{w_1}{RTT_1} = \frac{1}{RTT_1}$$

Thus, $fNTG = \frac{TG_i}{\frac{1}{RTT_i}}$

After substitution and simplification, we get

$$f_{NTG} = \frac{RTT_i}{(W_i - W_i - 1)} \left(\frac{W_i}{RTT_i} - \frac{W_i - 1}{RTT_i - 1}\right)$$

The NTG loss-predictor is a congestion avoidance technique. It uses throughput values to determine the cause of a packet loss. As the traffic load changes NTG value varies over the range [1, 0]. Under light traffic, the NTG value is around 1. NTG value decreases gradually for the increase in traffic load, and reaches 0 when the path is saturated. When the resources captured by a TCP session are released, the NTG value increases substantially. Without loss of generality we choose NTG threshold value as 0.5.

i) If $f_{NTG} < \frac{1}{2}$, the loss of next packet is due to congestion.

ii) if $f_{NTG} \ge 1/2$, the loss of next packet is due to transmission.

When the source detect that loss is due congestion, NTG parameter, β =0. The threshold value of sender window is fixed to half of the current window size and slow start phase started. When source detects that loss is due to wireless transmission, a loss recovery module called Immediate-recovery algorithm gets invoked, where sender window size is added with β -times the sending rate in the previous RTT.

3. Time-delay feedback control system

In this section, we study the asymptotic stability of TCP in wireless network system. The aim is to save the network system from congestion and decrease the packet losses accruing due to congestion. The implement methodology involves (i) linearizing the system models, (ii) using the Hermite matrix for timedelay control system, explicit conditions under which the system is asymptotically stability are obtained. A relationship between RED parameter P_{max} and NTG parameter β is derived. The stability regions for Pmax and β in terms of wireless TCP parameters are obtained.

3.1. Linear model derivation

Let x(t) be a general non-linear function defined by, x(t) = f(u(t), v(t), t), where, u(t) represents the sender window dynamics, and v(t) represent the queue dynamics at the bottleneck link. Assuming that f(u(t), v(t), t) has smooth and continuous derivatives around the equilibrium point, $Q_0 = (w_0, R_0, q_0, p_0)$. Using Taylor's series expansion, the linear function of non-linear function, ignoring second and higher order partial derivatives is,

$$f(u(t), v(t), t) = f(u_0(t), v_0(t), t_0) + f_u(t)\delta_u(t) + f_v(t)\delta_v(t) + O(\delta_u(t), \delta_v(t))$$

where $(t) = \frac{\partial f}{\partial u}|(u_0, v_0), f_v(t) = \frac{\partial f}{\partial v}|(u_0, v_0).$

The linear models of the equations (2) to (4) are derived around the equilibrium point $Q_0 = (w_0, R_0, q_0, p_0)$. Let N be the number of TCP flows and R be the round trip time which are considered as constants. At the equilibrium point Q_0 , the steady state conditions of equations (2) and (3) are given by $\dot{w}(t) = 0, \dot{q}(t) = 0$.

The estimation algorithm is based on small signal behaviour dynamics, therefore, at the equilibrium point, without loss of generality, we can assume,

$$w(t) = w(t - R(t)) = w_0, q(t) = q(t - R(t)) = q_0, p(t) = p(t - R(t)) = p_0,$$

$$R(t) = R(t - R(t)) = R_0$$
(5)

Using equations (5) in $\dot{w}(t) = 0$, $\dot{q}(t) = 0$, and after simplification we get,

$$p_{0} = \frac{2\beta w_{0} + 2}{w_{0}^{2}}, w_{0} = \frac{R_{0}C_{d}}{N}, N = \frac{R_{0}C_{d}}{w_{0}}$$
Let, $w_{R} = w(t - R(t)), p_{R} = p(t - R(t))$. From (2) we get,
 $u(w, w_{R}, q, p_{R}) = \frac{1}{R(t)} - \frac{w(t)w_{R}(t)}{2R(t)}p(t - R(t)) + \beta \frac{w_{R}(t)}{R(t)}$
(6)
To linearize equation (6) find all the partial derivatives of $u(w, w_{R}, q, p_{R})$ with respect to the variables of

To linearize equation (6), find all the partial derivatives of u(w,wR,q,pR) with respect to the variables at the equilibrium point and defining,

$$\delta w(t) = w(t) - w_0, \, \delta q(t) = q(t) - q_0, \, \delta p(t) = p(t) - p_0$$

$$\delta \dot{w}(t) = \dot{w}(t) - w_0, \qquad \delta \dot{q}(t) = \dot{q}(t) - q_0$$

$$\delta p(t) = \frac{L}{B} (q(t) - t_{min}) - p_0 , \text{ where } B = W_{max} - t_{min}, \quad L = p_{max}$$

$$\delta \dot{w}(t) = -\left(\frac{\beta}{R_0} + \frac{2N}{R_0^2 C_d}\right) \delta w - \frac{C_d^2 R_0}{2N^2} \, \delta p_R \tag{7}$$

218 N.G.Goudru et al.: Enhancement of Performance of TCP Using Normalised Throughput Gradient in Wireless Networks

$$\delta \dot{q}(t) = \frac{N}{R_0} \delta w(t) - \frac{1}{R_0} \delta q(t)$$
(8)

$$\delta p(t - R_0) = \frac{L}{B} \left(\delta q(t - R_0) \right) + \frac{L}{B} (q_0 - t_{min}) - p_0 \tag{9}$$

Using equation (9) in (7), we get

$$\delta \dot{w}(t) = -\left(\frac{R_0}{w_0} + \frac{2N}{R_0^2 C_d}\right) \delta w(t) - \frac{LC_d^2 R_0}{2N^2 B} \delta q(t - R_0) + \frac{LC_d^2 R_0}{2N^2 B} (t_{min} - q_0)$$

$$+ \frac{R_0 C_d^2 p_0}{2N^2}$$
(10)

Denote,
$$x(t) = \begin{bmatrix} \delta w(t) \\ \delta q(t) \end{bmatrix}$$
, then $\dot{x}(t) = Ax(t) + Ex(t - R(t)) + F$ (11)
Where

wnere,

$$A = \begin{bmatrix} -\left(\frac{\beta}{R_0} + \frac{2N}{R_0^2 C_d}\right) & 0\\ \frac{N}{R_0} & \frac{-1}{R_0} \end{bmatrix} \quad E = \begin{bmatrix} 0 & \frac{-LR_0C_d^2}{2N^2B}\\ 0 & 0 \end{bmatrix}, F = \begin{bmatrix} \frac{-LR_0C_d^2}{2N^2B}(t_{min} - q_0) + \frac{R_0C_d^2p_0}{2N^2}\\ 0 \end{bmatrix}$$

Solving linear differential equation (11) using Laplace transform technique with $L{x(t)} = x(s)$, we get

$$(sI - A - Ee^{-R_0 s})x(s) = \frac{F}{s}$$
(12)

The characteristic equation of (12) is, $|sI - A - Ee^{-R_0 s}| = 0$ (13)After simplification,

$$s^{2} + \left(\frac{\beta}{R_{0}} + \frac{2N}{R_{0}^{2}C_{d}} + \frac{1}{R_{0}}\right)s + \left(\frac{\beta}{R_{0}^{2}} + \frac{2N}{R_{0}^{3}C_{d}} + \frac{LC_{d}^{2}}{2NB}e^{-R_{0}s}\right) = 0$$
(14)

The characteristic equation (14) determines the stability of the closed-loop time-delay wireless system in terms of the state variables $\delta w(t)$ and $\delta q(t)$.

3.2. Stability analysis

Denote,
$$P(s, e^{-R_0 s}) = s^2 + \left(\frac{\beta}{R_0} + \frac{2N}{R_0^2 c_d} + \frac{1}{R_0}\right)s + \left(\frac{\beta}{R_0^2} + \frac{2N}{R_0^3 c_d} + \frac{LC_d^2}{2NB}e^{-R_0 s}\right)$$

Let $e^{-R_0 s} = z$, and $a_0 = 1$, $a_1 = \frac{\beta}{R_0} + \frac{2N}{R_0^2 c_d} + \frac{1}{R_0}$, $a_2 = \frac{\beta}{R_0^2} + \frac{2N}{R_0^3 c_d} + \frac{LC_d^2}{2NB}e^{-R_0 s}$
 $P(s, z) = a_0 s^2 + a_1 s + a_2$ (15)

The Hermit matrix for time-delay control system of equation (15) is

$$H = \begin{bmatrix} (0,1) & (0,2) \\ (0,2) & (1,2) \end{bmatrix}$$
$$(0,1) = 2a_0a_1 = \frac{2(\beta+1)}{R_0} + \frac{4N}{R_0^2C_d}$$
$$(0,2) = -2a_2 Im(z) = \frac{-Lc_d^2}{NB} Im(z)$$
$$(1,2) = 2a_1Re(a_2) = 2(\frac{\beta}{R_0} + \frac{2N}{R_0^2C_d} + \frac{1}{R_0})(\frac{\beta}{R_0^2} + \frac{2N}{R_0^3C_d} + \frac{LC_d^2}{2NB})$$
Put, $x_1 = \frac{\beta}{R_0} + \frac{2N}{R_0^2C_d} + \frac{1}{R_0}, x_2 = \frac{LC_d^2}{NB}, x_3 = \frac{\beta}{R_0^2} + \frac{2N}{R_0^3C_d}$ Let, $z = e^{i\omega}$, $z = cos\omega + isin\omega$, $Re(z) = cos\omega$, $Im(z) = sin\omega$, then
 $H(e^{i\omega}) = \begin{bmatrix} 2x_1 & -x_2 sin\omega\\ -x_2 sin\omega & 2x_1(x_3 + \frac{x_2}{2} cos\omega) \end{bmatrix}$.

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3.3. Derivation of stability conditions

The time-delayed control system (2) to (4) is asymptotically stable in terms of stable variables $\delta w(t)$ and $\delta q(t)$, if and only if the following two conditions are satisfied. **Condition 1**

The Hermit matrix $H(1) = H(e^{i0})$ is positive.

$$H(1) = \begin{bmatrix} 2x_1 & 0\\ 0 & 2x_1(x_3 + \frac{x_2}{2}) \end{bmatrix}$$

From the determinant, $4x_1^2\left(x_3 + \frac{x_2}{2}\right) > 0$, after simplification, $L = p_{max} > -\left(\frac{2\beta NB}{R_2^2 C_1^2} + \frac{4N^2B}{R_2^2 C_1^2}\right)$

$$\beta = p_{max} > -(\frac{2\beta NB}{R_0^2 C_d^2} + \frac{4N^2 B}{R_0^3 C_d^3})$$
$$\beta > -(\frac{p_{max} C_d^2 R_0^2}{2NB} + \frac{2N}{R_0 C_d})$$

Condition 2

For all $\omega \in [0, 2\pi]$, det $H(e^{i\omega}) > 0$, leads to the following inequality.

$$(2x_1)\left(2x_1\left(x_3 + \frac{x_2}{2}\right)\right)\cos\omega - x_2^2\sin^2\omega > 0$$

$$H(e^{i\omega}) = x_2^2\cos^2\omega + 2x_1^2x_2\cos\omega + 4x_1^2x_3 - x_2^2 > 0$$
(16)

The necessary condition for (16) to be true is the discriminate, $\Delta > 0$

$$\cos\omega = \frac{-x_1^2 \pm \sqrt{x_1^2(x_1^2 - 4x_3) + x_2^2}}{x_2}$$

By the properties of cosine function, for $\omega \in [0,2\pi]$, $\cos \omega \in [-1,1]$ The conditions can be written as,

$$\frac{-x_1^2 - \sqrt{x_1^2(x_1^2 - 4x_3) + x_2^2}}{x_2} > 1 \tag{17}$$

$$\frac{-x_1^2 + \sqrt{x_1^2(x_1^2 - 4x_3) + x_2^2}}{x_2} < -1 \tag{18}$$

By direct manipulation, there is no solution for the inequality (17). From (18), we obtain

$$0 < p_{max} < \left(\frac{2\beta NB}{R_0^2 C_d^2} + \frac{4N^2 B}{R_0^3 C_d^3}\right)$$
(19)

$$0 < \beta < \left(\frac{p_{max} C_d^2 R_0^2}{2NB} + \frac{2N}{R_0 C_d}\right)$$
(20)

3.4. Theorem 1

Given the wireless network parameters Cd (down link capacity), Cu (up link capacity), N (number of TCP sessions), R0, and the RED parameter B (minimum minus maximum threshold values), the wireless network system given by (2) to (4) is asymptotically stable in terms of the state variables $\delta w(t)$ and $\delta q(t)$ if and only if the RED control parameters Pmax (maximum packet discarding probability) and β (NTG loss-predictor parameter) satisfies,

$$0 < p_{max} < \left(\frac{2\beta NB}{R_0^2 C_d^2} + \frac{4N^2 B}{R_0^3 C_d^3}\right)$$
$$0 < \beta < \left(\frac{p_{max} C_d^2 R_0^2}{2NB} + \frac{2N}{R_0 C_d}\right)$$

4. Convergence analysis of dynamic queue

In this section, we discussion the convergence of the buffer queue length in the router. From equation (12),

$$x(s) = (s I - A - E e^{-R_0 s})^{-1} \frac{F}{s}$$
(21)

$$(s I - A - E e^{-R_0 s})^{-1} = \frac{1}{P(s, e^{-R_0 s})} \begin{bmatrix} y_1 & y_2 \\ y_3 & y_4 \end{bmatrix}$$

Where, $y_1 = s + \frac{1}{R_0}$, $y_2 = -\frac{LR_0 C_d^2}{2N^2 B} e^{-R_0 s}$, $y_3 = \frac{N}{R_0}$, $y_4 = s + \frac{\beta}{R_0} + \frac{2N}{R_0^2 C_d}$

Equation (21) can be written as

$$x(s) = \frac{F}{s \, p(s, e^{-R0s})} \begin{bmatrix} y_1 & y_2 \\ y_3 & y_4 \end{bmatrix}$$

After simplification, $x(s) = \frac{y_5}{s p(e^{-R_0 s})} \begin{bmatrix} y_1 \\ y_3 \end{bmatrix}$, Or

$$\begin{bmatrix} \delta W(s) \\ \delta Q(s) \end{bmatrix} = y_5 \begin{bmatrix} y_1 \\ y_3 \end{bmatrix}$$
(22)

Where,

$$y_5 = \frac{LR_0C_d^2 \frac{(t_{min} - q_0)}{2N^2B} + \frac{R_0C_d^2p_0}{2N^2}}{s \, p(s, e^{-R_0s})}$$

From equation (22),

$$\delta Q(s) = \frac{\frac{N}{R_0} \left(\frac{LR_0 C_d^2}{2N^2 B} (t_{min} - q_0) + \frac{R_0 C_d^2 p_0}{2N^2}\right)}{s^P(s, e^{-R_0 s})}$$

$$\delta Q(s) = \frac{\left(\frac{C_d^2}{2N}\right) \left(\frac{L}{B} (t_{min} - q_0) + p_0\right)}{sp(s, e^{-R_0 s})}$$

$$\delta Q(s) = \frac{\frac{1}{s} \frac{C_d^2}{2N} \left(\frac{L}{B} (t_{min} - q_0) + p_0\right)}{s^2 + \left(\frac{\beta}{R_0} + \frac{2N}{R_0^2 C_d} + \frac{1}{R_0}\right) s + \left(\frac{\beta}{R_0^2} + \frac{2N}{R_0^3 C_d}\right) + \left(\frac{LC_d^2}{2N B}\right) e^{-R_0 s}}$$

$$\delta Q(s) = \frac{y_6}{s^2 + x_1 s + x_3 + \frac{x_2}{2} e^{-R_0 s}}$$
(23)

Where,

$$y_6 = \frac{1}{s} \frac{C_d^2}{2N} \left(\frac{L}{B}(t_{min} - q_0) + p_0\right)$$

4.1. Theorem 2

Given the wireless network parameters C_d , C_u , N, R_0 and B, wireless network system given in (2) to (4), has the approximate solution of q(t) given by,

$$q(t) = q_0 + y_6 \left[\frac{e^{xt}}{(x - y)x} - \frac{e^{yt}}{(x - y)y} + \frac{1}{xy}\right]$$

Provided Pmax and β satisfies the conditions as

$$0 < p_{max} \le \frac{-\xi_2 + \sqrt{\xi_2^2 - 4\xi_1\xi_3}}{2\xi_1}$$
$$0 < \beta \le \frac{-A_2 + \sqrt{A_2^2 - 4A_1A_3}}{2A_1}$$

Where ξi 's and Ai 's are as given in (29).

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Proof

Expanding $e^{-R_0 s}$, using Taylor's theorem about the stability point discarding the terms of order three and higher,

$$e^{-R_0s} = 1 - R_0s + \frac{R_0^2s^2}{2}$$

From (16),
$$s^{2} + x_{1}s + x_{3} + \frac{x_{2}}{2}e^{-R_{0}s}$$
. This simplifies to
 $s^{2} + \left(\frac{\beta}{R_{0}} + \frac{2N}{R_{0}^{2}C_{d}} + \frac{1}{R_{0}}\right)s + \left(\frac{\beta}{R_{0}^{2}} + \frac{2N}{R_{0}^{3}C_{d}} + \frac{LC_{d}^{2}}{2NB}(1 - R_{0} + \frac{R_{0}^{2}s^{2}}{2})\right)$
 $\left(1 + \frac{LR_{0}^{2}C_{d}^{2}}{4NB}\right)s^{2} + \left(\frac{\beta}{R_{0}} + \frac{2N}{R_{0}^{2}C_{d}} + \frac{1}{R_{0}} - \frac{LR_{0}C_{d}^{2}}{2NB}\right)s + \left(\frac{\beta}{R_{0}^{2}} + \frac{2N}{R_{0}^{3}C_{d}} + \frac{LC_{d}^{2}}{2NB}\right)$
Put $\eta 1 = \left(1 + \frac{LR_{0}^{2}C_{d}^{2}}{4NB}\right)$, $\eta 2 = \frac{\beta}{R_{0}} + \frac{2N}{R_{0}^{2}C_{d}} + \frac{1}{R_{0}} - \frac{LR_{0}C_{d}^{2}}{2NB}$, $\eta 3 = \frac{\beta}{R_{0}^{2}} + \frac{2N}{R_{0}^{3}C_{d}} + \frac{LC_{d}^{2}}{2NB}$
 $\eta_{1}s^{2} + \eta_{2}s + \eta_{3}$
(25)

If $\Delta > 0$, equation (25) has distinct roots given by,

$$x, y = \frac{-\eta_2 \pm \sqrt{\eta_2^2 - 4\eta_1 \eta_3}}{2\eta_1} \tag{26}$$

where, x takes + and y takes - of \pm

The discriminate of (26) is positive. After simplification for the discriminant, and expressing in terms of L and β , we get,

$$\xi_1 L^2 + \xi_2 L + \xi_3 > 0 \tag{27}$$

$$A_1\beta^2 + A_2\beta + A_3 > 0 \tag{28}$$

where $\xi_1 = \left(\frac{C_d^2 R_0}{2NB}\right)^2$, $\xi_2 = \frac{2\beta c_d^2}{NB} + \frac{4C_d}{R_0B} + \frac{C_d^2}{NB}$

$$\xi_3 = \frac{4\beta}{R_0^2} - \left(\frac{\beta}{R_0} + \frac{1}{R_0} - \frac{2N}{R_0^2 C_d}\right)^2$$
(29)

$$A_{1} = \frac{1}{R_{0}^{2}}$$

$$A_{2} = \frac{4N}{R_{0}^{3}C_{d}} - \frac{2}{R_{0}^{2}} - \frac{2LC_{d}^{2}}{NB}$$

$$A_{3} = \frac{1}{R_{0}} + \frac{4N^{2}}{R_{0}^{4}C_{d}^{2}} - \frac{4N}{R_{0}^{3}C_{d}} - \left(\frac{LC_{d}^{2}R_{0}}{2NB}\right)^{2} - \frac{LC_{d}^{2}}{NB} - \frac{4LC_{d}}{R_{0}B}$$

Solving (27) for L=Pmax and (28) for β , we get

$$0 < p_{max} \le \frac{-\xi_2 + \sqrt{\xi_2^2 - 4\xi_1\xi_3}}{2\xi_1} \tag{30}$$

$$0 < \beta \le \frac{-A_2 + \sqrt{A_2^2 - 4A_1A_3}}{2A_1} \tag{31}$$

From Equation (24) and (26),

$$\delta Q(s) = \frac{y_6}{s(\eta_1 \, s^2 + \eta_2 \, s + \eta_3)}$$

$$\delta Q(s) = \frac{y_6}{s(s - x)(s - y)}$$

Using partial fraction,

$$\delta Q(s) = y_6 \left[\frac{1}{x(x-y)} \frac{1}{(s-x)} - \frac{1}{y(x-y)} \frac{1}{s-y} + \frac{1}{xy} \frac{1}{s} \right]$$

Taking Laplace transform,

222 N.G.Goudru et al.: Enhancement of Performance of TCP Using Normalised Throughput Gradient in Wireless Networks

$$\delta q(t) = y_6 \left[\frac{e^{xt}}{x(x-y)} - \frac{e^{yt}}{y(x-y)} + \frac{1}{xy} \right]$$

$$q(t) = q_0 + y_6 \left[\frac{e^{xt}}{x(x-y)} - \frac{e^{yt}}{y(x-y)} + \frac{1}{xy} \right]$$
(32)

4.2. Theorem 3

Given the wireless network parameters C_d, C_u, N, R0 and B, the instantaneous queue length converges to the target, $T = \frac{(t_{min}+w_{max})}{2}$ if and only if RED control parameter, P_{max} and NTG parameter β satisfies the conditions as

$$P_{max} = K_1 K_2 + K_3$$
$$\beta = K_4 K_5$$

where K_i 's are given by (36).

Proof

From equation (24),

$$\lim_{t \to \infty} \delta q(t) = \lim_{s \to 0} sQ(s) = \lim_{s \to 0} \frac{y_6}{s^2 + x_1 s + x_3 + \frac{x_2}{2} e^{-R_0 s}}$$
$$\lim_{t \to \infty} \delta q(t) = \lim_{s \to 0} sQ(s) = \frac{y_6}{x_3 + x_2/2} + q_0$$
$$\lim_{t \to \infty} q(t) = q_0 + \lim_{t \to \infty} \delta q(t)$$
$$\lim_{t \to \infty} q(t) = q_0 + \frac{y_6}{x_3 + \frac{x_2}{2}}$$
(33)

Let
$$\lim_{t \to \infty} q(t) = \frac{t_{min} + w_{max}}{2}$$
$$P_{max} = K_1 K_2 + K_3$$
(34)

$$\beta = K_4 K_5 \tag{35}$$

Where

$$K_{1} = \frac{4N^{2}}{R_{0}^{3} C_{d}^{3}} + \frac{2\beta N}{R_{0}^{2} C_{d}^{2}}$$

$$K_{2} = 2q_{0} - t_{min} - w_{max}$$

$$K_{3} = \frac{4\beta N}{R_{0}C_{d}} + \frac{4N^{2}}{R_{0}^{2} C_{d}^{2}}$$

$$K_{4} = \frac{P_{max} R_{0}^{2} C_{d}^{2} - 4N^{2}}{2N + 4NR_{0}C_{d}}$$

$$K_{5} = 1 + \frac{1}{R_{0}C_{d}} (2q_{0} - t_{min} - w_{max})$$
(36)

4.3. Theorem 4

The wireless network system given by equations (2)-(4) is asymptotically stable in terms of the state variable $\delta w(t)$ and $\delta q(t)$, if the queue level q_0 at equilibrium point satisfies

$$\frac{t_{min} + w_{max}}{2} - \frac{R_0 C_d}{2} < q_0 \le \frac{t_{min} + w_{max}}{2} - \frac{R_0 C_d}{2} - \frac{\xi_2}{\xi_7} + \frac{\xi_8}{\xi_7} \sqrt{\xi_6}$$

Proof

Using (34) in (30)

$$0 < K_1 K_2 + K_3 \le \frac{-\xi_2 + \sqrt{\xi_2^2 - 4\xi_1\xi_3}}{2\xi_1}$$

Subtitling for K1, K2 and K3 from (36), and simplifying for q_0 , we get

$$\frac{t_{min} + w_{max}}{2} - \frac{R_0 C_d}{2} < q_0 \le \frac{t_{min} + w_{max}}{2} - \frac{R_0 C_d}{2} - \frac{\xi_2}{\xi_7} + \frac{\xi_8}{\xi_7} \sqrt{\xi_6}$$
(37)

$$\xi_{6} = \frac{C_{d}^{4}}{N^{2}B^{2}} \left[\left(2\beta + 1 + \frac{4N}{R_{0}C_{d}} \right)^{2} + (\beta - 1)^{2} + \frac{4N}{R_{0}C_{d}} \left(\frac{N}{R_{0}C_{d}} - \beta - 1 \right) \right]$$

$$\xi_{7} = \frac{2C_{d}^{3}}{NB^{2}} \left(\beta + \frac{2N}{R_{0}C_{d}} \right)$$

$$\xi_{8} = \frac{C_{d}^{2}}{NB}.$$
(38)

5. Simulation and performance analysis

A number of simulation experiments are conducted to evaluate the performance of TCP with NTG losspredictor. All simulations are performed using Matlab R2009b. The network model is as illustrated in Figure 1. S_1 and S_2 are the sources, R_1 and R_2 are the routers, and D_1 and D_2 are the mobile stations which are the destinations. We have TCP Reno connections from sources to destinations. These connections share the link R_1 and R_2 . The TCP which has been modeled represent the last hop transmission between R_2 and the destinations D_1 and D_2 .

5.1. Experiment 1: un-stable network system

In this experiment, the link between R_1 and R_2 is the bottleneck link. Packet size is 1000 bytes, P_{max} =0.05, queue buffer at the router has a minimum threshold value, t_{min} =200 packets, maximum threshold, W_{max} = 500 packets, initial RTT= 50 ms, C_d =10 Mbps, Cu = 500Kbps, N =10.





If the packet loss is due to congestion, TCP should react such that it minimises congestion in the network. If the loss is due to transmission, TCP should continue with its operations. When transmission loss is treated as a congestion loss, the sender will unnecessarily reduce its offered load, causing reduction in the throughput. To overcome this problem, we use NTG loss-predictor. The loss-predictor function, $f_{NTG} < 0.5$ the monitored packet is lost due to congestion in the network. In the model, this situation is described by using NTG parameter β . When $\beta=0$, congestion control algorithm is invoked to reduce the flow rate. The

loss-predictor function, $f_{NTG} \ge 0.5$ the monitored packet is lost due to wireless transmission then β =0.1 (conveniently selected value) immediate-recovery algorithms is invoked. The advantage of introducing β is within a short span of time the sender window size recovered from one packet to the window size whose flow rate was that of previous rtt. The graph in fig.2 illustrates the traces of immediate-recovery algorithm when $f_{NTG} \ge 0.5$.

5.1.1. Analysis of performance metrics of fNTG

This experiment was conducted to analyse the performance ability of NTG loss-predictor to distinguish congestion loss from wireless transmission loss. Important performance analysis metrics are, (i) Frequency of congestion loss prediction (FCP): FCP is obtained by dividing the number of times the loss-predictor predicts that the next loss will be due to congestion, by the total number of times the predictor was evaluated during the TCP connection. Two counters are introduced to count the number of times the congestion and the transmission losses accrues. The estimated values were used in the calculation of FCP, FWP, Ac and Aw. In this experiment, number of times the NTG loss-predictor predicts that the next loss will be due to congestion=812. Total number of times the predictor is evaluated during a TCP connection=2000. FCP=812/2000=0.406(approximately 40%). (ii) Frequency of wireless loss prediction (FWP): FWP is obtained by dividing the number of times the NTG loss-predictor predicts that the next loss will be due to wireless transmission error, by the total number of times the predictor was evaluated during the TCP connection. Number of times the loss-predictor is predicted that the next loss will be due to wireless=1188. Total number of times the predictor is evaluated during a TCP connection=2000. FWP= 1188 /2000=0.594 (approximately 60%). (iii) Congestion loss prediction (Ac): Ac is the fraction of packet losses due to congestion, diagnosed by NTG loss-predictor. (iv) Wireless loss prediction (Aw): Aw is the fraction of packet losses due to wireless transmission errors diagnosed by NTG loss-predictor.



Fig. 3: Variation of FCP, FWP, Ac and Aw w.r.t time.

In experiment 1, during the simulation, approximately 40% of the losses are due to congestion and 60% of the losses are due to wireless transmission as predicted by the loss-predictor. The graph of Fig. 3 shows the performance metric with respect to (w.r.t) time in seconds.





The round trip time (RTT) is the total time taken by a packet to travel from source to destination and the ACK from destination to source. In our work, RTT includes variable queuing delay and a constant propagation delay. In the experiment, we assume that both wired and wireless medium has a constant propagation delay of 50 ms. Thus, RTT varies over the range [50, 250] milliseconds. Since RTT depends on queue length, we observe that increase in queue length leads to increase in RTT value and vice versa. Larger the queue length implies higher the congestion. Fig.4 describe the performance of FCP and Ac. FCP increase slowly in the beginning and becomes high over the RTT range 200 to 250. Increase in congestion is responsible for increase in FCP. The graph is also associated with the fraction of packet losses due to congestion diagnosed by NTG loss-predictor. Fig.5 illustrates the traces of FWP and Aw with respect to rtt-delay. FWP is almost constant till rtt-delay reach 220 ms, suddenly increases when delay is 220-245 ms. This may be because of burst in traffic and wireless transmission impairments. The graph is also associated with Aw, the fraction of packet losses due to wireless transmission errors diagnosed by NTG loss-predictor.

Queue length is measured at the bottleneck link of R_2 . We consider a variable queue which depends on the components such as (i) traffic due to arrival of the packets from TCP- sources,(ii) decrease in queue length due to servicing of the packets by the router and delay of the packet departure. NTG loss-predictor built based on congestion avoidance strategy, so f_{NTG} is increasing with increase in queue length. As a result, FCP increases slowly in the beginning and high with the increase queuing-delay. The traces of Fig. 6 and Fig. 7 illustrate the variation of FCP, FWP, Ac and Aw w.r.t queuing-delay. Sum of the fraction of packet losses due to congestion (Ac) is 11 packets and sum of the fraction of packet losses due to transmission (Aw) is 7 packets for a total of 119520 packets transported during simulation.







Fig.7: queuing-delay versus FWP and AW.

5.2. Experiment 2: stable network system

The objective of stability analysis is to minimise the occurrence of queue overflow and underflow, thus reducing the packet loss and maximising the link utilisation. Majority of the end-point control techniques regulate the traffic demands according to the resources available in the wireless network. In an end-to-end control scheme, stability analysis can achieve optimality. A time-delay control theory is applied for the analysis of FCP, Ac, FWP, and Aw. Knowing the wireless network parameters such as C_d, Cu, N, R₀ (round trip time), and B (RED parameter, determined by maximum minus minimum queue buffer threshold values). we derive a stability range for P_{max} (maximum packet dropping probability) and β (f_{NTG} parameter for immediate- recovery). The relations are illustrated in equation (19) and (20) with reference to theorem-1. Given, Cd=10Mbps, Cu=500Kbps, B=300packets, N=10, R₀=100 ms, at the equilibrium point we get $P_{max}=0.0068$, and $\beta=0.1538$.

The graphs in Fig.8 and Fig. 9 describe the stable queue length and sender window size respectively with respect to time. Queue length in the bottleneck link fluctuates over the range of [498, 500] packets. The sender window fluctuates over the range of [26, 29] packets. Since queue length stabilises over a definite range based on the available resources, the packet losses due to congestion is very small. The packet losses due to wireless transmission remain unchanged. The traces of Fig. 10 illustrate congestion and wireless losses with respect to time. Fig. 11 presents the immediate- recovery of the sender window size when the loss predictor predicts that next packet loss is due to transmission.



Fig. 9: Stable sender window.



Fig. 11:Stable immediate-recovery of source w.r.t f_{NTG}





Number of times the loss predictor predicted that the next loss is due to congestion= 652.Total number of times the predictor evaluated during a TCP connection=2000. FCP=605/2000=0.3025 (approximately 30%). Number of times the loss predictor predicted that the next loss is due to wireless=1394. Total number of times the predictor evaluated during a TCP connection=2000. FWP=1395/2000= 0.6975 (approximately 70%). Figure 12 and Fig. 13 illustrate the NTG prediction of congestion loss and transmission losses

respectively. Sum of the fraction of packet losses due to congestion (Ac) is 6 packets and sum of the fraction of packet losses due to transmission (Aw) is 9 packets for a total of 52276 packets transported. Fig. 14 and Fig. 15 illustrate the traces of FCP, FWP, Ac and Aw with respect to queue length.



Fig.15: FWP and Aw w.r.t. queue.

5.3. Experiment 3: queue convergence analysis

In a bust traffic, sender window and queue length exhibit oscillatory behaviour. The objective of analysis of convergence of the queue length is to minimise oscillatory nature of RED router. Using q0 as the initial value, queue length converges faster to the given target value. This helps in improving the fairness of the system. Knowing the wireless network parameters C_d , C_u , N, R_0 , and B, we derive a range of convergence for P_{max} , β , and q_0 using the relations (34),(35) and (37). Using (37) estimate q_0 , by using this value of q_0 and (34) find p_{max} , by using p_{max} and (35) find β . Given C_d =4Mbps, C_u =500Kbps, t_{min} = 200 packets, w_{max} = 500 packets, N=10, R_0 =100 ms, B=300 packets, taget value, T=350 packets, we get, q_0 =313, P_{max} =0.0054, and β =0.0018. The range of convergence of queue length is (305.12, 320.8). The traces of the graph of Fig. 16 illustrate the convergence of queue length for a given target value.

Maximum duration of the simulation time, the value of loss-predictor function f_{NTG} is greater than 0.5, predicting FCP is negligible and Ac is near to zero. Majority of the predictions are wireless transmission losses. The traces of Fig. 17 and Fig.18 describe the performance of f_{NTG} when queue convergence scheme is deployed.



Fig.16: Queue convergence to target value=350 packets.

6. Conclusion

The work presented in this paper is a novel research to improve the performance of TCP in wireless networks. The proposed transport model gives and end-to-end solution. The main problem of TCP has an implicit assumption that all packet loss is due to congestion which is resolved by introducing NTG loss-predictor. The NTG loss-predictor is a congestion avoidance method. NTG loss-predictor parameter β is integrated with the system model, and can tune depending on the prediction of the losses. Statistical analysis of FCP,FWP, Ac and Aw are made. After a successful implementation of loss detection in TCP, we tried to minimise the packet loss due to congestion by introducing the concept of stability. The stability boundary of two important tuning parameter P_{max} and β is derived in terms wireless network parameters. Knowing the wireless network resources and tuning these parameters, the system becomes asymptotically stable. After successful control on congestion loss, we discuss convergence analysis of queue length at the ingress point of the router. Condition for queue length convergence at the router is established. This helps in resolving the oscillatory problem of RED router. Thus, the model presented in this work is efficient, fair and adaptable to the wireless environment.



Fig.18: Performance of FWC and Aw.

7. References

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