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Tracking the State of the Hindmarsh-Rose Neuron by Using the Coullet Chaotic System Based on a Single Input

Jiawei Zhou[1], Xingping Zhu[2], Jin Liu[1], Yi Zhai[1], Zuolei Wang [1*]
[1]*School of Mathematical Sciences, Yancheng Teachers University,*
Yancheng, 224002, China, E-mail:wangzuolei@163.com
[2] *Department of Elementary Education, Yancheng Higher Normal School,*
Yancheng, 224002, China.
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Abstract. Based on Lyapunov stability theory, a partial synchronization scheme is proposed to track the signal of Hindmarsh-Rose neuron using the Coullet system via only one single controller. Summation for the series of error variables are employed to detect the degree of synchronization. Three cases are considered to verify the proposed partial synchronization scheme. To demonstrate the effectiveness of the proposed method, some simulation results are given. It is found that the arithmetic product of the gain coefficients dominate the process and speed of synchronization of the two systems. The larger arithmetic product of the two gain coefficients is used, the less time is required.

Keywords: chaos; Hindmarsh-Rose system; Coullet system; partial synchronization.

1. Introduction

Chaos synchronization has attracted the attention of research community since Carroll and Pecora discovered it [1]. The problem has been studied due to its potential applications, such as in secure communication [2], in biological systems [3], in robotics [4]. Recently, synchronization is applied into complex networks [5, 6].

Chaos synchronization is to make two chaotic systems identical after transient initial states. For a sufficiently strong coupling, complete synchronization of chaotic systems can occur [7]. It is found that synchronization is useful and has many potential applications in many domains [2-6], especially, synchronization in physical or biological system is a fascinating subject attracting many renewed attention [5]. In recent years, synchronizations of coupled neuronal systems, such as Hindmarsh-Rose (HR) model[8,9], FitzHugh-Nagumo (FHN) model[10] and Chay model[11], have been studied.

In general, synchronization research has been focused on two areas: one is related with the employment of state observers, where the main applications lie on the synchronization of non-linear oscillators with the same structure and order, but different initial conditions and/or parameters[12–16]; the other is about the use of control laws to achieve the synchronization between non-linear oscillators with different structures and orders, where the variable states of the slave system are forced to follow the trajectories of the master system. The second approach can be seen as a tracking problem [17–21]. It transforms the tracking problem to a regulation problem with the origin (zero) as the corresponding set point. Some suitable controllers are designed to achieve the synchronization. Synchronous motion is often considered as the equality of corresponding variables of two systems. In other words, the trajectories of two systems will follow the same path after some transient. However, this situation is not the only commonly understood synchronization. Other different relationships between coupled systems can be considered synchronous. The concept of partial synchronization between two or more similar chaotic systems has been studied [22, 23]. Vieira and Lichtenberg [24] showed that partial (in their notation “weak”) synchronization does not necessarily precede complete synchronization. Taborov et al. reported on partial synchronization in a system of three coupled logistic maps [25]. These results show that partial synchronization depends on the type of basic map constituting the coupled system. However, the mechanism of the occurrence of partial synchronization remains unclear.

In this paper, we will study the problem of tracking the **Hindmarsh-Rose neuron** signal by controlling the **Coullet chaotic system** with improved adaptive control scheme via single controller. In this scheme, changeable gain coefficients are introduced into Lyapunov function, which is composed of error variable between the outputs and the external standard signal. The controller is approached analytically. The main difference between the scheme and the previous ones is that the gain coefficient in the controller and Lyapunov function is changeable with time. Three cases are considered and some numerical simulations are presented to demonstrate the effectiveness of the proposed scheme.

2. System description

2.1. Hindmarsh-Rose neuron system

The **Hindmarsh-Rose (HR)** neuronal model was first proposed by Hindmarsh and Rose as a mathematical representation of the firing behavior of neurons. It was originally introduced to give a bursting type with long inters pike intervals of real neurons [26]. The form of HR system is given by

$$\begin{aligned} \dot{x}_1 &= ax_1^2 - bx_1^3 + x_2 - x_3 + I_{ext}, \\ \dot{x}_2 &= c - dx_1^2 - x_2, \\ \dot{x}_3 &= r(S(x_1 + k) - x_3), \end{aligned} \tag{1}$$

where a, b, c, d, r, S, k, I_{ext} are real constants.

2.2. Coullet chaotic system

The Coullet chaotic system, proposed by Coullet and Arneodo [27], is one of paradigms of chaos for it captures many features of chaotic systems. It includes a simple cube part and three simple ordinary differential equations that depend on three positive real parameters. The dynamic equations of the system can be written as following:

$$\begin{aligned} \dot{y}_1 &= y_2, \\ \dot{y}_2 &= y_3, \\ \dot{y}_3 &= a_1 y_1 - a_2 y_2 - y_3 - y_1^3, \end{aligned} \tag{2}$$

where a_1, a_2 are real constants.

3. Control schemes

In this section, we will propose a systematic design procedure to simulate the bursting activity of HR neuron system by using the Coullet system and the control scheme can be approached via an improved adaptive track. The mechanism can be understood as partial synchronization between dynamical models. This method needs only one single controller. A single control input u_1 is added to the second equation of system (2) and the first state y_1 of the controlled Coullet chaotic model is used to simulate the dynamical properties of the HR neuron system state. Thus, the controlled Coullet chaotic system is given as following

$$\begin{aligned} \dot{y}_1 &= y_2, \\ \dot{y}_2 &= y_3 + u_1, \\ \dot{y}_3 &= a_1 y_1 - a_2 y_2 - y_3 - y_1^3, \end{aligned} \tag{3}$$

The error is denoted as

$$e = x - y_1, \tag{4}$$

where x is one of the observed states of system (1). The error function $e = x - y_1$ will be stabilized to certain value when the output variables x and y_1 is close to each other, which indicates a kind of partial synchronization. To realize the partial synchronization between the two systems is now transformed to how to choose a control law u_1 and make e generally converge to zero with time increasing.

To realize partial synchronization, Lyapunov function is chosen as following:

$$V = \alpha e^2 + (\dot{e} + \beta e)^2, \tag{5}$$

where α and β are positive gain coefficients, the over dot denotes the differential variable e of time. The differential coefficient of the above Lyapunov function as shown in Eq.(5) of time is approached by

$$\dot{V} = 2\alpha e\dot{e} + 2(\dot{e} + \beta e)(\ddot{e} + \beta\dot{e})$$
$$= -2\beta V + 2\beta V + 2\alpha e\dot{e} + 2(\dot{e} + \beta e)(\ddot{e} + \beta\dot{e})$$
$$= -2\beta V + 2\beta[\alpha e^2 + (\dot{e} + \beta e)^2] + 2\alpha e\dot{e} + 2(\dot{e} + \beta e)(\ddot{e} + \beta\dot{e})$$
$$= -2\beta V + 2\alpha e(\dot{e} + \beta e) + 2\beta(\dot{e} + \beta e)^2 + 2(\dot{e} + \beta e)(\ddot{e} + \beta\dot{e})$$
$$= -2\beta V + 2(\ddot{e} + 2\beta\dot{e} + \alpha e + \beta^2 e)(\dot{e} + \beta e). \tag{6}$$

If the following condition can be achieved

$$2(\ddot{e} + 2\beta\dot{e} + \alpha e + \beta^2 e)(\dot{e} + \beta e) = 0. \tag{7}$$

Then

$$\frac{dV}{dt} = -2\beta V < 0. \tag{8}$$

According to Lyapunov stability theory, the errors of corresponding variables will be stabilized to a certain threshold. As a result, the observed state x of system (1) and the salved state y_1 of the system (2) with a controller will reach synchronization completely. In the following, three cases will be considered and some numerical simulation results will be presented to demonstrate the effectiveness of the proposed scheme. In all of following numerical simulations, the fourth-order Runge–Kutta algorithm is used for calculating the nonlinear equations with time step h = 0.001. The system parameters are chosen as a = 3.0, b = 1.0, c = 1.0, d = 5.0, r = 0.006, S = 4.0, k = 1.6, I_{ext} = 3.0, a_1 = 5.5 and a_2 = 3.5.The initial conditions of the HR system and the Coullet system are set to be (0.1, 0.9, 0.8) and (0.1, 0.3, 0.2), respectively.

Case 1 Simulating the bursting activity of x_1 using y_1.

In this case, the first state y_1 of the controlled Coullet chaotic system is used to simulate the dynamical properties of the chaotic bursting behavior state x_1. Thus, the error of corresponding variable is denoted by $e_1 = x_1 - y_1$. According to the formulas (7), the corresponding controller u_1 can be deduced as

$$u_1 = (2ax_1 - 3bx_1^2)(ax_1^2 - bx_1^3 + x_2 - x_3 + I_{ext}) + (c - dx_1^2 - x_2)$$
$$-[r(S(x_1 + k) - x_3)] - y_3 + 2\beta(ax_1^2 - bx_1^3 + x_2 - x_3 + I_{ext} - y_2) + (\alpha + \beta^2)(x_1 - y_1). \tag{9}$$

The evolutions of the outputs of x_1 and y_1, as well as the corresponding error variable e_1, are calculated under fixed constants ($\alpha = \beta$ =0.2) to demonstrate the effectiveness of the proposed partial synchronization scheme. The results are shown in Fig. 1 and Fig. 2.

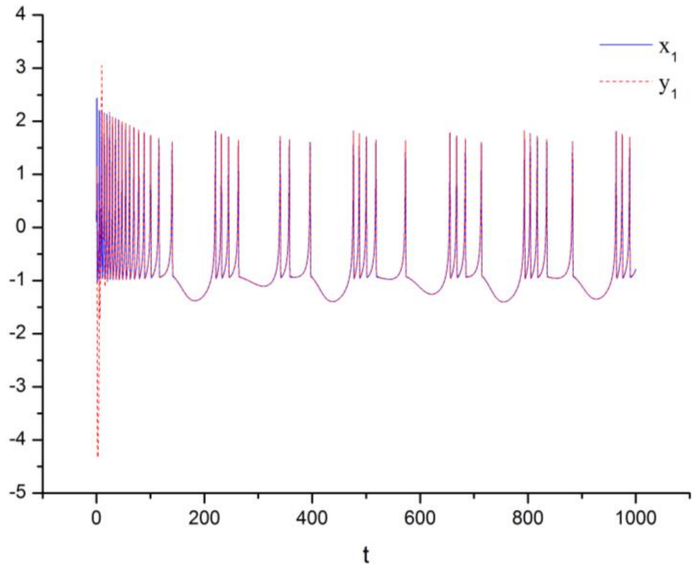

Fig.1. The evolution of x_1 and y_1 at fixed constants ($\alpha = \beta$ =0.2).

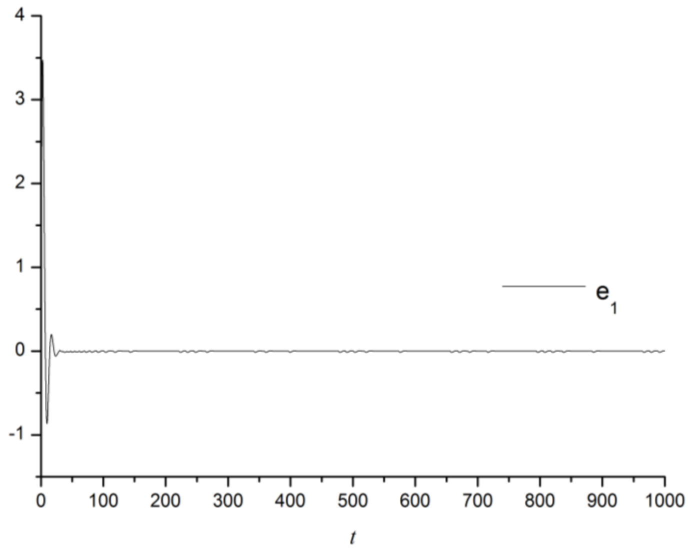

Fig.2. Synchronization error of e_1.

Clearly, the two positive gain coefficients α and β are involved, which will affect the rate of synchronization. To detect the degree of synchronization, the summation for the series of error variables are employed, which can be described as (10).

$$\varphi(e_1) = \sum_{i=1}^{n} e_1^2(i)\,. \tag{10}$$

The distribution of the summation $\varphi(e_1)$ in the two-gain coefficients phase space (α , β) can be obtained by numerical method (see Fig. 3). Fig. 3 gives a visualized and vivid explanation for the effects of gain coefficients on the systems. It indicates that the larger arithmetic product of the two gain coefficients is used and the less time to get synchronization is required. Therefore, the arithmetic product of the two gain coefficients dominates the process and speed of synchronization of the two systems. Complete synchronization can be realized with short transient period when larger gain coefficients are used (see Fig. 4 and Fig.5).

Case 2 Simulating the bursting activity of x_2 using y_1.

In this case, the state x_2 of the HR neuron system will be considered as the drive signal, and the first state y_1 of the controlled Coullet chaotic model is used as the salve signal to simulate the chaotic bursting behavior. Thus, the error of is denoted by $e_2 = x_2 - y_1$. Accordingly, the corresponding controller u_2 can be derived as

$$\begin{aligned} u_2 = & -2dx_1(ax_1^2 - bx_1^3 + x_2 - x_3 + I_{ext}) - (c - dx_1^2 - x_2) \\ & -y_3 + 2\beta(c - dx_1^2 - x_2 - y_2) + (\alpha + \beta^2)(x_2 - y_1)\,. \end{aligned} \tag{11}$$

The evolutions of x_2 and y_1, as well as the corresponding error variable e_2, are calculated under fixed constants ($\alpha = \beta$ =0.2) to demonstrate the effectiveness of the proposed partial synchronization. The results are shown in Fig. 6 and Fig. 7.

The distribution of the summation $\varphi(e_2)$ in the two-gain coefficients phase space (α , β) can be obtained by numerical method (see Fig. 8). Fig. 8 gives a visualized and vivid explanation for the effects of gain coefficients in the systems. It indicates that the larger arithmetic product of the two gain coefficients is used and the less time to get synchronization is approached. Therefore, the arithmetic product of the two gain coefficients dominates the process and speed of synchronization of the two systems. Complete synchronization can be realized with short transient period when larger gain coefficients are used (see Fig. 9 and Fig. 10).

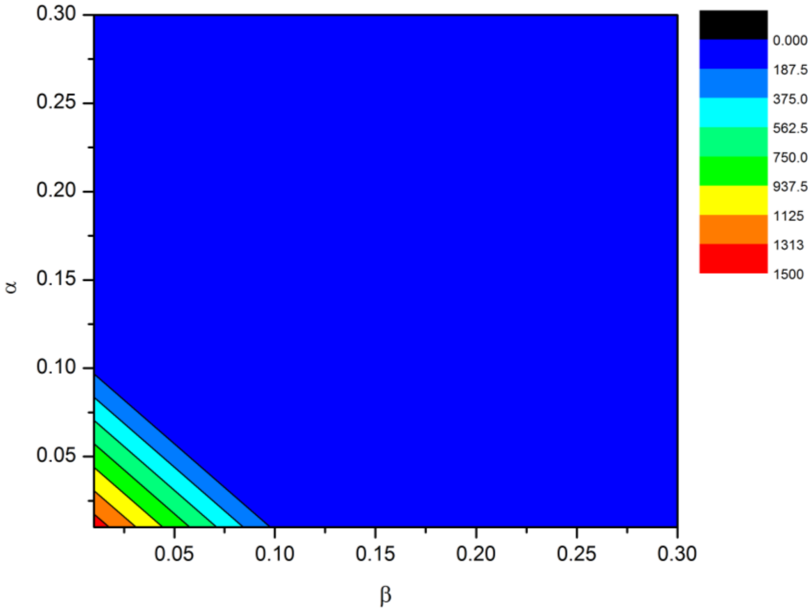

Fig.3. The distribution of $\varphi(e_1)$ as described in Eq. (10) in the two-gain coefficients phase space (α, β), the summation of errors function is calculated from t=100 to 500 time units (t = i*h).

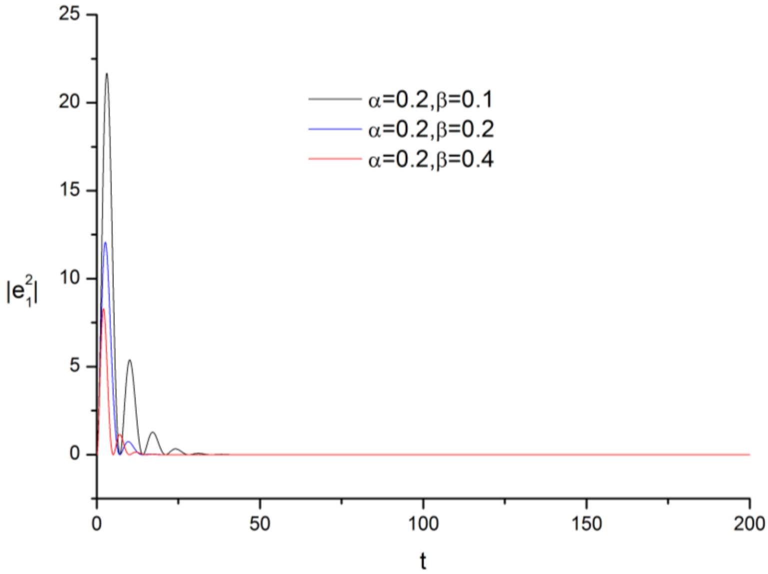

Fig.4. The evolutions of $|e_1^2|$ when α = 0.2 and β =0.1, 0.2, 0.4，respectively.

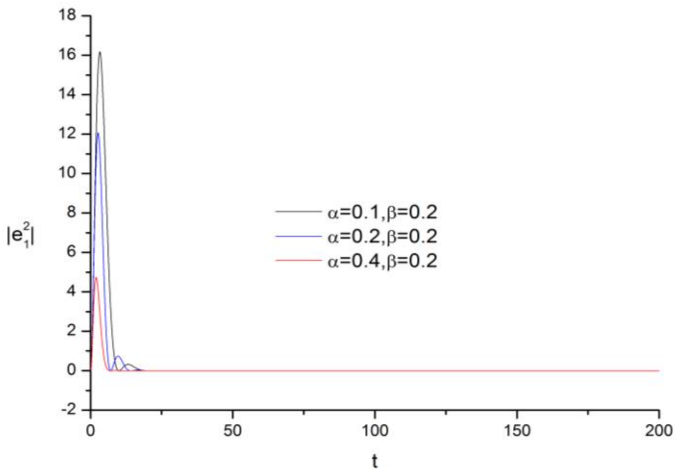

Fig.5. The evolutions of $|e_1^2|$ when β = 0.2 and α =0.1, 0.2, 0.4，respectively.

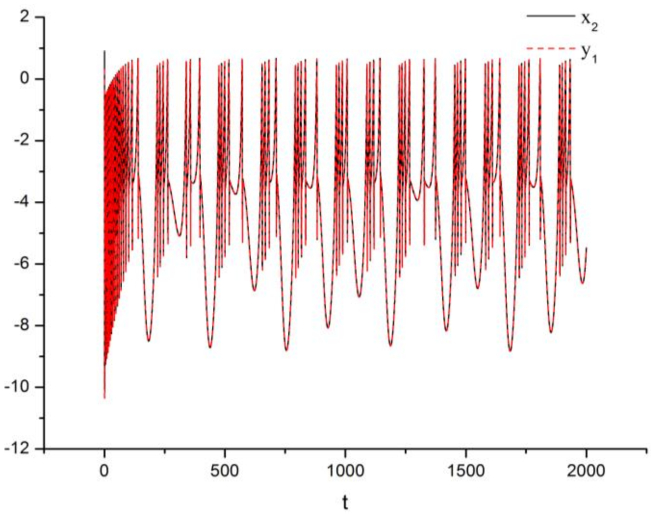

Fig.6. The evolution of x_2 and y_1 at fixed constants ($\alpha = \beta$ =0.2).

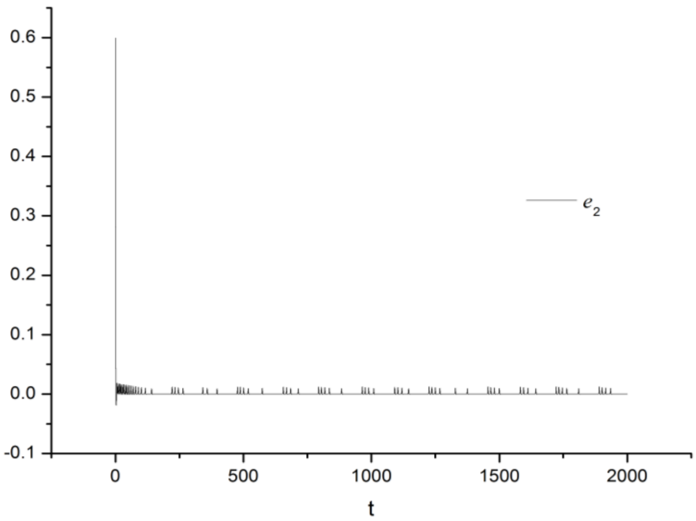

Fig.7. Synchronization error of e_2.

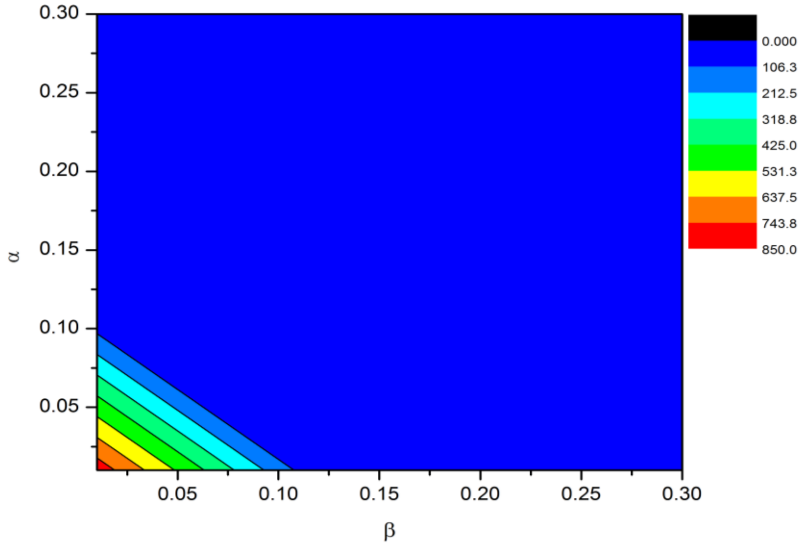

Fig.8. The distribution of $\varphi(e_2)$ in the two-gain coefficients phase space (α , β), the summation of errors function is calculated from t=100 to 500 time units (t = i*h).

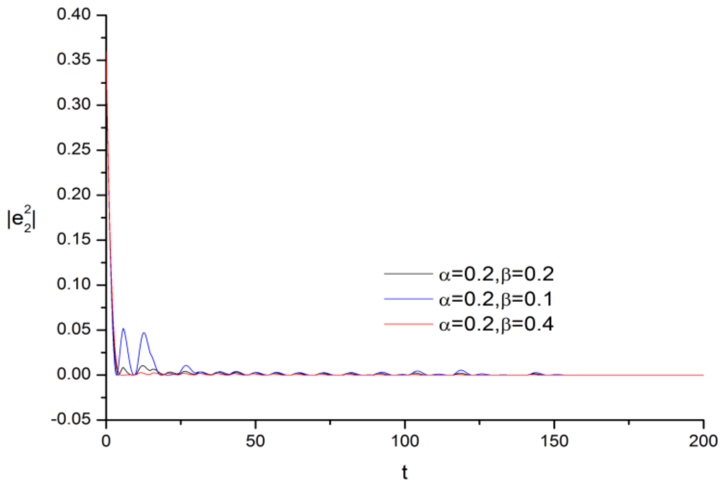

Fig.9. The evolutions of $\left|e_2^2\right|$ when α = 0.2 and β =0.1, 0.2, 0.4，respectively.

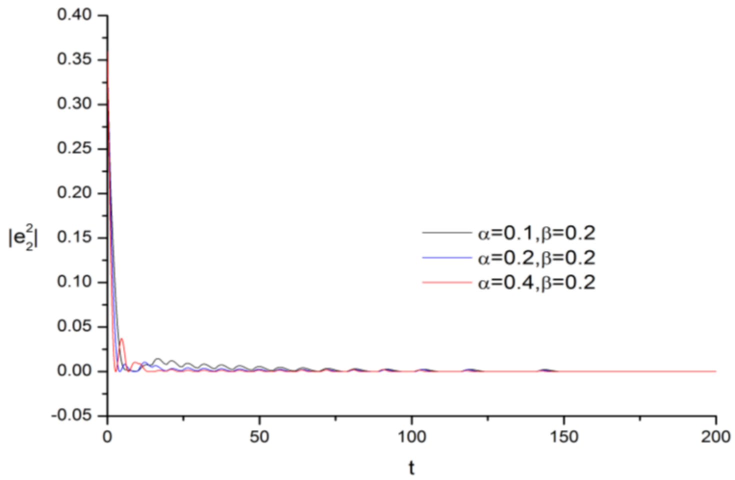

Fig.10. The evolutions of $\left|e_2^2\right|$ when β = 0.2 and α =0.1, 0.2, 0.4，respectively.

Case 3 Simulating the bursting activity of x_3 using y_1.

In this case, y_1 is used to simulate the state x_3 of the HR neuron system. The error is denoted by $e_3 = x_3 - y_1$. According to the conditions in (7), the controller can be written as

$$\begin{aligned} u_3 &= rS(ax_1^2 - bx_1^3 + x_2 - x_3 + I_{ext}) - r^2[S(x_1+k) - x_3] \\ &\quad - y_3 + 2\beta[r(S(x_1+k) - x_3) - y_2] + (\alpha+\beta^2)(x_3 - y_1). \end{aligned} \tag{12}$$

The evolutions of the outputs of x_3 and y_1, as well as the corresponding error variable e_3, are calculated under fixed constants ($\alpha = \beta$ =0.2) to demonstrate the effectiveness of the proposed partial synchronization scheme. The results are shown in Fig. 11 and Fig. 12.

The distribution of the summation $\varphi(e_3)$ in the two-gain coefficients phase space (α, β) can be obtained by numerical method (see Fig. 13). Fig. 13 gives a visualized and vivid explanation for the effects of gain coefficients in the systems. It indicates that larger arithmetic product of the two gain coefficients is used and less time to get synchronization is approached. Therefore, the arithmetic product of the two gain coefficients dominates the process and speed of synchronization of the two systems. Complete synchronization can be realized with short transient period when larger gain coefficients are used (see Fig.14 and Fig. 15).

4. Conclusion

An improved scheme is proposed to control the nonlinear dynamical system to generate any selectable signals by tracking the external standard signal within short transient periods. The changeable gain coefficient is introduced into the error function or Lyapunov function and the controller is adaptive with the variable of the system. The power consumption of controller is estimated according to the dimensionless model. It is found that larger power is spent when the external standard signal to be tracked is within larger amplitude and/or high angular frequency. It could be useful for designing the controller in circuit and generating useful signals from the signal generators.

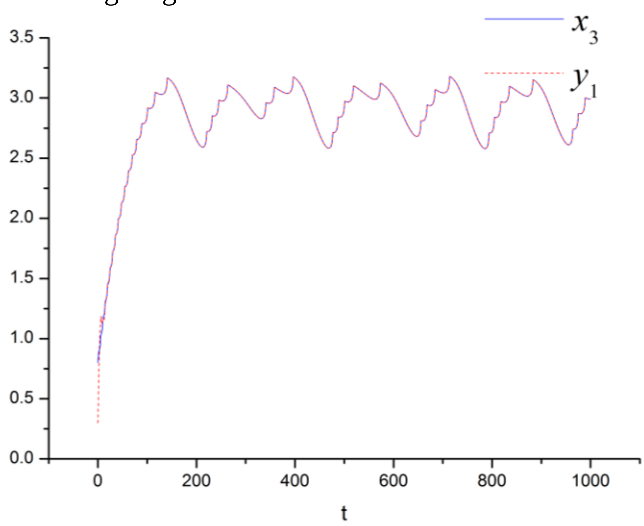

Fig.11 .The evolution of x_3 and y_1 at fixed constants ($\alpha = \beta$ =0.2).

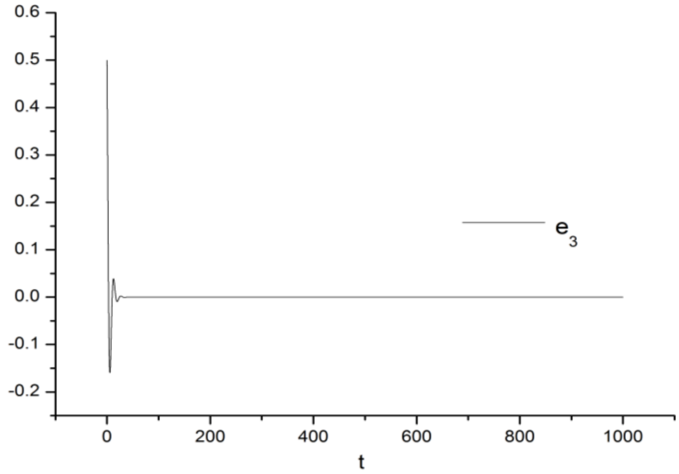

Fig.12. Synchronization error of e_3 .

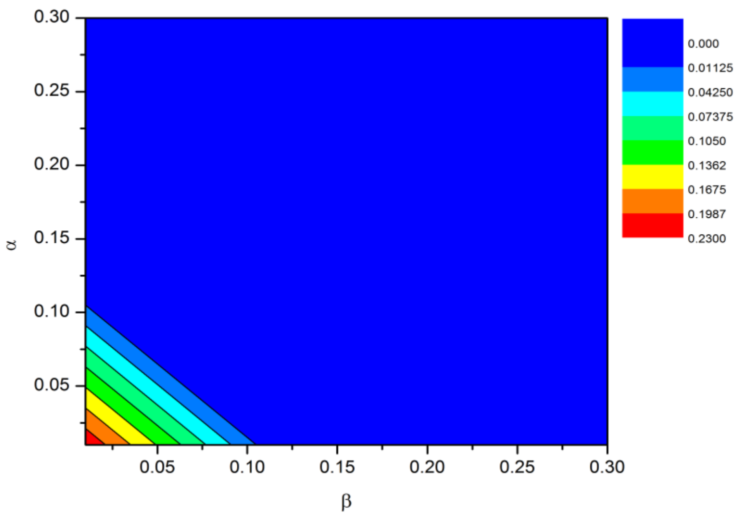

Fig.13. The distribution of $\varphi(e_3)$ in the two-gain coefficients phase space (α , β), the summation of errors function is calculated from t=100 to 500 time units (t = i*h).

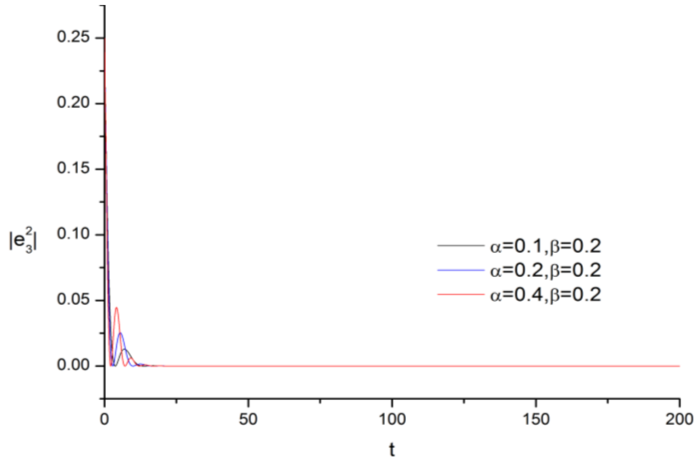

Fig.14. The evolutions of $\left|e_3^2\right|$ when $\beta = 0.2$ and α =0.1, 0.2, 0.4， respectively.

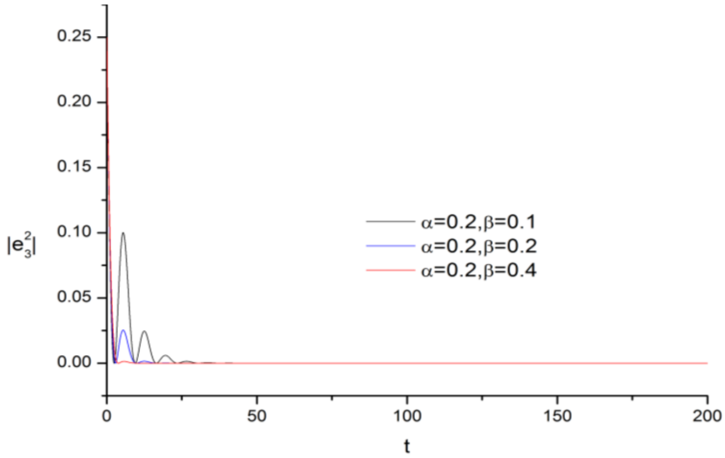

Fig.15. The evolutions of $\left|e_3^2\right|$ when $\alpha = 0.2$ and β =0.1, 0.2, 0.4， respectively.

5. Acknowledgement

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