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Stability and hybrid synchronization of a time-delay financial hyperchaotic system

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Abstract. This paper studies stability and hybrid synchronization of a time-delay financial hyperchaotic system. Based on Lyapunov stability theorem and differential inequalities, stability is obtained by intermittent linear state feedback control. Furthermore, hybrid synchronization method is firstly proposed to synchronize a financial hyperchaotic system and globally synchronization is obtained by proper hybrid controllers and Lyapunov stability theorem. The corresponding numerical simulations are performed to verify and illustrate the effectiveness and correctness of proposing methods.

Keywords: time-delay financial hyperchaotic system, intermittent linear state feedback control, hybrid synchronization, hybrid control.

1. Introduction

In nonlinear science, chaos and hyperchaos study has attracted much attention from scientists and engineers for its applications in diverse areas, such as physical systems, biological networks, secure communications and so on [1,2]. Hyperchaotic systems have more complex behaviors and abundant dynamics than chaotic system because of possessing at least two positive Lyapunov exponents. Therefore, it is extremely important to research on hyperchaotic systems nowadays. Some classical hyperchaotic systems have been proposed, such as the hyperchaotic Chen system, the hyperchaotic Lü system, etc. Due to its applications in many areas, the studies of hyperchaos have not only emphasized on proposing and analyzing new interesting hyperchaotic systems, but also studying hyperchaos control and synchronization. Chaos control is an important subject and it could control system to a predictive target, many methods which include adaptive control, linear feedback control, nonlinear feedback control, fuzzy control, time delay control have been used to achieve it. Up to now, various kinds of synchronization have been proposed to synchronized different systems or identity system with different values, like phase synchronization, lag synchronization, projective synchronization, hybrid synchronization, complete synchronization, anti-synchronization and so on[3-9].

Recently, economy is the hot topic. Basing on the global economic crisis in 2007, we firstly construct a financial hyperchaotic system in [10]. Its dynamical behaviors are more complex and more effective controls are proposed to control it. In fact, hyperchaos system has a strong sensitivity to initial value, so discrete control method is more in line with the actual situation. The intermittent control is active in work time and rest in other time(rest time) [11-13].It reduce control input and save cost. So, we main investigate intermittent control in this paper. Nowadays, most works focus on studying the same kind synchronization between drive system and response system that is the states of response system synchronized to the states of drive system by the same kind synchronization. Whether it has the same phenomenon by two or more kinds of synchronization which defined as hybrid synchronization or not? There is no doubt that it is an interesting problem. Some scholars has investigate hybrid synchronization, [14] study the alternating between complete synchronization and hybrid synchronization of hyperchaotic Lorenz system with time delay, [15] investigate hybrid synchronization of time-delay hyperchaotic 4D systems via partial variables, However, up to now, fewer scholars investigate hybrid synchronization by hybrid control which combine continuous control with discrete control. As a result, in this paper, periodically intermittent linear control is firstly proposed to stabilize the financial hyperchaotic system which we introduce. Basing on Lyapunov stability thermo and differential inequalities, stability of financial hyperchaotic system achieves. We also proposed hybrid synchronization scheme which is firstly applied to financial hyperchaotic system.

This paper is organized as follows. In section 2, describing the time-delay financial hyperchaotic system model. In section 3, presenting stability scheme of a time-delay financial hyperchaotic system. In section 4, hybrid synchronization method of a time-delay financial hyperchaotic system is shown. In section 5, corresponding numerical simulation are given. Finally, making a conclusion in section 6.

2. The time-delay financial hyperchaotic system model

Yu, Cai, etc proposed a financial hyperchaotic system without time-delay in [10], however, time-delay phenomenon often happed in actual situation. Therefore, in this paper, we add time-delay τ to the system that has been proposed in [10], we got a novel time-delay system as follows:

$$\begin{cases} \dot{x} = z + (y-a)x + w(t-\tau) \\ \dot{y} = 1 - by - x^2 \\ \dot{z} = -x - cz \\ \dot{w} = -dxy - kw(t-\tau) \end{cases} \tag{1}$$

where the interest rate *x*, the investment demand *y*, the price exponent *z*, the average profit margin *w*, they are the state variables, τ is time delay, *a*, *b*, *c*, *d*, *k* are the positive parameters of the system(1). When parameters a=0.9, b=0.2, c=1.5, d=0.2 and k=0.17, the four Lyapunov exponents of the system (1) calculated with Wolf algorithm are L_1=0.034432, L_2=0.018041, L_3=0 and L_4=-1.1499. There are three unstable equilibrium points $P_1\left(0,\frac{1}{b},0,0\right), P_{2,3}\left(\pm\theta,\frac{k+ack}{c(k-d)},\mp\frac{\theta}{c},\frac{d\theta(1+ac)}{c(d-k)}\right)$, where $\theta=\sqrt{\frac{kb+abck}{c(d-k)}+1}$. According to the hyperchaotic parameter values given above, the equilibrium points are calculated as: P_1 (0, 5, 0, 0), P_2 (1.66, -8.87, -1.11, 17.4) and P_3 (-1.66, -8.87, 1.11, -17.4). Figure1 shows the Lyapunov exponents of system (1). Figure 2(a)-(d) shows the 3-dimensional phase portraits of financial hyperchaotic system (1).

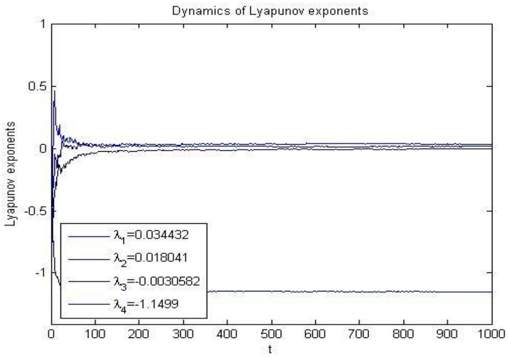
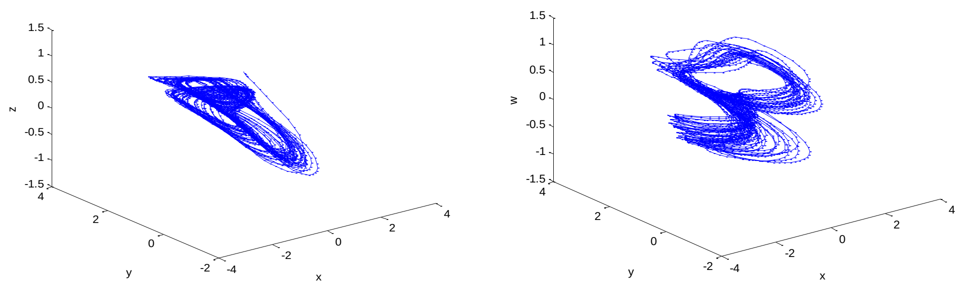
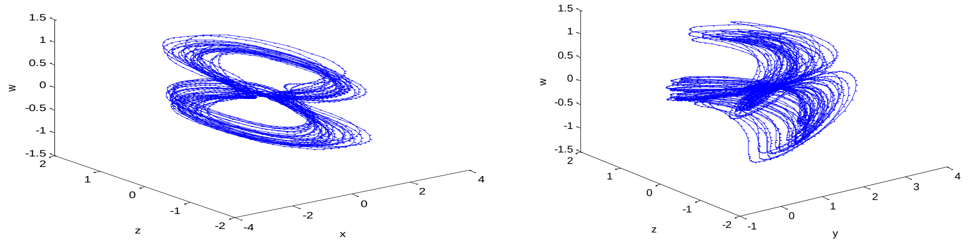


Fig.1: Lyapunov exponents spectrum of system (1) with τ=0.



(a) 3D view in the *x-y-z* space (b) 3D view in the *x-y-w* space



(c) 3D view in the *x-z-w* space (d) 3D view in the *y-z-w* space

Fig. 2: Phase portraits of system (1) with τ=0.

3. Stability schemes of a time-delay financial hyperchaotic system via intermittent linear state feedback control

In this section, we will give the control scheme of a time-delay financial system in detail. At first, we will give a stability scheme of a class of time-delay hyperchaotic system by intermittent linear state feedback control. Secondly, this method will be applied to a time-delay financial hyperchaotic system. Finally, stability is obtained by Lyapunov stability and proper controllers.

3.1. Stability scheme of a time-delay financial hyperchaotic system via intermittent control

Consider a class of certain hyperchaotic system with time delay described by:

$$\dot{x}(t) = Ax(t) + f(x(t)) + g(x(t-\tau)) \tag{2}$$

The controlled system is designed as

$$\dot{x}(t) = Ax(t) + f(x(t)) + g(x(t-\tau)) + u(t) \tag{3}$$

where $x \in R^n$ is the state vector of the systems (11) and (12), $A \in R^{n\times n}$ is the constant matrix, $f: R^{n\times n} \to R^{n\times n}$ and $g: R^{n\times n} \to R^{n\times n}$ are continuous nonlinear vector functions satisfying $f(0)=0$, $g(0)=0$, τ is the time delay and $u(t)$ is the intermittent linear state feedback controller that is defined as the following:

$$u(t) = \begin{cases} Kx(t), nT \le t \le nT+\delta, \\ 0, nT+\delta < t \le (n+1)T, \end{cases} \tag{4}$$

where $K \in R^{n\times n}$ is a constant control gain, $T>0$ is the control period, and $\delta>0$ is called the control width. Our goal is to design suitable δ, T, K such that the system (3) stable in this section.

Assumption 1. We assume $f(x)$, $g(x(t-\tau))$ are bounded functions. That is exist constants matrices L and P, for any x, such that

$$\|f(x)\|^2 \le L\|x\|^2 = x^T Lx, \|g(x(t-\tau))\|^2 \le P\|x(t-\tau)\|^2 = x^T(t-\tau)Px(t-\tau)$$

where $L = \mathrm{diag}\{l_1, l_2, ..., l_n\} \ge 0$, with $P = \mathrm{diag}\{p_1, p_2, \ldots, p_n\} \ge 0$.

Assumption 2. We assume x is bounded variables. That is exist constants D, for any x, such that $|x| \le D$.

Lemma 1[11]. For any vectors $x, y \in R^n$ and a positive definite matrix $Q \in R^{n\times n}$, the following matrix inequality holds: $2x^T y \le x^T Qx + y^T Q^{-1} y$.

Lemma 2[11]. $V(t) \le M\exp(-\frac{\rho}{\omega}t)$, $M = \|V(0)\|_\tau e^{(v_1+v_2)\omega} e^{\rho}$, $t>0$. If there exist positive constants u_1, u_2, v_1, v_2 such that the following condition holds:

$(a)\ \rho = r(\delta-\tau) - (v_1+v_2)(\omega-\delta) > 0$,

$$(b)\begin{cases} \dot{V}(t) \le -u_1 V(t) + u_2 V(t-\tau), n\omega \le t < n\omega+\delta \\ \dot{V}(t) \le v_1 V(t) + v_2 V(t-\tau), n\omega+\delta \le t < (n+1)\omega \end{cases},$$

where r is the unique positive solution to $-r=-u_1+u_2 e^{r\tau}$, τ is the time delay, δ is the control width, ω is the control period.

Theorem 1. (stabilization criterion) The system (3) is stabilized, if there exist positive constants $\alpha_1>\alpha_2$, c_1, c_2, c_3 and positive matrix L which defined in Assumption 1, such that the following conditions hold:

$(a)\ A^T+A+2KI+\alpha_1^{-1}I+\alpha_2^{-1}I+\alpha_1 L+c_1 I\le 0$,

$(b)\ A^T+A+2KI+\alpha_1^{-1}I+\alpha_1 L-c_3 I\le 0$,

$(c)\ \eta_1=r(\delta-\tau)-\left(c_3+c_2\right)\left(T-\delta\right)>0$,

where r is the unique positive solution to $-r=-c_1+c_2e^{r\tau}$, I is the identity matrix.

Proof. Let us choose the following Lyapunov function:

$$V(t)=\left\|x(t)\right\|^2=x^T(t)x(t) \tag{5}$$

and calculate the derivation $\dot{V}(t)$ with respect to time to along the trajectories of the controlled system (3).

For $nT\le t<nT+\delta$, in addition to Assumption1, Lemma1 and condition (a) in the Theorem 1 are used to get the following estimate:

$$\begin{aligned}
\dot{V}(t)&=\dot{x}^T(t)x(t)+x(t)\dot{x}^T(t)\\
&=\left(Ax(t)+f(x(t))+g\left(x(t-\tau)\right)+Kx(t)\right)^T x(t)+x^T(t)(Ax(t)+f(x(t))+g\left(x(t-\tau)\right)\\
&\quad+Kx(t))\\
&=x^T(t)\left(A^T+A+K^T\right)x(t)+2f^T(x(t))x(t)+2g^T\left(x(t-\tau)\right)x(t)\\
&\le x(t)^T\left(A^T+A+K^T+K\right)x(t)+g^T\left(x(t-\tau)\right)Qg\left(x(t-\tau)\right)+x^T(t)Q^{-1}x(t)\\
&\quad+f^T(x)\Psi f(x(t))+x^T(t)\Psi^{-1}x(t)\\
&\le x(t)^T\left(A^T+A+K+K^T+Q^{-1}+\Psi^{-1}\right)x(t)+x^T(t)L\Psi x(t)+PQx^T(t-\tau)x(t-\tau)\\
&\le x(t)^T\left(A^T+A+K+K^T+\alpha_2^{-1}+\alpha_1^{-1}+\alpha_1 L+c_1 I\right)x(t)+c_2V(t-\tau)-c_1V(t)\\
&\le -c_1V(t)+c_2V(t-\tau).
\end{aligned}$$

Notice that $c_1>0, c_2=PQ$, $\Psi=\alpha_1 I, Q=\alpha_2 I, \alpha_1,\alpha_2>0.$

Namely, we have

$\dot{V}(t)\le -c_1V(t)+c_2V(t-\tau)$, for $nT\le t<nT+\delta$.

For $nT+\delta\le t<(n+1)T$, in addition to Assumption 2 and condition (b) in the Theorem 1 are used to get the following estimate:

$$\begin{aligned}
\dot{V}(t)&=\dot{x}^T(t)x(t)+x(t)\dot{x}^T(t)\\
&=\left(Ax(t)+f(x(t))+g\left(x(t-\tau)\right)\right)^T x(t)+x^T(t)\left(Ax(t)+f(x(t))+g\left(x(t-\tau)\right)\right)\\
&=x^T(t)\left(A^T+A\right)x(t)+2f^T(x(t))x(t)+2g^T\left(x(t-\tau)\right)x(t)\\
&\le x(t)^T\left(A^T+A\right)x(t)+f^T(x(t))\Psi f(x(t))+x^T(t)\Psi^{-1}x(t)\\
&\quad+g^T\left(x(t-\tau)\right)Qg\left(x(t-\tau)\right)+x^T(t)Q^{-1}x(t)\\
&\le x^T(t)\left(A^T+A+\alpha_1^{-1}+\alpha_2^{-1}+\alpha_1 L-c_3 I\right)x(t)+c_3V(t)+c_2x^T(t-\tau)x(t-\tau)\\
&\le c_3V(t)+c_2V(t-\tau).
\end{aligned}$$

Notice that $PQ=c_2$, $\Psi=\alpha_1 I, Q=\alpha_2 I, \alpha_1,\alpha_2>0.$

Namely, we have

$\dot{V}(t)\le c_3V(t)+c_2V(t-\tau)$, for $nT+\delta\le t<(n+1)T$.

By Lemma 2, we have

$$V(t)\le M_1 e^{-\frac{\eta_1}{T}t}, \text{ for } t>0, \tag{6}$$

where $M_1=\left\|V(0)\right\|_\tau e^{(c_2+c_3)T}e^{\eta_1}$, $\eta_1=r(\delta-\tau)-(c_2+c_3)(T-\delta)$.

By Eq. (5) and Eq.(6), we have

$$\left\|x(t)\right\|^2 \le M_1 e^{-\frac{\eta_1}{T}t}.$$

Therefore, we obtained the following:

$$\left\|x(t)\right\| \le \sqrt{M_1 e^{-\frac{\eta_1}{T}t}}, \text{ for } t>0.$$

This means that the control system is get stabilization. The proof is complete.

3.2. Periodically intermittent linear state feedback control with time delay stabilized P_1(0,1/b,0,0)

At first, we transform system (1) in P_1, let $x_1=x$, $x_2=y-1/b$, $x_3=z$, $x_4=w$. So, the Eq. (1) changes into Eq. (7) and unstable equilibrium point P_1 changes into $P_1'(0,0,0,0)$.

$$\begin{cases} \dot{x}_1 = x_3 + x_1x_2 + \left(\frac{1}{b} - a\right)x_1 + x_4(t-\tau) \\ \dot{x}_2 = -bx_2 - x_1^{\ 2} \\ \dot{x}_3 = -x_1 - cx_3 \\ \dot{x}_4 = -dx_1x_2 - \frac{d}{b}x_1 - kx_4(t-\tau) \end{cases} \tag{7}$$

The controlled system is designed as

$$\begin{cases} \dot{x}_1 = x_3 + \left(\frac{1}{b} - a\right)x_1 + x_4(t-\tau) + x_1x_2 + u_1 \\ \dot{x}_2 = -bx_2 - x_1^{\ 2} + u_2 \\ \dot{x}_3 = -x_1 - cx_3 + u_3 \\ \dot{x}_4 = -\frac{d}{b}x_1 - kx_4(t-\tau) - dx_1x_2 + u_4 \end{cases} \tag{8}$$

The controllers are designed as

$$u_i = \begin{cases} k_i x_i(t), nT < t \le nT + \delta \\ 0, nT + \delta \le t < (n+1)T \end{cases}, i = 1,2,3,4.$$

In system (8), the parameters a=0.9, b=0.2, c=1.5, d=0.2 and k=0.17, therefore, linear matrix is calculated as:

$$A = \begin{bmatrix} 4.1 & 0 & -1 & -1 \\ 0 & -0.2 & 0 & 0 \\ 1 & 0 & -1.5 & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix},$$

nonlinear function vector $f(x) = \left(x_1x_2, -x_1^{\ 2}, 0, -0.2x_1x_2\right)^T$, and the time delay function $g(x(t-\tau)) = \left(x_4(t-\tau), 0, 0, -0.17x_4(t-\tau)\right)^T$, We choose positive matrices L=P=0.5I, α_1=α_2=1, c_1=1, c_2=0.5, c_3=1, then system (8) satisfied all conditions in Theorem1. It has proved that system (8) stabilized in equilibrium point $P_1'(0,0,0,0)$. Therefore, the system (1) stabilized in P_0(0, 1/b, 0,0).

4. Hybrid synchronization scheme of a time-delay financial hyperchaotic system via hybrid control

In this section, we will discuss hybrid synchronization of a time-delay financial system by hybrid control. The definitions of hybrid synchronization and hybrid control will be given. Base on proper hybrid controllers, hybrid synchronization will be achieve between drive system and response system.

Definition 1. It is defined as hybrid synchronization, if $e_i(t)=y_i(t)-x_i(t)$, $e_j(t)=y_j(t)+x_j(t)$, $i\neq j$, $i, j=1, 2,\dots, n$, where $y_i(t)$, $x_i(t)$ are the state variables of response system and drive system respectively.

Definition 2. $U(t)$ is called hybrid controller, if there exist different kinds of controllers, such that $U(t)=U_i(t)+U_j(t)$, $i, j=1, 2,\dots, n$, $i\neq j$.

Rewritten system (1) as:

$$\begin{cases}\dot{x}_1 = x_3+(x_2-a)x_1+x_4(t-\tau)\\ \dot{x}_2=1-bx_2-x_1^2\\ \dot{x}_3=-x_1-cx_3\\ \dot{x}_4=-dx_1x_2-kx_4(t-\tau)\end{cases} \tag{9}$$

Choosing system (9) as the drive system and response system is designed as:

$$\begin{cases}\dot{y}_1 = y_3+(y_2-a)y_1+y_4(t-\tau)+u_1\\ \dot{y}_2=1-by_2-y_1^2+u_2\\ \dot{y}_3=-y_1-cy_3+u_3\\ \dot{y}_4=-dy_2y_1-ky_4(t-\tau)+u_4\end{cases} \tag{10}$$

where u_1, u_2, u_3, u_4 are controllers to be constructed.

The goal of this section is to choose proper controllers, so that the state variables y_1, y_2, y_4 in response system are complete synchronized to x_1, x_2, x_4 in drive system respectively, while the state variable y_3 in response system is anti-synchronized to x_3 in drive system. For this target, let error vector $e_1(t)=y_1(t)-x_1(t)$, $e_2(t)=y_2(t)-x_2(t)$, $e_3(t)=y_3(t)+x_3(t)$, $e_4(t)=y_4(t)-x_4(t)$, $e_4(t-\tau)=y_4(t-\tau)-x_4(t-\tau)$, from systems (9) and (10), we get error system:

$$\begin{cases}\dot{e}_1=-ae_1+e_4(t-\tau)+y_3+y_1y_2-x_3-x_1x_2+u_1\\ \dot{e}_2=-be_2-y_1^2+x_1^2+u_2\\ \dot{e}_3=-ce_3-y_1-x_1+u_3\\ \dot{e}_4=-ke_4(t-\tau)-dy_1y_2+dx_1x_2+u_4\end{cases} \tag{11}$$

The purpose is to propose hybrid input controllers so that the state errors in (11) satisfy:

$$\lim_{t\to\infty} e_i(t)=0.\ i=1, 2, 3, 4.$$

With this in mind, the proper hybrid controllers will be designed as follows:

$$u_i=u_{i1}+u_{i2},\ (i=1, 2, 3, 4), \tag{12}$$

where

$$u_{i1}(t)=\begin{cases}-b_ie_i,\ nT\le t\le nT+\delta,\\ 0,\ nT+\delta<t\le(n+1)T,\end{cases}\ i=1,2,3,4 \tag{13}$$

and

$$\begin{cases}u_{12}=-y_3+x_3-y_1y_2+x_1x_2\\ u_{22}=y_1{}^2-x_1{}^2\\ u_{32}=y_1+x_1\\ u_{42}=dy_1y_2+dx_1x_2\end{cases} \tag{14}$$

where b_i (i=1, 2, 3, 4) are positive intermittent linear state feedback control gain.

Theorem 2. Hybrid synchronization can be achieved between systems (9) and (10) with above hybrid control law(12) and proper positive constants m_1, m_2, m_3, m_4, d_1, d_2, d_3,satisfied the following conditions:

(a) $\min\{a+b_1+\frac{1}{2}, b+b_2, c+b_3,\ b_4+\frac{k}{2}, \frac{1}{2}-\frac{k}{2}\}>0$,

(b) $\min\{a+m_1-\frac{1}{2}, b+m_2, c+m_3, \frac{k}{2}+m_4\}>0$,

(c) $\rho_1 = r(\delta-\tau)-(d_2+d_3)(T-\delta)>0$,

where r is the unique positive solution to $-r=-d_1+d_2e^{r\tau}$, τ is the time delay, δ is the control width, T is the control period.

Proof. Constructing a positive definite Lyapunov function as following:

$$V=\frac{1}{2}\left(e_1^2+e_2^2+e_3^2+e_4^2\right) \tag{15}$$

Calculating the time derivative of the Lyapunov function (15) along the trajectory of system (11) arrives at:

$$\begin{aligned}\dot{V} &= e_1\dot{e}_1+e_2\dot{e}_2+e_3\dot{e}_3+e_4\dot{e}_4 \\ &= e_1(-ae_1+e_4+y_3+y_1y_2-x_3-x_1x_2+u_1)+e_2(-be_2-y_1^2+x_1^2+u_2) \\ &\quad +e_3(-ce_3-y_1-x_1+u_3)+e_4(-ke_4-dy_1y_2+dx_1x_2+u_4)\end{aligned} \tag{16}$$

For $nT \le t < nT+\delta$, Adding Eq.(12) to Eq.(16), by Lemma 1 and condition (a) in Theorem 2,we obtain

$$\begin{aligned}\dot{V} &= -(a+b_1)e_1^2-(b+b_2)e_2^2-(c+b_3)e_3^2-b_4e_4^2-ke_4e_4(t-\tau)+e_1e_4(t-\tau) \\ &\le -(a+b_1+\frac{1}{2})e_1^2-(b+b_2)e_2^2-(c+b_3)e_3^2-(b_4+\frac{k}{2})e_4^2+(\frac{1}{2}-\frac{k}{2})e_4^2(t-\tau) \\ &\le -d_1V(t)+d_2V(t-\tau)\end{aligned}$$

where $d_1=\max\{a+b_1+\frac{1}{2}, b+b_2, c+b_3, b_4+\frac{k}{2}\}>0,\ d_2=\frac{1}{2}-\frac{k}{2}>0$

For $nT+\delta \le t<(n+1)T$, by Assumption 3 and condition (b) in Theorem 2, we get

$$\begin{aligned}\dot{V} &= -ae_1^2-be_2^2-ce_3^2-ke_4e_4(t-\tau)+e_1e_4(t-\tau) \\ &\le -(a+\frac{1}{2})e_1^2-be_2^2-ce_3^2-\frac{k}{2}e_4^2+(\frac{1}{2}-\frac{k}{2})e_4^2(t-\tau) \\ &\le -(a+m_1-\frac{1}{2})e_1^2-(b+m_2)e_2^2-(c+m_3)e_3^2-(\frac{k}{2}+m_4)e_4^2+(\frac{1}{2}-\frac{k}{2})e_4^2(t-\tau) \\ &\quad +m_1e_1^2+m_2e_2^2+m_3e_3^2+m_4e_4^2 \\ &\le d_3V(t)+d_2V(t-\tau)\end{aligned}$$

where $d_3=\max\{m_1,m_2,m_3,m_4\}>0,\ d_2=\frac{1}{2}-\frac{k}{2}>0$

Therefore,

$$\dot{V} \le \begin{cases}-d_1V(t)+d_2V(t-\tau),\ nT\le t<nT+\delta \\ d_3V(t)+d_2V(t-\tau),\ nT+\delta\le t<(n+1)T\end{cases}$$

By Lemma 2, we get

$$V\le M_2\exp\{-\frac{\rho_1 t}{T}\},\ M_2=\|V(0)\|_\tau \exp\{(d_2+d_3)T+\rho_1\},$$

Therefore, by Eq.(16), we get

$$\|e_i(t)\| \le \sqrt{M_2 e_i\{-\frac{\rho_1 t}{T}\}},\ i=1,2,3,4.$$

that is ,if $t\to\infty$, then $e_i\to 0, i=1,2,3,4.$

Therefore, if appropriate intermittent linear state feedback control gains and d_1, d_2, d_3 are selected appropriately, it can be obtained that the error system (11) will be convergence to zero and globally asymptotically stable, while the coexistence of anti-synchronization and complete synchronization of systems (9) and (10) with the hybrid control (12) can be achieved.
This completes the proof.

5. Numerical simulation

To verify stability scheme of financial hyperchaotic system via intermittent linear state feedback control, we choose system (8) as the controlled system. The parameters are chosen as a=0.9, b=0.2, c=1.5, d=0.2, k =0.17. The initial values of the controlled system ($x_1(0)$, $x_2(0)$, $x_3(0)$, $x_4(0)$)=(1,2,3,4), the time delay τ=0.5 and control gain K is designed as 6. Assuming the controllers are switched on the T=1s and δ=0.8s. Using MATLAB, we get the time response of states for the controlled system (8) in Fig.3. The state variables of controlled system (8) converge to zero when t>2s, therefore, the unstable equilibrium point P_0 get stabilization.

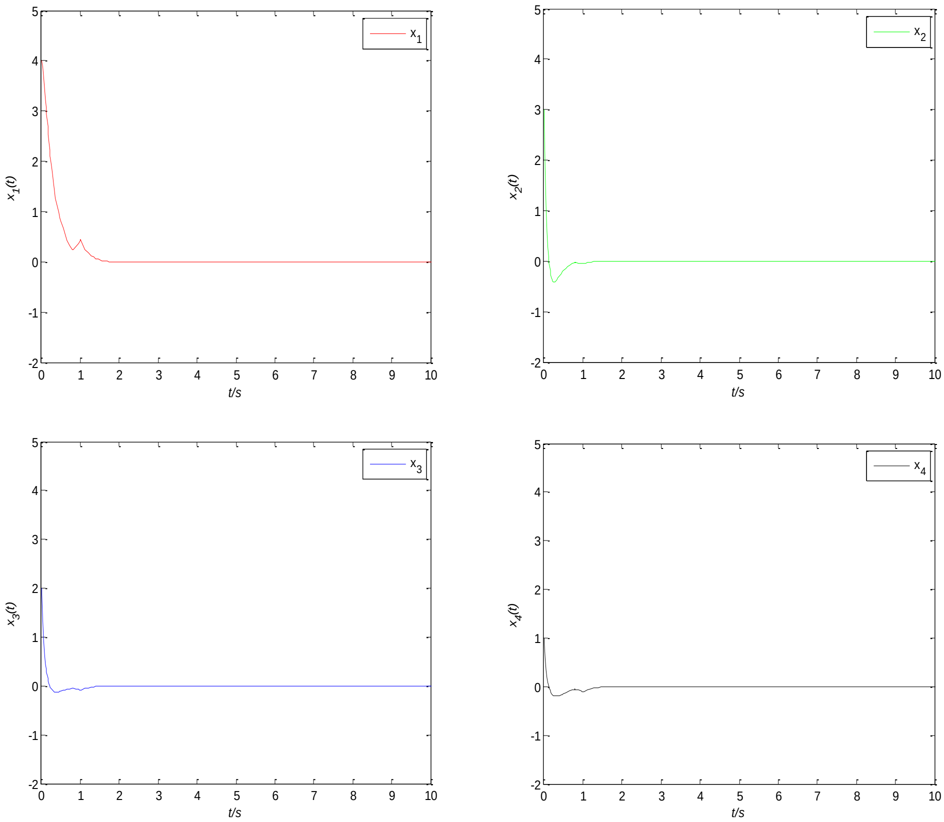


Fig.3: Time response of $x(t)=(x_1(t),x_2(t),x_3(t),x_4(t))^T$ of the controlled system (8) with intermittent linear state feedback control.

To verify hybrid synchronization scheme of financial hyperchaotic system, we choose system (9) as the drive system, system (10) as the response system. The parameters are chosen as a=0.9, b=0.2, c=1.5, d=0.2, k =0.17. The initial values of the drive system and the response system are ($x_1(0)$, $x_2(0)$, $x_3(0)$, $x_4(0)$)=(-1, -1, -1, -1) and ($y_1(0)$, $y_2(0)$, $y_3(0)$, $y_4(0)$)=(2, 1, 2, 1). Control gain b_i (i=1, 2, 3, 4) are designed as 2, the time delay $\tau=1$, Assuming the controllers are switched on the T=1s and δ=0.8s. Using MATLAB, we get the time evolution of states for the error system (11) in Fig.4 and the time evolution of state variables in drive system (9) and response system (10) in Fig.5. In Fig.4, the states for the error system (11) converge to zero quickly and the fluctuation is small, it also need less energy by intermittent linear state feedback control. In Fig.5, it is easy to find that the states $x_1(t)$, $x_2(t)$, $x_4(t)$ complete synchronized to $y_1(t)$, $y_2(t)$, $y_4(t)$ and $x_3(t)$ anti-synchronized to $y_3(t)$. All of those have been illustrated the effectiveness and correctness of hybrid synchronization method.

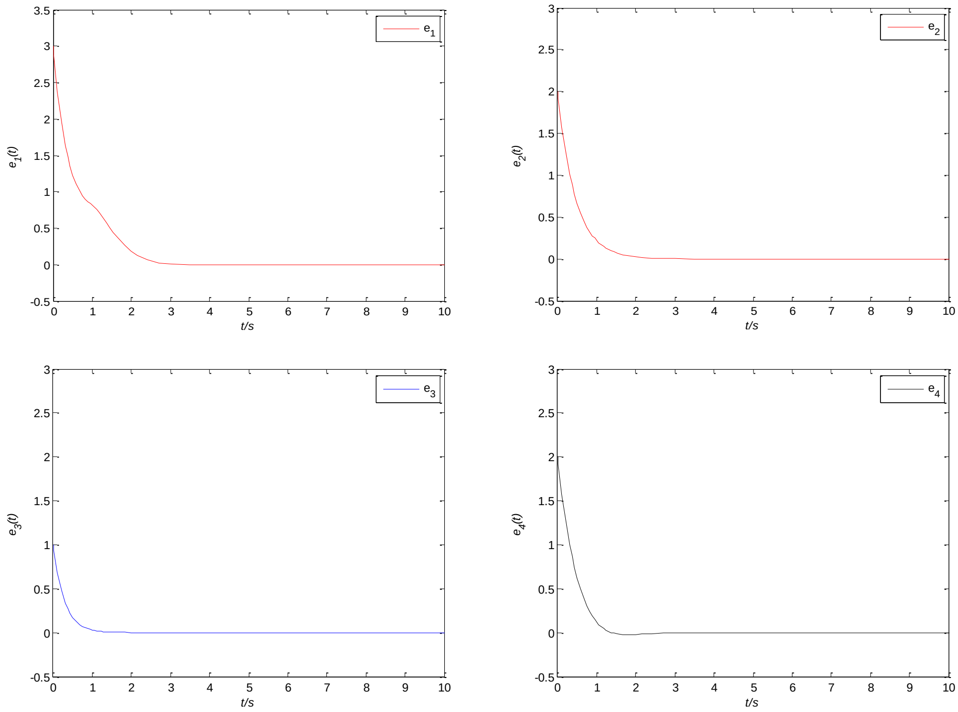


Fig.4: The time evolution of errors system (11).

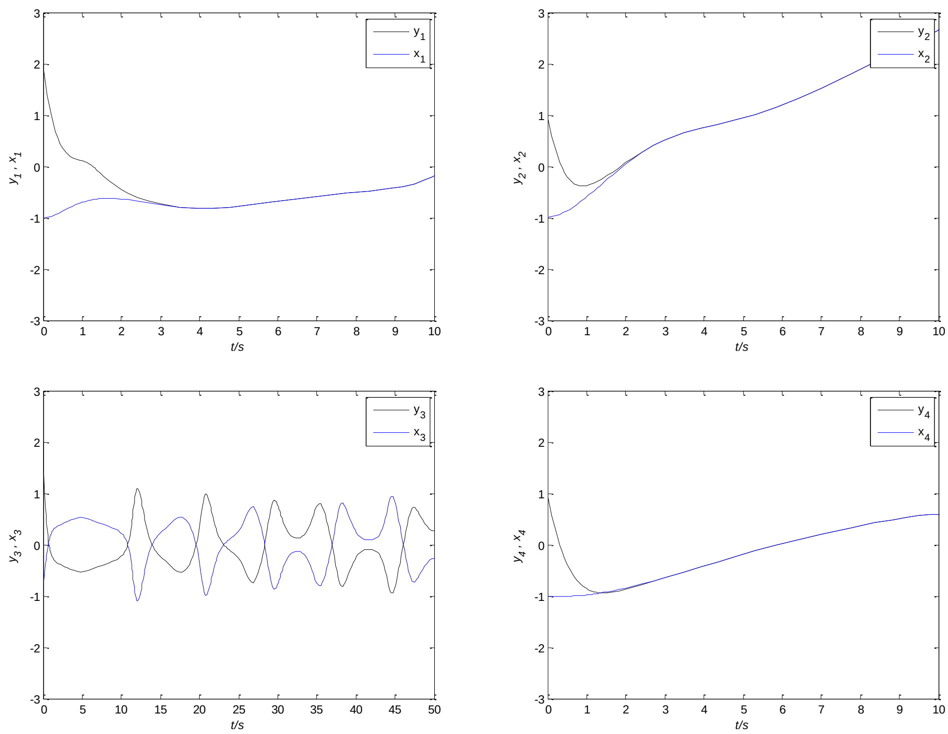


Fig.5: The time evolution of state variables in drive system (9) and response system (10).

6. Conclusion

In this paper, a new time-delay financial hyperchaotic system is introduced and analysis its stability by intermittent linear state feedback control with time-delay τ. Hybrid synchronized method is firstly proposed to a financial hyperchaotic system. By proper hybrid control and Lyapunov stability theory, hybrid synchronization is achieve between drive system and response system, the state variables y_1, y_2, y_4 in response system are complete synchronized to x_1, x_2, x_4 in drive system respectively, while the state variable y_3 in response system is anti-synchronized to x_3 in drive system. Corresponding numerical simulation are offered to show the effectiveness and correctness of proposing methods.

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