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Hybrid synchronization of the hyperchaotic 4D systems via impulsive coupling

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Abstract. Hybrid synchronization of the hyperchaotic 4D systems with different initial conditions is investigated via impulsive coupling. Based on the Lyapunov stability theory, sufficient conditions are given to get the hybrid synchronization by constructing a Lyapunov function. It is proved that some partial state variables of two hyperchatic systems are anti-synchronized, while other state variables are complete synchronized, when impulsive coupling controllers are imposed on the response system. Numerical simulation results are presented to demonstrate the effectiveness of the proposed chaos synchronization scheme.

Keywords: hybrid synchronization; hyperchaotic system; impulsive coupling

1. Introduction

Starting from the pioneering work of Pecora and Carroll [1], synchronization of chaos has attracted much attention [2-3] due to its many applications in physics, secure communication, chemical reactor, control theory, telecommunications, biological networks, artificial neural networks, etc. Several different types of synchronization of coupled chaotic oscillators have been described theoretically and observed experimentally, such as complete synchronization[4], anti-synchronization[5], phase synchronization[6], generalized synchronization[7], partially synchronization[8], time-scale synchronization[9], projective synchronization[10], Q-S synchronization[11], and even cluster synchronization [12], etc. Many different methods have been proposed to study synchronization such as active control, feedback control, observer control[13], etc.

As far as we know, in drive-response synchronization, the research results reported on the same synchronization regime of the response and drive systems. Recently, a class of new synchronization phenomenon, hybrid synchronization in chaotic systems had been investigated intensively [14-16]. In hybrid synchronization scheme, one part of the system is anti-synchronized and the other is completely synchronized so that complete synchronization (CS) and anti-synchronization (AS) co-exist in the system. In this paper, the coexistence of CS and AS between two identical hyperchaotic systems will be invenstigated via impulsive coupling controller.The rest of this paper is organized as follows. Section 2 describes the Hyperchaotic 4D system. In Section 3, based on the Lyapunov stability theory, the Hybrid synchronization of the hyperchaotic 4D systems via impulsive coupling is presented and the stability of the error dynamic system is derived. In Section 4, some numerical illustrative examples are provided to illustrate the effectiveness of the proposed scheme. Finally, conclusions are presented in Section 5.

2. System description of hyperchaotic 4D system

The considered hyperchaotic 4D system is described as

$$\begin{cases} \dot{x}_1 = a(x_2 - x_1), \\ \dot{x}_2 = x_1 + bx_2 - x_1x_3 + x_4, \\ \dot{x}_3 = x_1^2 - cx_3, \\ \dot{x}_4 = -dx_1. \end{cases} \tag{1}$$

where x_i $(i=1,2,3,4)$ are state variables. a, b, c and d are real constants. the system (1) is the hyperchaotic system constructed by Cai et al.[39]. Therefore, in the following sections, we will investigate the coexistence of anti-phase and complete synchronization of the hyperchaotic 4D systems via impulsive coupling controller.

3. Hybrid synchronization of the hyperchaotic 4D system via unidirectional impulsive coupling

For convenience, the drive hyperchaotic 4D system is chosen as (1) and the response system is given as the following:

$$\begin{cases}\dot{y}_1 = a(y_2 - y_1),\ t \neq t_k, \\ \dot{y}_2 = y_1 + by_2 - y_1y_3 + y_4, t \neq t_k, \\ \dot{y}_3 = y_1^2 - cy_3, t \neq t_k, \\ \dot{y}_4 = -dy_1, t \neq t_k.\end{cases} \tag{2a}$$

$$y_i(t_k^+) - y_i(t_k^-) = \alpha_i(y_i(t_k^-) - x_i(t_k^-)), i=1,2,3,4, t = t_k, \tag{2b}$$

where $\alpha = (\alpha_1, \alpha_2, \alpha_3, \alpha_4)^T$ are positive constant vector, $0 \le t_0 < t_1 < t_2 \cdots < t_k < t_{k+1} < \cdots$ and $t_k \to \infty$ as $k \to \infty$.

The goal is to find some conditions on the control gains α_i, and the impulse distances $t_{k+1} - t_k = \zeta_k$, so that the state variables y_1, y_2 and y_4 in response system are anti-synchronized to x_1, x_2 and x_4 in drive system, respectively, while the third state variable y_3 in response system is complete-synchronized to x_3 in drive system. For this purpose, let

$$e_1 = y_1 + x_1,\ e_2 = y_2 + x_2,\ e_3 = y_3 - x_3,\ e_4 = y_4 + x_4. \tag{3}$$

It follows from (1) and (2) that the errors system (3) are governed by the following dynamical system:

$$\begin{cases}\dot{e}_1 = a(e_2 - e_1),\ t \neq t_k, \\ \dot{e}_2 = e_1 + be_2 - e_1y_3 + x_1e_3 + e_4, t \neq t_k, \\ \dot{e}_3 = (y_1 - x_1)e_1 - ce_3, t \neq t_k, \\ \dot{e}_4 = -de_1, t \neq t_k.\end{cases} \tag{4a}$$

$$e_i(t_k^+) = e_i(t_k^-) - \alpha_i e_i(t_k^-), t = t_k, i=1,2,3,4, \tag{4b}$$

which can be rewritten as

$$\dot{e} = Ae + f(x,y),\ t \neq t_k \tag{5a}$$

$$e_i(t_k^+) = e_i(t_k^-) - \alpha_i e_1(t_k^-), t = t_k, i=1,2,3,4, \tag{5b}$$

where $e = (e_1, e_2, e_3, e_4)^T$, $A = \begin{pmatrix} -a & a & 0 & 0 \\ 1 & b & 0 & 1 \\ 0 & 0 & -c & 0 \\ -d & 0 & 0 & 0 \end{pmatrix}$, $f(x,y) = Be$, $B = \begin{pmatrix} 0 & 0 & 0 & 0 \\ -y_3 & 0 & x_1 & 0 \\ y_1 - x_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$.

$$J_1 = A + B,\ J_2 = \begin{pmatrix} 1-\alpha_1 & 0 & 0 & 0 \\ 0 & 1-\alpha_2 & 0 & 0 \\ 0 & 0 & 1-\alpha_3 & 0 \\ 0 & 0 & 0 & 1-\alpha_4 \end{pmatrix}, \mu = \lambda_{\max}\left(\frac{J_1^T + J_1}{2}\right), \nu = \sigma_{\max}\left(J_2^T J_2\right).$$

$\lambda_{\max}(.)$ and $\sigma_{\max}(.)$ denote the maximal eigenvalue and the maximal singular value of matrix, respectively.

The goal is to propose simple input controllers so that the state errors in (5) satisfy

$$\lim_{t \to \infty} e_i(t) = 0, i=1,2,3,4. \tag{6}$$

We propose the following theorem:

Theorem 1. Consider the unidirectional impulsive coupling (1) and (2), suppose that the average impulsive interval of the impulsive sequence $\xi = \{t_0, t_1, t_2, \ldots\}$ is less than T_a. Then, the system (4) is globally exponentially stable with convergence rate η if

$$\eta \ge \frac{1}{T_a}\ln|\nu| + \mu. \tag{7}$$

Proof: Let $E = (e_1, e_2, e_3, e_4)^T$, choose the Lyapunov functional candidate as following

$$V = \frac{1}{2}E^T E. \tag{8}$$

When $t \in [t_{k-1}, t_k)$, the time derivative of V along trajectories of error dynamical (5) can be given by

$$\begin{aligned}\dot{V} &= \frac{1}{2}\left(\dot{E}^T E + E^T \dot{E}\right) \\ &= \frac{1}{2} E^T (J_1^{\ T} + J_1) E \\ &\le \mu E^T E = \mu V .\end{aligned} \tag{9}$$

When $t = t_k$, the following equality can be obtained

$$\begin{aligned}V(t_k) &= \frac{1}{2} E^T(t_k) E(t_k) \\ &= \frac{1}{2} E^T(t_k^-) J^T{}_2 J_2 E(t_k^-) \\ &\le \nu V(t_k^-) .\end{aligned} \tag{10}$$

So when $t \in [t_0, t_1)$, according to $\dot{V} \le \mu V$, we can get

$$V(t) \le V(t_0) e^{\mu(t-t_0)}, \tag{11}$$

$$V(t_1) \le \nu V(t_0) e^{\mu(t_1-t_0)}. \tag{12}$$

Similarly, for $t \in [t_1, t_2)$, the following equalities can be given

$$\begin{aligned}V(t) &\le V(t_1) e^{\mu(t-t_1)} \\ &\le \nu V(t_0) e^{\mu(t-t_0)},\end{aligned} \tag{13}$$

$$V(t_2) \le \nu V(t_0) e^{\mu(t_2-t_0)}. \tag{14}$$

In general, for $t \in [t_{k-1}, t_k)$, we have

$$V(t) \le \nu^{k-1} V(t_0) e^{\mu(t-t_0)}. \tag{15}$$

Let $N_\xi(t_0, t)$ be the number of impulsive times of the impulsive sequence ξ on the interval (t_0, t). Hence, for any $t \in R$,

$$V(t) \le V(t_0) \nu^{N_\xi(t_0,t)} e^{\mu(t-t_0)} . \tag{16}$$

Since the average impulsive interval of the impulsive sequence $\xi = \{ t_0, t_1, t_2, \dots \}$ is less than T_a , we have

$$N_\xi(t_0, t) \ge \frac{t - t_0}{T_a} - N_0 , \forall T \ge t \ge 0 . \tag{17}$$

Since $|\nu| < 1$, according to (16) and (17), it can be gotten that

$$\begin{aligned}V(t) &\le V(t_0) \nu^{\frac{1}{T_a}(t-t_0)-N_0} e^{\mu(t-t_0)} \\ &\le V(t_0) \nu^{-N_0} e^{\frac{1}{T_a}\ln|\nu| \ (t-t_0)} e^{\mu(t-t_0)} \\ &= V(t_0) \nu^{-N_0} e^{(\frac{1}{T_a}\ln|\nu| \ +\mu)(t-t_0)} .\end{aligned} \tag{18}$$

It follows from (18) that there exists constant $M_0 = \nu^{-N_0}$, such that

$$V(t) \le V(t_0) M_0 e^{(\frac{1}{T_a}\ln|\nu| \ +\mu)(t-t_0)} , \tag{19}$$

which further implies that

$$\frac{1}{2}\|E\|^2 \le V(t_0) M_0 e^{\eta(t-t_0)}, \ \eta \ge \frac{1}{T_a} \ln|\nu| + \mu . \tag{20}$$

Since $\eta < 0$, we can conclude that the error dynamical system (5) can be globally exponentially stabilized to the equilibrium point zero. Then the unidirectional impulsive coupling hyperchaotic 4D systems (1) and (2) achieved hybrid synchronization. Theorem 1 is proved completely.

When fixed $\alpha_i = (\alpha, \alpha, \alpha)^T$ and $t_k - t_{k-1} = \varsigma$, based on Theorem 1, we can get the following corollary directly.

Corollary 1 Considering the unidirectional impulsive coupling hyperchaotic 4D systems (1) and (2), let $\alpha_i = \alpha$ and $t_k - t_{k-1} = \varsigma$ for all $k = 1, 2, 3....$, if

$$\eta \geq \frac{1}{T_a} \ln|\nu| + \mu, \tag{21}$$

and then, the systems (1) and (2) achieve hybrid synchronization, where $\mu = \mu = \lambda_{\max}\left(\frac{J_1^T + J_1}{2}\right)$, $\nu = \sigma_{\max}\left(J_2^T J_2\right)$.

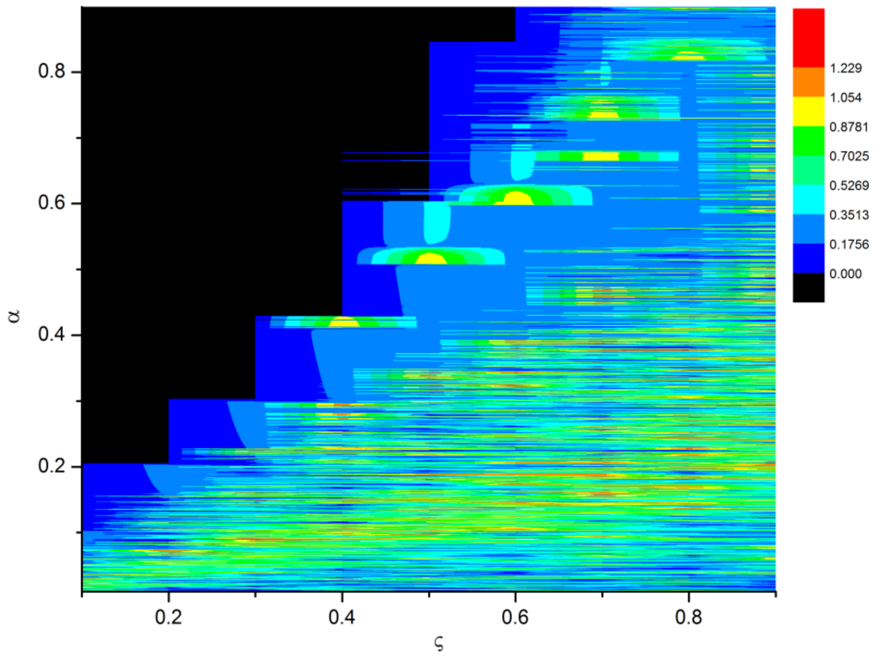

Fig.1. The distribution of the average of errors described in Eq. (22) in the two-parameter phase space (α , ς), where t=10000 to 15000 time units (t = i*h). The simulation step of ς is chosen as 0.1.

4. Illustrative numerical simulation examples

In this section, some numerical examples are presented to illustrate the theoretical analysis. In the following numerical simulations, the system parameters are selected as a =20, b =10.6, c = 2.8, d =3.7. The initial values of drive system and the response system are chosen as (-2.0, 3.0, 0.4, 0.2) and (5.0, 8.0, 2.5, 1.1), respectively. With above selected values, the chaotic behavior of system (1) can be observed. From condition (21), the synchronization of the impulsively coupled system is related to the impulsive coupling strength α_i, impulsive reset period ς. In order to explore the synchronization behavior of the scheme, for simplicity we fix $\alpha_i = \alpha$ and $t_k - t_{k-1} = \varsigma$. α and ς are taken as bifurcation parameters to see how the degree of the hybrid synchronization evolves with the variation of the parameters. The distribution about the summation of errors function on corresponding variable as following (22) in the two-gain coefficients phase space (α , ς) is illustrated in Fig.1.

$$\varphi(e_x) = \sum_{i=1}^{500,000} |e_x(i)| \tag{22}$$

The summation of errors function is calculated from t=400 to 500 time units(t = i*h, h is the time step). From the bifurcation diagram plotted in Fig.2, it could be concluded that the degree of the hybrid synchronization with impulsive coupling could be dependent on the selection of the two gain coefficients α and ς (see Fig. 2). The hybrid synchronization can be achieved with suitable parameters of α , ς (see Fig.3 and Fig.4), where $\alpha = \varsigma$ =0.3.

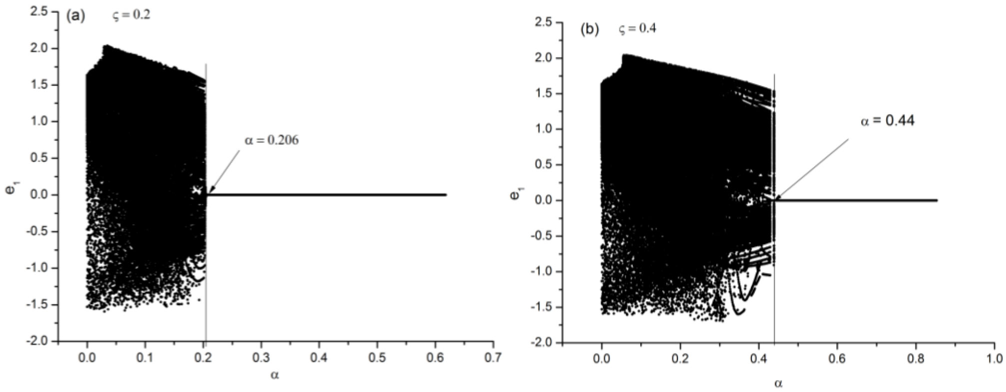

Fig. 2. Bifurcation diagram between e_1 and α, (a) ς =0.2, (b) ς =0.4.

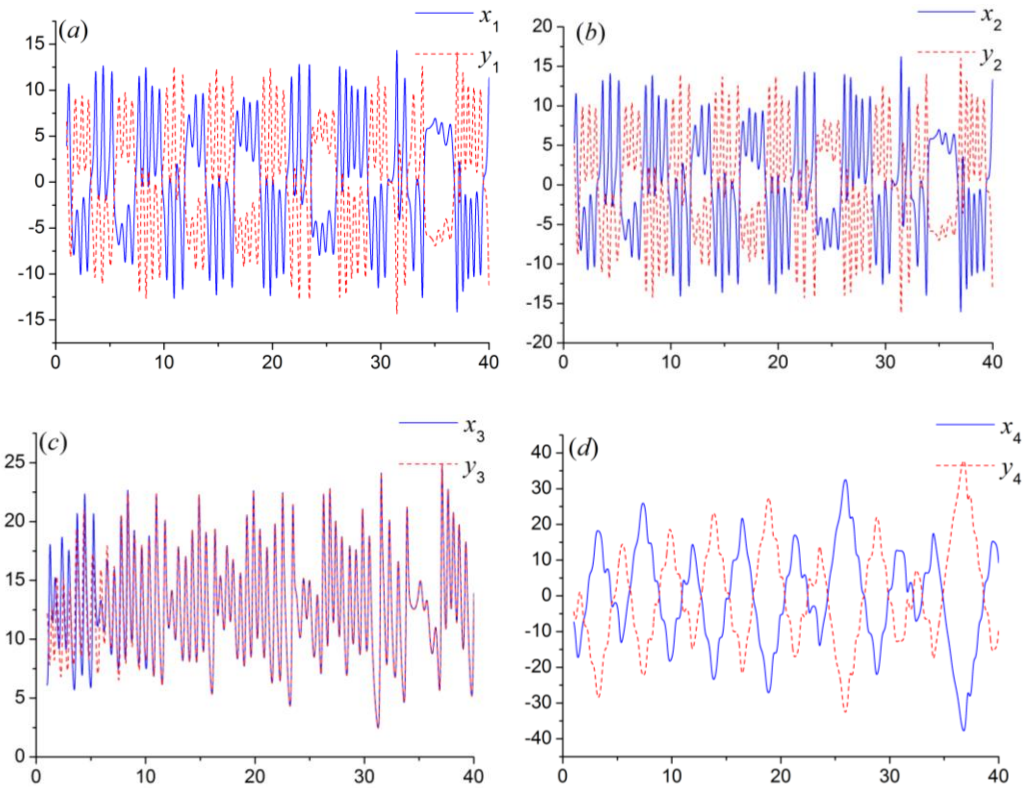

Fig.3 Time evolution curves of two systems with the impulsive controllers, where $\alpha = \varsigma = 0.3$

(a) (x_1, y_1), (b) (x_2, y_2), (c) (x_3, y_3), (d) (x_4, y_4).

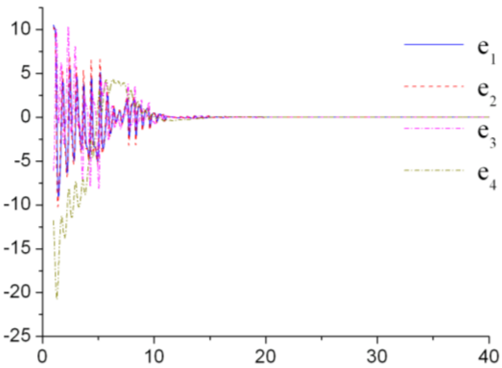

Fig.4 Dynamics of synchronization error states e_i ($i = 1, 2, 3, 4$) when $\alpha = \varsigma = 0.3$.

5. Conclusions

In this paper, the hybrid synchronization of two hyperchaotic systems is investigated via impulsive coupling. By constructing Lyapunov function, the sufficient conditions are given to get the hybrid synchronization. Theoretical analysis results show that under suitable conditions, two complex hyperchaotic systems can realize the hybrid synchronization when impulsive controllers are imposed on the response system unidirectional. That is to say, the state variables x_2, y_2 and w_2 in response system are anti-synchronized to x_1, y_1 and w_1 in drive system, respectively, while the third state z_2 in the response system is complete synchronized to z_1 in the drive system, and the corresponding parameter observers will also be approached analytically.

Acknowledgements

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