Some Fixed Point Theorems in G-Metric and Fuzzy Metric Spaces using E.A Property

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Abstract. Fuzzy logic is a new scientific field which is employed in mathematics, computer science and Engineering. Fuzzy logic calculates the extent to which a proposition is correct and allows computer to manipulate the information. The aim of this paper is to prove some common fixed point theorems using the property E.A. in the setting of G-metric and Fuzzy metric space by taking a set of three conditions for self mappings. Here the notion of semi compatibility and occasionally compatibility is used to replace the assumption of continuity and completeness of space. Our result generalizes the result of Tanmony Som [1] and Mujahid Abbas [2].

1. Introduction

The concept of Fuzzy is initially given by Lofti A Zadeh [3]. Fuzzy sets and Fuzzy logic are powerful mathematical tools for modelling and controlling uncertain systems in industry, humanity, nature and easily be implemented on a standard computer. Fuzzy metric space and fixed point in this space have been studied by many authors [4]-[7]. In 2007 R.K Saini [8] gave fixed point result in fuzzy metric space using expansion mapping. Further R-weakly commuting mapping has been employed by R.K Saini et. al. to obtain results in [9].

Mustafa [10] gave generalized metric called G-metric and define many concepts like convergence, continuity, completeness, compactness, product of spaces in the setting of G-metric space and stated that every G-metric space is topologically equivalent to a metric space. Again in 2009 Mustafa [11] showed fixed point results without using the completeness property but with other sufficient conditions. Further he employ a new concept of expansive mapping to prove results in [12].

In 1986, Jungck [13] introduced the concept of compatible mapping. This concept was frequently used to prove existence theorems in common fixed point theory. Vishal Gupta et. al. [14] gave a result in G-metric space using weakly compatibility.

In 2002, Aamir and Moutawakil [16] studied a new property for pair of maps, the property E.A, which is generalization of concept of non compatible maps. Further Pant and Pant [17] studied the common fixed points of a pair of non compatible maps and property E.A in Fuzzy metric space. Recently, Vishal Gupta and A. Kanwar[15] proved Fixed Point Theorem in Fuzzy Metric Spaces Satisfying E.A Property.

The purpose of this paper is to prove common fixed point results for four self mappings using E.A property and compatibility.

Definition 1.1: Let X be a non empty set and let $G : X \times X \times X \to R^+$ be a function satisfying the following,
1) $G(x, y, z) = 0$ if $x = y = z$
2) $0 > G(x, x, y)$ for all $x, y \in X$ with $x \neq y$
3) $G(x, x, y) < G(x, y, z)$ for all $x, y, z \in X$ with $z \neq y$
4) $G(x, y, z) = G(x, z, y) = G(y, z, x) = ...$
5) $G(x, y, z) \leq G(x, a, a) + G(a, y, z)$ for all $x, y, z, a \in X$

Then the function G is called a generalized metric or a G-metric on X and the pair (X, G) is called G-metric space.

Definition 1.2: Let A and S be self mappings on a G-metric space (X,G). Then the mappings are said to be weakly compatible if they commute at their coincidence point, that is $Ax = Sx$ implies that $ASx = SAx$.

Definition 1.3: A binary operation $* : [0,1] \times [0,1] \to [0,1]$ is a continuous t-norm if $*$ is satisfying the following conditions:
(i) $*$ is commutative and associative
(ii) $*$ is continuous
(iii) $a * 1 = a$ for all $a \in [0,1]$
(iv) $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$ and $a,b,c,d \in [0,1]$.

Definition 1.4: The 3-tuple $(X,M,*)$ is said to be Fuzzy metric space if X is an arbitrary set, $*$ is continuous t-norm and M is a Fuzzy set in $X^2 \times [0,\infty)$ satisfying the following conditions:
(i) $M(x,y,0) = 0$
(ii) $M(x,y,t) = 1$ $\forall t > 0$ if and only if $x = y$
(iii) $M(x,y,t) = M(y,x,t)$
(iv) $M(x,y,t) * M(y,z,s) \leq M(x,z,t+s)$
(v) $M(x,y,.) : [0,\infty) \to [0,1]$ is left continuous
(vi) $\lim_{n \to \infty} M(x,y,t) = 1$

Definition 1.5: Two self mappings A and S of a Fuzzy metric space $(X,M,*)$ are said to be compatible if and only if $M(ASx_n, SAx_n, t) \to 1$ for all $t > 0$, whenever $\{x_n\}$ is a sequence in X such that $Sx_n, Ax_n \to p$ for some p in X as $n \to \infty$.

Definition 1.6: Two self mappings A and S of a Fuzzy metric space $(X,M,*)$ are said to be semi-compatible if and only if $M(ASx_n, Sp, t) \to 1$ for all $t > 0$, whenever $\{x_n\}$ is a sequence in X such that $Sx_n, Ax_n \to p$ for some p in X as $n \to \infty$.

Definition 1.7: Two self mappings A and S of a Fuzzy metric space $(X,M,*)$ are said to be occasionally weakly compatible if and only if there is a point x in X which is coincidence point of A and S at which A and S commute.

Definition 1.8: Two self mappings A and S of a Fuzzy metric space $(X,M,*)$ are said to satisfy the property E.A if there exists sequence $\{x_n\}$ in X such that $\lim_{n \to \infty} Ax_n = \lim_{n \to \infty} Sx_n = z$ for some $z \in X$.

Theorem 1.1[1]: Let S and T be two continuous self mappings of a complete fuzzy metric space $(X,M,*)$. Let A be a self mapping of X satisfying (A,S) and (A,T) are R-weakly commuting and

$$A(X) \subseteq S(X) \cap T(X) \quad \text{and} \tag{1.1}$$
$$M(Ax, Ay, t) \geq r\left[\min \left\{ M(Sx,Ty,t), M(Sx,Ax,t), M(Sx,Ay,t), M(Ty,Ay,t) \right\} \right] \tag{1.2}$$

for all $x, y \in X$, where r is a continuous function such that

$$r(t) > t \text{ for each } t < 1 \text{ and } r(t) = t \text{ for } t = 1 \tag{1.3}$$

The sequences $\{x_n\}$ and $\{y_n\}$ in X are such that $x_n \to x, y_n \to y, t > 0$ implies $M(x_n, y_n, t) \to M(x,y,t)$. Then A, S, T have a unique common fixed point in X.

Theorem 1.2[2]: Let X be a complete G-metric space. Suppose that $\{f,S\}$ and $\{g,T\}$ be point wise R-weakly commuting pairs of self-mappings on X satisfying

$$G(fx, fx, gy) \leq h \max \left\{ G(Sx,Sx,Ty), G(fx,fx,Sx), G(gy,gy,Ty), \left[G(fx,fx,Ty) + G(gy,gy,Sx) \right]/2 \right\} \tag{1.4}$$

and

$$G(fx, gy, gy) \leq h \max \left\{ G(Sx,Ty,Ty), G(fx,Sx,Sx), G(gy,Ty,Ty), \left[G(fx,Ty,Ty) + G(gy,Sx,Sx) \right]/2 \right\} \tag{1.5}$$

for all $x, y \in X$, where $h \in (0,1)$. Suppose that $fX \subseteq TX$, $gX \subseteq SX$ and one of the pair $\{f,S\}$ and $\{g,T\}$ is compatible. If the mappings in the compatible pair be continuous, then f, g, S and T have a unique common fixed point.

2. Main Result

Theorem 2.1: Let (X,G) be a G-metric space and $A,B,S,T:X \to X$ be four self mappings such that

(i) $A(X) \subseteq T(X)$ and $B(X) \subseteq S(X)$

(ii)   One of the pair (A,S) and (B,T) satisfy the property E.A

(iii) $G(Ax,By,By) \le h \max \left\{ G(Sx,Ty,Ty), G(Ax,Sx,Sx), G(By,Ty,Ty), \left[\dfrac{G(Ax,Ty,Ty)+G(By,Sx,Sx)}{2} \right] \right\}$

for all $x,y \in X$, where $h \in (0,1)$.   (2.1)

(iv)   One of $A(X), B(X), S(X)$ and $T(X)$ is a complete subset of X.

Then the pairs (A,S) and (B,T) have a coincidence point. Further if (A,S) and (B,T) are weakly compatible, then A, B, S and T have a unique common fixed point in X.

Proof: Suppose that the pair (B,T) satisfies the property E.A. Then there exist a sequence $\{x_n\}$ in X such that $\lim\limits_{n\to\infty} Bx_n = \lim\limits_{n\to\infty} Tx_n = t$ for some $t \in X$. Since $B(X) \subseteq S(X)$, there exists a sequence $\{y_n\}$ in X such that $Bx_n = Sy_n$. Hence $\lim\limits_{n\to\infty} Sy_n = t$.

We claim that $\lim\limits_{n\to\infty} Ay_n = t$. To contrary suppose that this is not true.

Then from (2.1), we have

$$G(Ay_n, Bx_n, Bx_n) \le h \max \left\{ G(Sy_n,Tx_n,Tx_n), G(Ay_n,Sy_n,Sy_n), G(Bx_n,Tx_n,Tx_n), \left[\frac{G(Ay_n,Tx_nTx_n)+G(Bx_n,Sy_n,Sy_n)}{2} \right] \right\}$$

Taking the limit as $n \to \infty$, we get

$$\lim_{n\to\infty} G(Ay_n,t,t) \le h \max \left\{ G(t,t,t), \lim_{n\to\infty} G(Ay_n,t,t), G(t,t,t), \left[\frac{\lim\limits_{n\to\infty} G(Ay_n,t,t)+G(t,t,t)}{2} \right] \right\}$$

Implies $\lim\limits_{n\to\infty} G(Ay_n,t,t) \le h \lim\limits_{n\to\infty} G(Ay_n,t,t) < G(Ay_n,t,t)$ (as $h \in (0,1)$)

a contradiction. Hence $\lim\limits_{n\to\infty} Ay_n = t$

Now suppose that $S(X)$ is a complete subset of X. Then $t = Su$ for some $u \in X$. Now we will show that $Au = Su = t$.

Again from (2.1) we have

$$G(Au, Bx_n, Bx_n) \le h \max \left\{ G(Su,Tx_n,Tx_n), G(Au,Su,Su), G(Bx_n,Tx_n,Tx_n), \left[\frac{G(Au,Tx_nTx_n)+G(Bx_n,Su,Su)}{2} \right] \right\}$$

taking the limit as $n \to \infty$, we get

$$G(Au,t,t) \le h \max \left\{ G(t,t,t), G(Au,t,t), G(t,t,t), \left[\frac{G(Au,t,t)+G(t,t,t)}{2} \right] \right\}$$

Implies $G(Au,t,t) \le hG(Au,t,t) < G(Au,t,t)$

        Hence $Au = Su$.   (2.2)

Therefore u is a coincidence point of the pair (A,S). The weak compatibility of A and S implies that $ASu = SAu$ and hence $AAu = ASu = SAu = SSu$.

Since $A(X) \subseteq T(X)$, there exists $v \in X$ such that

        $Au = Tv$ (2.3)

Again from (2.1), we have

$$G(Au, Bv, Bv) \le h \max \left\{ G(Su, Tv, Tv), G(Au, Su, Su), G(Bv, Tv, Tv), \left[\frac{G(Au, Tv, Tv) + G(Bv, Su, Su)}{2} \right] \right\}$$

On using (2.2) and (2.3) we get

$$G(Tv, Bv, Bv) \le h \max \left\{ G(Au, Au, Au), G(Au, Au, Au), G(Bv, Tv, Tv), \left[\frac{G(Tv, Tv, Tv) + G(Bv, Tv, Tv)}{2} \right] \right\}$$

Implies $Tv = Bv$

Thus $Au = Su = Tv = Bv = t$ (2.4)

Now take $x = t$ and $y = v$ in (2.1), we get

$$G(At, Bv, Bv) \le h \max \left\{ G(St, Tv, Tv), G(At, St, St), G(Bv, Tv, Tv), \left[\frac{G(At, Tv, Tv) + G(Bv, St, St)}{2} \right] \right\}$$

On using (2.4), we obtain $At = t$ .Since the pair (A, S) is weakly compatible, therefore $ASu = SAu = At = St$. Hence we have $At = St = t$ .The same result can be obtained for the pair (B, T) .For uniqueness, let z be another fixed point of mappings A, B, S, T .

Take $x = t$ and $y = z$ in (2.1)

$$G(At, Bz, Bz) \le h \max \left\{ G(St, Tz, Tz), G(At, St, St), G(Bz, Tz, Tz), \left[\frac{G(At, Tz, Tz) + G(Bz, St, St)}{2} \right] \right\}$$

$$G(t, z, z) \le h \max \left\{ G(t, z, z), G(t, t, t), G(z, z, z), \left[\frac{G(t, z, z) + G(z, t, t)}{2} \right] \right\}$$

implies $t = z$. Hence the result.

Theorem 2.2: Let A, B, S, T be self mappings of a Fuzzy metric space $(X, M, *)$ satisfying the condition

$$M(Ax, By, t) \ge r \left[\min \left\{ M(Sx, Ty, t), M(Sx, Ax, t), M(Sx, By, t), M(Ty, Ax, t) \right\} \right]$$ (2.5)

For all $x, y \in X$ where $r : [0,1] \to [0,1]$ is a continuous function such that

$$r(t) > t \text{ for each } t < 1 \text{ and } r(t) = t \text{ for } t = 1$$ (2.6)

Also suppose the pair (A, S) and (B, T) share the common property (E.A) and $S(X)$ and $T(X)$ are closed subsets of X .Then the pair (A, S) as well as (B, T) have a coincidence point. Further A, B, S, T have a unique common fixed point provided the pair (A, S) is semi-compatible and (B, T) is occasionally weakly compatible.

Proof: Since the pair (A, S) and (B, T) share the common property (E.A) then there exist two sequences $\{x_n\}$ and $\{y_n\}$ in X such that

$$\lim_{n \to \infty} Ax_n = \lim_{n \to \infty} Sx_n = \lim_{n \to \infty} By_n = \lim_{n \to \infty} Ty_n = z, \text{ for some } z \in X$$

Since $S(X)$ is closed subset of X , then $\lim_{n \to \infty} Sx_n = z \in S(X)$, therefore there is a point u in X such that $Su = z$.

Now we claim that $Au = z$, if not then by using (2.5), we have

$$M(Au, By_n, t) \ge r \left[\min \left\{ M(Su, Ty_n, t), M(Su, Au, t), M(Su, By_n, t), M(Ty_n, Au, t) \right\} \right]$$

On taking the limit as $n \to \infty$, we get

$$M(Au, z, t) \ge r \left[\min \left\{ M(z, z, t), M(z, Au, t), M(z, z, t), M(z, Au, t) \right\} \right]$$

$$= r \left[M(Au, z, t) \right] > M(Au, z, t) \text{ [by using (2.6)]}$$

a contradiction which implies $Au = z$. Thus we have $Au = Su = z$ i.e. u is a coincidence point of the pair (A, S).

Again $T(X)$ is a closed subset of X, then $\lim_{n\to\infty} Ty_n = z \in T(X)$, therefore there exists $w \in X$ such that $Tw = z$. Again using (2.5) we obtain

$$M\left(Ax_n, Bw, t\right) \geq r\left[\min\left\{M\left(Sx_n, Tw, t\right), M\left(Sx_n, Ax_n, t\right), M\left(Sx_n, Bw, t\right), M\left(Tw, Ax_n, t\right)\right\}\right]$$

On taking limit as $n \to \infty$, we get

$$M\left(z, Bw, t\right) \geq r\left[\min\left\{M\left(z, z, t\right), M\left(z, z, t\right), M\left(z, Bw, t\right), M\left(z, z, t\right)\right\}\right]$$
$$= r\left[M\left(z, Bw, t\right)\right] > M\left(z, Bw, t\right)$$
$$\Rightarrow Bw = z \text{ and hence } Tw = Bw = z.$$

That shows w is a coincidence point of the pair (B, T).

Also is semi-compatible pair, so $\lim_{n\to\infty} ASx_n = Sz$ and $\lim_{n\to\infty} ASx_n = Az$.

Since the limit in Fuzzy metric space is unique so $Sz = Az$.

Now, we claim that z is a common fixed point of the pair (A, S).

Again from (2.5), we have

$$M\left(Az, Bw, t\right) \geq r\left[\min\left\{M\left(Sz, Tw, t\right), M\left(Sz, Az, t\right), M\left(Sz, Bw, t\right), M\left(Tw, Az, t\right)\right\}\right]$$

On taking the limit as $n \to \infty$, we get

$$M\left(Az, z, t\right) \geq r\left[\min\left\{M\left(Az, z, t\right), M\left(Az, Az, t\right), M\left(Az, z, t\right), M\left(z, Az, t\right)\right\}\right]$$
$$= r\left[M\left(Az, z, t\right)\right] > M\left(Az, z, t\right)$$

$\Rightarrow Az = z$ and thus $Az = z = Sz$.

Now, since w is a coincidence point of B and T and the pair (B, T) is occasionally weakly compatible so we have $BTw = TBw \Rightarrow Bz = Tz = z$.

Hence z is the common fixed point of A, S, B and T.

For uniqueness let v be another common fixed point of A, S, B and T. On taking x = z and $y = v$ in (2.5), we get

$$M\left(Az, Bv, t\right) \geq r\left[\min\left\{M\left(Sz, Tv, t\right), M\left(Sz, Az, t\right), M\left(Sz, Bv, t\right), M\left(Tv, Az, t\right)\right\}\right]$$

On applying $n \to \infty$, we have

$$M\left(z, v, t\right) \geq r\left[\min\left\{M\left(z, v, t\right), M\left(z, z, t\right), M\left(z, v, t\right), M\left(v, z, t\right)\right\}\right]$$
$$= r\left[M\left(z, v, t\right)\right] > M\left(z, v, t\right)$$

implies z = v. Thus z is the unique common fixed point of mappings A, S, B and T.

Theorem 2.3: Let A, S, B and T be four self mappings of a Fuzzy metric space $(X, M, *)$ satisfying the following conditions:

(i)   The pairs (A, S) and (B, T) share the common property (E.A)

(ii) $S(X)$ and $T(X)$ are closed subsets of X

(iii) $qM\left(Ax, By, t\right) \geq aM\left(Ty, Sx, t\right) + bM\left(Sx, By, t\right) + cM\left(Ax, By, t\right) + \max\left\{M\left(Ax, Sx, t\right), M\left(By, Ty, t\right)\right\}$

(2.7)

for all $x, y \in X$, $a, b, c \geq 0$, $q > 0$ and $q < a + b + c$. Then the pairs (A, S) and (B, T) have a point of coincidence each. Also if the pair is semi-compatible and (B, T) is occasionally weakly compatible then A, S, B and T have a unique common fixed point.

Proof: As the pairs (A, S) and (B, T) share the common property (E.A) then there exist two sequences $\{x_n\}$ and $\{y_n\}$ in X such that

$$\lim_{n \to \infty} Ax_n = \lim_{n \to \infty} Sx_n = \lim_{n \to \infty} By_n = \lim_{n \to \infty} Ty_n = z, \text{ for some } z \in X$$

Since $S(X)$ is closed subset of X, then $\lim_{n \to \infty} Sx_n = z \in S(X)$, therefore there is a point u in X such that $Su = z$. Using the condition (2.7) we get

$$qM(Au, By_n, t) \geq aM(Ty_n, Su, t) + bM(Su, By_n, t) + cM(Au, By_n, t) + \max\{M(Au, Su, t), M(By_n, Ty_n, t)\}$$

On taking limit as $n \to \infty$, we obtain

$$qM(Au, z, t) \geq aM(z, z, t) + bM(z, z, t) + cM(Au, z, t) + \max\{M(Au, z, t), M(z, z, t)\}$$

$$(q - c)M(Az, z, t) \geq (a + b)M(z, z, t) + 1 > (a + b)M(z, z, t)$$

$$\Rightarrow M(Au, z, t) > \frac{a + b}{q - c} > 1, \text{ for all } t > 0$$

Hence $Au = z$ and $Au = Su$ which shows that u is the coincidence point of (A, S).

Again $T(X)$ is closed subset of X, therefore there is a point w in X such that $Tw = z$. Now take $x = x_n$ and $y = w$ in condition (2.7), we get

$$qM(Ax_n, Bw, t) \geq aM(Tw, Sx_n, t) + bM(Sx_n, Bw, t) + cM(Ax_n, Bw, t) + \max\{M(Ax_n, Sx_n, t), M(Bw, Tw, t)\}$$

On applying the limit as $n \to \infty$, we obtain

$$qM(z, Bw, t) \geq aM(z, z, t) + bM(z, Bw, t) + cM(z, Bw, t) + \max\{M(z, z, t), M(Bw, z, t)\}$$

$$(q - b - c)M(z, Bw, t) \geq aM(z, z, t) + 1 > aM(z, z, t)$$

$$\Rightarrow M(z, Bw, t) > \frac{a}{q - b - c} > 1, \text{ for all } t > 0$$

Implies $Bw = z$ and hence $Tw = Bw = z$. Thus w is the coincidence point of (B, T).

Further we assume (A, S) is semi-compatible pair, so $\lim_{n \to \infty} ASx_n = Sz$ and $\lim_{n \to \infty} ASx_n = Az$.

Since the limit in Fuzzy metric space is unique so $Sz = Az$. Now we claim that z is a common fixed point of A and S.

From condition (2.7), we have

$$qM(Az, Bw, t) \geq aM(Tw, Sz, t) + bM(Sz, Bw, t) + cM(Az, Bw, t) + \max\{M(Az, Sz, t), M(Bw, Tw, t)\}$$

on taking $n \to \infty$, we obtain

$$qM(Az, z, t) \geq aM(z, Az, t) + bM(Az, z, t) + cM(Az, z, t) + \max\{M(Az, Az, t), M(z, z, t)\}$$

$$(q - a - b - c)M(z, Bw, t) \geq \frac{1}{q - a - b - c} > 1 \text{ for all } t > 0$$

implies $Az = z$. Hence $Az = z = Sz$.

Now since w is a coincidence point of B and T and the pair (B, T) is occasionally weakly compatible so we have $BTw = TBw$ implies $Bz = Tz = z$.

Hence z is the common fixed point of mappings A, B, S and T. The uniqueness of fixed point follows from taking $x = z$ and $y = v$ in condition (2.7).

On taking $A = B$ in the above theorem we get the following corollary

Corollary 2.1: Let A, S and T be four self mappings of a Fuzzy metric space (X, M^*) satisfying the following conditions:

(i) The pairs (A, S) and (A, T) share the common property (E.A)

(ii) $S(X)$ and $T(X)$ are closed subsets of X

(iii) $qM\left(Ax,Ay,t\right) \geq aM\left(Ty,Sx,t\right) + bM\left(Sx,Ay,t\right) + cM\left(Ax,Ay,t\right) + \max\left\{M\left(Ax,Sx,t\right),M\left(Ay,Ty,t\right)\right\}$

for all $x,y \in X$, $a,b,c \geq 0$, $q > 0$ and $q < a+b+c$

Then the pairs $\left(A,S\right)$ and $\left(A,T\right)$ have a point of coincidence each. Further if the pair $\left(A,S\right)$ is semi-compatible and $\left(A,T\right)$ is occasionally weakly compatible then A, S and T have a unique common fixed point.

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