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Application of HPM for determination of an unknown function in a semi-linear parabolic equation

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Abstract. In this work, the homotopy perturbation method, a powerful technique, is applied to obtain an unknown time-dependent function in a semi-linear parabolic equation with given initial and boundary conditions. This kind of problem plays a very important role in many branches of science and engineering. Using the homotopy perturbation method a rapid convergent sequence can be constructed which tends to the exact solution of the problem. Some examples are presented to illustrate the strength of the method.

Keywords: Inverse problem, Semi-linear parabolic equation, Homotopy perturbation method.

1. Introduction

Most of physical and engineering problems are nonlinear and in most cases it is difficult to solve them, especially analytically. A number of analytical methods are available in the literature for the investigation of these problems, such as Adomian decomposition method [1, 2], the δ-expansion method [3], the homotopy analysis method [4-6], the variational iteration method [7-10] and the homotopy perturbation method (HPM) [11-13]. The HPM was proposed by Ji-Huan He in 1999. The essential idea of this method is to introduce a homotopy parameter, say m, which takes values from 0 to 1. When $m=0$, the systems of equations usually reduced to a sufficiently simplified form, which normally admits a rather simple solution. As m is gradually increased to 1, the system goes through a sequence of deformations, the solution for each of which is close to that at the previous stage of deformation. Eventually at $m=1$, the systems takes the original form of the equation and the final stage of deformation gives the desired solution.

One of the most considerable features of the HPM is that usually just few perturbation terms are sufficient for obtaining a reasonably accurate solution.

The HPM is an effective solution method for a broad class of problems. This technique was applied to nonlinear oscillators with discontinuities [14], nonlinear wave equations [15], nonlinear boundary value problems [16], a nonlinear convection- radiative cooling equation, a nonlinear heat equation [17], limit cycle and bifurcation of nonlinear problems [18, 19], nonlinear fractional partial differential equations [20], inverse heat conduction problem [21] and some other subjects [22-26].

In this work, we consider the inverse problem of finding a pair of function (T, p) in the following semi-linear parabolic equation:

$$\frac{\partial T}{\partial t}(X,t)=\Delta T(X,t)+p(t)T(X,t)+\phi(X,t); \qquad X\in\Gamma,\ 0<t<t_{\max}, \tag{1}$$

with initial and boundary conditions:

$$T(X,0)=f(X); \qquad X\in\Gamma, \tag{2}$$

$$T(X,t)=h(X,t); \qquad X\in\partial\Gamma, \quad 0<t\le t_{\max}, \tag{3}$$

and an additional condition as an over specification at a point in the spatial domain in the following form:

$$T(X_0,t)=E(t); \qquad X_0\in\Gamma,\ 0<t\le t_{\max}, \tag{4}$$

where Δ is Laplace operator, $t_{\max}$ is final time, $\Gamma=[0,1]^d$ is spatial domain of the problem for $d=1,2,3$, $X=(x_1,\dots,x_d)$, $\partial\Gamma$ is the boundary of Γ and ϕ, f, h and E are known functions.

The existence, uniqueness and continuous dependence of the solution upon the data for this problem are demonstrated in [27-31].

These kinds of problems have many important applications in heat transfer, thermoelasticity, control theory and chemical diffusion. Equation (1) can be used to describe a heat transfer process with a source parameter $p(t)$ and (4) to represent the temperature $T(X,t)$ at a specific point X_0 in the spatial domain at

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any time.

In [32, 33] the finite difference techniques are used to approximate the solution of this problem. The Adomian decomposition method for the problem (1)-(4) is proposed in [34]. Authors of [35, 36] applied the variational iteration method to obtain the analytical solution for this problem. Also Sinc-collocation method has been used in [37] for solving the one dimensional parabolic inverse problem with a source control parameter.

In this paper, we use the HPM to derive an analytical solution for the problem (1)-(4).The organization of the paper is as follows: In Section 2, the homotopy perturbation method is presented. Section 3 is devoted to some examples. Conclusion is finally discussed in Section 4.

2. Homotopy perturbation method (HPM)

To clarify the basic ideas of HPM, consider the following nonlinear differential equation:

$$A(T)-f(r)=0; \qquad r\in\Gamma, \tag{5}$$

subject to the boundary condition:

$$B(T,\frac{\partial T}{\partial n})=0; \qquad r\in\partial\Gamma, \tag{6}$$

where $T=T(X,t)$ is the dependent variable to be solved, A is a general differential operator, B is a boundary operator and $f(r)$ is a known analytic function.

The operator A can be divided into two parts, which are L and N, where L is a linear and N is a nonlinear operator. Equation (5) can be, therefore, written as:

$$L(T)+N(T)-f(r)=0. \tag{7}$$

By using homotopy technique, one can construct a homotopy

$$v(r,m):\Gamma\times[0,1]\rightarrow\Re \tag{8}$$

which satisfies

$$H(v,m)=(1-m)[L(v)-L(T_0)]+m[A(v)-f(r)]=0, \tag{9}$$

or

$$H(v,m)=L(v)-L(T_0)+mL(T_0)+m[N(v)-f(r)]=0, \tag{10}$$

where $m\in\left[0,1\right]$ is an embedding parameter and T_0 is the initial approximation of equation (5) which satisfies the boundary conditions. Clearly, from equations (9) and (10), we have

$$H(v,0)=L(v)-L(T_0)=0, \tag{11}$$

$$H(v,1)=A(v)-f(r)=0. \tag{12}$$

Changing the process of m from zero to unity is just that of $v(r,m)$ changing from $T_0(r)$ to $T(r)$. In topology, this is called deformation and also, $L(v)-L(T_0)$ are called homotopic. According to the homotopy perturbation method, the parameter m is considered as a small parameter and the solution of equations (11) and (12) can be given as a series in m in the form [11-13]:

$$v=T_0+mT_1+m^2T_2+\cdots, \tag{13}$$

and setting $m=1$ results in the approximate solution of equation (5) as:

$$T=\lim_{m\rightarrow 1}v=T_0+T_1+T_2+\cdots. \tag{14}$$

If we limit the sum to the first $n+1$ components, we obtain so-called $n-$order approximate solution of equation (1):

$$T=T_0+T_1+\cdots T_n. \tag{15}$$

The major advantage of HPM is that the perturbation equation can be freely constructed in many ways (therefore is problem dependent) by homotopy in topology and the initial approximation can also freely selected.

For the convergence of the series obtained via HPM, we recall Banach's theorem:

Theorem. Assume that X is a Banach space and $N:X\rightarrow X$ is a nonlinear mapping and suppose that

$$\forall v,\tilde{v}\in X;\ \|N(v)-N(\tilde{v})\|\leq\gamma\|v-\tilde{v}\|, \quad 0<\gamma<1.$$

Then N has a unique fixed point. Furthermore, the sequence

$$V_{n+1}=N(V_n)$$

with an arbitrary choice of $V_0 \in X$ converges to the fixed point of N and

$$\|V_k - V_l\| \le \|V_1 - V_0\| \sum_{j=l-1}^{k-2} \gamma_j .$$

The sequence generated by HPM will be regarded as:

$$V_0 = T_0, \ \ V_n = N(V_{n-1}), \ \ N(V_{n-1}) = \sum_{i=0}^{n} T_i, \ \ i = 1, 2, \cdots.$$

According to the above theorem, for the nonlinear mapping N a sufficient condition for the convergence of HPM is strictly contraction of N.

Before applying the HPM to equation (1), we employ two following transformations:

$$w(X,t) = T(X,t)\exp(-\int_0^t p(s)ds), \tag{16}$$

$$r(t) = \exp(-\int_0^t p(s)ds). \tag{17}$$

Transformation (16) allows us to eliminate the unknown term $p(t)$ from equation (1) and to obtain a new non-classic partial differential equation which has suitable form to apply the HPM. Now using trensformations (16) and (17), we can write (1)-(4) as follows:

$$\frac{\partial w}{\partial t}(X,t) = \Delta w(X,t) + r(t)\phi(X,t); \quad X \in \Gamma, \ 0 < t < t_{\max}, \tag{18}$$

$$w(X,0) = f(X); \quad X \in \Gamma, \tag{19}$$

$$w(X,t) = r(t)h(X,t); \quad X \in \partial\Gamma, \quad 0 \le t \le t_{\max}, \tag{20}$$

$$w(X_0,t) = r(t)E(t); \quad X_0 \in \Gamma, \ 0 \le t \le t_{\max}. \tag{21}$$

Assume $E(t) \ne 0$, then the later is equivalent to:

$$r(t) = \frac{w(X_0,t)}{E(t)}. \tag{22}$$

According to HPM, we construct the following homotopy [11-13]

$$\frac{\partial w}{\partial t}(X,t) - \frac{\partial w_0}{\partial t}(X,t) = m\{\Delta w(X,t) + \frac{w(X_0,t)}{E(t)}\phi(X,t) - \frac{\partial w_0}{\partial t}(X,t)\}. \tag{23}$$

The solution of equation (23) is assumed in the form [11-13]:

$$w = w_0 + m w_1 + m^2 w_2 + m^3 w_3 + \cdots. \tag{24}$$

Substituting (24) into (23) and collecting coefficients of the same power of m yield:

$$m^0 : \frac{\partial w_0}{\partial t} - \frac{\partial w_0}{\partial t} = 0, \tag{25}$$

$$m^1 : \frac{\partial w_1}{\partial t} = \Delta w_0 + \frac{\phi}{E} w_0(X_0,t) - \frac{\partial w_0}{\partial t}, \tag{26}$$

$$m^2 : \frac{\partial w_2}{\partial t} = \Delta w_1 + \frac{\phi}{E} w_1(X_0,t), \tag{27}$$

$$\vdots \tag{28}$$

We let

$$w_0(X,t) = f(X), \tag{29}$$

then all the above linear equations can be easily solved. The solution of (23) can be obtained by putting $m = 1$ in equation (24) as follows:

$$w = w_0 + w_1 + w_2 + \cdots. \tag{30}$$

Now from (16), we compute:

$$T(X,t) = \frac{w(X,t)}{r(t)}, \tag{31}$$

and from (17), we obtain:

$$p(t) = -\frac{r'(t)}{r(t)}, \tag{32}$$

where $r(t)$ is given by (22).

The numerical results in Section 3 indicate that the proposed scheme is efficient.

3. Numerical examples

In this section, we present some examples to show the high accuracy of HPM for solving the inverse problem (1)-(4).

Example 1. Consider the following problem [35, 39, 40, 41, 42]:

$$\frac{\partial T}{\partial t} = \Delta T + p(t)T + (\pi^2 - (t+1)^2)\exp(-t^2)(\cos(\pi x) + \sin(\pi x)); \qquad 0 < x < 1,\ 0 < t < 1, \tag{33}$$

$$T(x,0) = \cos(\pi x) + \sin(\pi x); \qquad 0 \le x \le 1,$$
$$T(0,t) = \exp(-t^2); \qquad 0 < t \le 1,$$
$$T(1,t) = -\exp(-t^2); \qquad 0 < t \le 1,$$
$$T(0.25,t) = \sqrt{2}\exp(-t^2); \qquad 0 < t \le 1.$$

The exact solution of this problem is:

$$T(x,t) = \exp(-t^2)(\cos(\pi x) + \sin(\pi x)),$$
$$P(t) = 1 + t^2.$$

Using the equation (26)-(29), we find

$$w_0(x,t) = f(x) = \cos(\pi x) + \sin(\pi x),$$
$$w_1(x,t) = (-\frac{t^3}{3} - t^2 - t)(\cos(\pi x) + \sin(\pi x)),$$
$$w_2(x,t) = \frac{(-\frac{t^3}{3} - t^2 - t)^2}{2}(\cos(\pi x) + \sin(\pi x)),$$
$$\vdots$$

Then from (30), we have the approximate solution in a series form as:

$$w(x,t) = (1 + (-\frac{t^3}{3} - t^2 - t) + \frac{(-\frac{t^3}{3} - t^2 - t)^2}{2} + \cdots)(\cos(\pi x) + \sin(\pi x))$$
$$= \exp(-\frac{t^3}{3} - t^2 - t)(\cos(\pi x) + \sin(\pi x)).$$

Now from (31) and (32) the exact values of $T(x,t)$ and $p(t)$ can be obtained. This is the same as obtained by Adomian's decomposition method and the variational iteration method [35].

We can compute $n-$order approximate solution of equation (33) from (15) as:

$$w(x,t) = (1 + (-\frac{t^3}{3} - t^2 - t) + \frac{(-\frac{t^3}{3} - t^2 - t)^2}{2} + \cdots + \frac{(-\frac{t^3}{3} - t^2 - t)^n}{n!})(\cos(\pi x) + \sin(\pi x)).$$

Then from (31) and (32) the $n-$order approximate values of $T(x,t)$ and $p(t)$ can be obtained.

Tables 1 and 2 show the comparison of absolute error of several methods in approximating $T(x,1)$ and $p(t)$, respectively, for problem 1. As we see the HPM has good accuracy in comparison with the other methods of [40, 41, 42]. In HPM we take $n = 25$.

Figures 1 and 2 presents the exact and numerical values of $T(x,1)$ and $p(t)$,respectively. In HPM we take $n = 10$ and $n = 25$.

Table 1. Comparison of absolute error of the several techniques in approximating $T(x,1)$ for test problem 1.

x	HPM	CFDM[40]	SaulyevII [41]	MOL[42]
0.1	5.5511×10^{-17}	1.3267×10^{-14}	8.0×10^{-3}	1.9219×10^{-7}
0.2	1.11022×10^{-16}	3.7192×10^{-14}	8.0×10^{-3}	5.7795×10^{-8}
0.3	0	9.6589×10^{-15}	8.0×10^{-3}	5.0769×10^{-8}
0.4	1.1102×10^{-16}	3.3862×10^{-15}	8.3×10^{-3}	1.3041×10^{-7}
0.5	5.5511×10^{-17}	1.3156×10^{-14}	8.8×10^{-3}	1.8180×10^{-7}
0.6	2.7755×10^{-17}	9.4091×10^{-15}	8.9×10^{-3}	2.0885×10^{-7}
0.7	1.3877×10^{-17}	1.2490×10^{-14}	8.5×10^{-3}	2.1800×10^{-7}
0.8	1.3877×10^{-17}	3.9829×10^{-15}	8.7×10^{-3}	2.1716×10^{-7}
0.9	2.7755×10^{-17}	1.4239×10^{-14}	8.9×10^{-3}	2.1463×10^{-7}

Table 2. Comparison of absolute error of the several techniques in approximating $p(t)$ for test problem 1.

t	HPM	CFDM[40]	SaulyevII [41]	MOL[42]
0.1	0	4.1325×10^{-9}	9.5×10^{-3}	6.3751×10^{-5}
0.2	2.2204×10^{-16}	4.1342×10^{-9}	9.3×10^{-3}	5.6986×10^{-5}
0.3	0	4.1374×10^{-9}	9.2×10^{-3}	3.3660×10^{-4}
0.4	2.2204×10^{-16}	4.1247×10^{-9}	9.1×10^{-3}	1.6461×10^{-4}
0.5	6.6613×10^{-16}	4.1371×10^{-9}	8.8×10^{-3}	4.0586×10^{-4}
0.6	8.8818×10^{-16}	4.1256×10^{-9}	8.8×10^{-3}	3.9383×10^{-5}
0.7	2.2204×10^{-15}	4.1451×10^{-9}	8.7×10^{-3}	4.6266×10^{-4}
0.8	1.7763×10^{-15}	4.1089×10^{-9}	8.6×10^{-3}	4.7802×10^{-4}
0.9	1.1768×10^{-14}	4.1589×10^{-9}	8.4×10^{-3}	2.1816×10^{-3}
1.0	1.6431×10^{-14}	4.0122×10^{-9}	8.3×10^{-3}	2.2165×10^{-4}

Example 2. In this example, let us consider the following problem [33, 35]:

$$\frac{\partial T}{\partial t} = \Delta T + p(t)T + \left(\frac{5\pi^2}{16} - 5t\right)\exp(t)\sin\left(\frac{\pi}{4}(x+2y)\right); \qquad 0<x,y<1,\ 0<t<1, \tag{34}$$

$$T(x,y,0) = \sin\left(\frac{\pi}{4}(x+2y)\right); \qquad 0 \le x,y \le 1,$$

$$T(0,y,t) = \exp(t)\sin\left(\frac{\pi}{2}y\right); \qquad 0 \le y \le 1,\ 0<t\le 1,$$

$$T(1,y,t) = \exp(t)\sin\left(\frac{\pi}{4}(1+2y)\right); \qquad 0 \le y \le 1,\ 0<t\le 1,$$

$$T(x,0,t) = \exp(t)\sin\left(\frac{\pi}{4}x\right); \qquad 0 \le x \le 1,\ 0<t\le 1,$$

$$T(x,1,t) = \exp(t)\sin\left(\frac{\pi}{4}(x+2)\right); \qquad 0 \le x \le 1,\ 0<t\le 1,$$

$$T(0.4,0.2,t) = \exp(t)\sin(0.2\pi); \qquad 0<t\le 1.$$

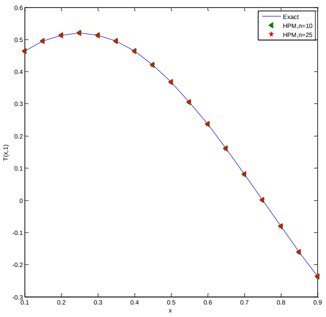

Fig. 1: The exact and numerical values of $T(x,1)$.

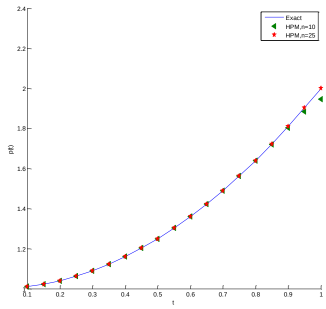

Fig. 2: The exact and numerical values of $p(t)$.

The exact solution of this problem is:

$$T(x,y,t)=\exp(t)\sin(\frac{\pi}{4}(x+2y)),$$

$$P(t)=1+5t.$$

Using the equation (26)-(29), we compute

$$w_0(x,y,t)=f(x,y)=\sin(\frac{\pi}{4}(x+2y)),$$

$$w_1(x,y,t)=-\frac{5}{2}t^2\sin(\frac{\pi}{4}(x+2y)),$$

$$w_2(x,y,t)=\frac{(-\frac{5}{2}t^2)^2}{2}\sin(\frac{\pi}{4}(x+2y)),$$

$$\vdots$$

So from (30), we have the approximate solution in a series form as:

$$w(x,y,t)=(1+(-\frac{5}{2}t^2)+\frac{(-\frac{5}{2}t^2)^2}{2}+\cdots)\sin(\frac{\pi}{4}(x+2y))=\exp(-\frac{5}{2}t^2)\sin(\frac{\pi}{4}(x+2y)).$$

One can compute the exact values of $T(x,y,t)$ and $p(t)$ from (31) and (32). This result is the same as obtained by Adomian's decomposition method [34] and the variational iteration method [35].

The $n-$order approximate solution of equation (34) can be obtained as follows:

$$w(x,t)=(1+(-\frac{5}{2}t^2)+\frac{(-\frac{5}{2}t^2)^2}{2}+\cdots+\frac{(-\frac{5}{2}t^2)^n}{n})\sin(\frac{\pi}{4}(x+2y)).$$

Now from (31) and (32) the $n-$order approximate values of $T(x,y,t)$ and $p(t)$ can be obtained.

Tables 3 and 4 show the comparison of absolute error of several methods in approximating $T(x,y,1)$ and $p(t)$, respectively, for problem 2. As we see the HPM has good accuracy in comparison with the other methods of [33]. In HPM we take $n=20$.

Figures 3, 4 and 5 presents the exact and numerical values of $T(x,y,1)$ and $p(t)$. In figure 3, we put $n=10$ and in figure 4 we take $n=20$.

Table 3. Comparison of absolute error of the several techniques in approximating $T(x, y, 1)$ for test problem 2.

x	HPM	(1,3) F.E. [33]	(9,9) F. I. [33]	(3,9) ADI[33]
0.1	1.1102×10^{-16}	5.5×10^{-6}	7.5×10^{-6}	6.4×10^{-6}
0.2	2.2204×10^{-16}	5.5×10^{-6}	7.4×10^{-6}	6.9×10^{-6}
0.3	4.4408×10^{-16}	5.4×10^{-6}	7.5×10^{-6}	7.0×10^{-6}
0.4	4.4408×10^{-16}	5.6×10^{-6}	7.8×10^{-6}	7.3×10^{-6}
0.5	4.4408×10^{-16}	5.7×10^{-6}	7.9×10^{-6}	7.2×10^{-6}
0.6	4.4408×10^{-16}	5.9×10^{-6}	7.6×10^{-6}	7.0×10^{-6}
0.7	0	6.0×10^{-6}	7.8×10^{-6}	6.9×10^{-6}
0.8	4.4408×10^{-16}	5.8×10^{-6}	7.7×10^{-6}	6.8×10^{-6}
0.9	4.4408×10^{-16}	5.9×10^{-6}	9.0×10^{-6}	6.5×10^{-6}

Table 4. Comparison of absolute error of the several techniques in approximating $p(t)$ for test problem 2.

t	HPM	(1,3) F.E. [33]	(9,9) F. I. [33]	(3,9) ADI[33]
0.1	0	4.2×10^{-5}	2.1×10^{-5}	3.4×10^{-5}
0.2	0	4.1×10^{-5}	2.3×10^{-5}	3.6×10^{-5}
0.3	0	4.0×10^{-5}	2.4×10^{-5}	3.7×10^{-5}
0.4	0.000000×10^{-9}	4.4×10^{-5}	2.5×10^{-5}	3.7×10^{-5}
0.5	0	4.5×10^{-5}	2.6×10^{-5}	3.5×10^{-5}
0.6	0.000001×10^{-9}	4.3×10^{-5}	2.6×10^{-5}	3.4×10^{-5}
0.7	0.000003×10^{-9}	4.5×10^{-5}	2.4×10^{-5}	3.6×10^{-5}
0.8	0.000102×10^{-9}	4.3×10^{-5}	2.3×10^{-5}	3.4×10^{-5}
0.9	0.018829×10^{-9}	4.1×10^{-5}	2.3×10^{-5}	3.3×10^{-5}
	2.277106×10^{-8}	4.0×10^{-5}	2.2×10^{-5}	3.5×10^{-5}

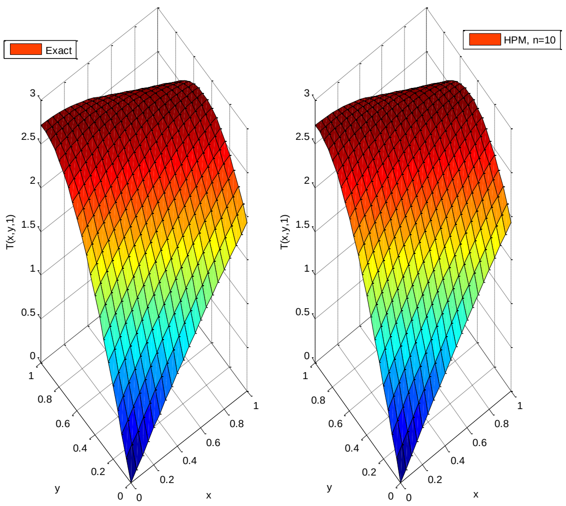

Fig. 3: The exact and numerical values of $T(x, y, 1)$.

Example 3. In this example, consider [36]:

$$\frac{\partial T}{\partial t} = \Delta T + p(t)T - yz(4x^2y^2 + 5y^2t - 4y^2 - y^2t^2 + 6)\exp(-2t - x^2); \quad 0 < x, y, z < 2, \ 0 < t < 1,$$

$$T(x, y, z, 0) = zy^3 \exp(-x^2); \quad 0 \le x, y, z \le 2,$$

$$T(0, y, z, t) = zy^3 \exp(-2t); \quad 0 \le y, z \le 2, \ 0 < t \le 1,$$

$$T(1, y, z, t) = zy^3 \exp(-1 - 2t); \quad 0 \le y, z \le 2, \ 0 < t \le 1,$$

$$T(x, 0, z, t) = 0; \quad 0 \le x, z \le 2, \ 0 < t \le 1,$$

$$T(x, 1, z, t) = z\exp(-x^2 - 2t); \quad 0 \le x, z \le 2, \ 0 < t \le 1,$$

$$T(x, y, 0, t) = 0; \quad 0 \le x, y \le 2, \ 0 < t \le 1,$$

$$T(x, y, 1, t) = y^3 \exp(-x^2 - 2t); \quad 0 \le x, y \le 2, \ 0 < t \le 1,$$

$$T(1, 2, 1, t) = 8\exp(-2t - 1); \quad 0 < t \le 1.$$

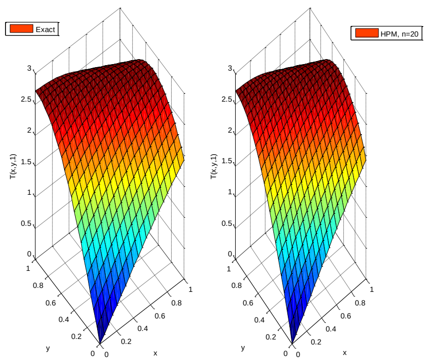

Fig. 4: The exact and numerical values of $T(x, y, 1)$.

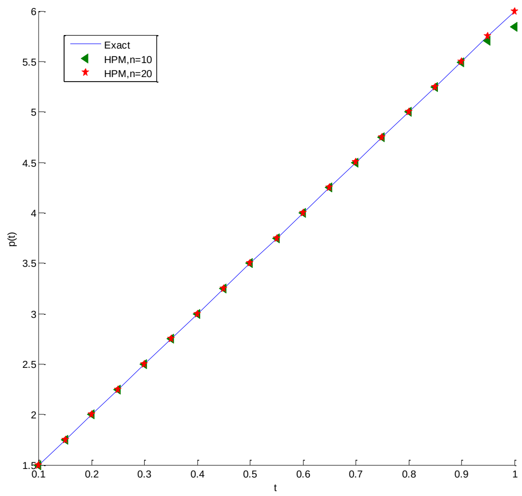

Fig. 5: The exact and numerical values of $p(t)$.

The exact solution of this problem is:

$$T(x, y, z, t) = zy^3 \exp(-x^2 - 2t),$$

$$P(t) = -t^2 + 5t - 4.$$

According to the equations (26)-(29), we obtain:

$$w_0(x, y, z, t) = zy^3 \exp(-x^2),$$

$$w_1(x, y, z, t) = zy^3 (2t - \frac{5}{2}t^2 + \frac{t^3}{3})\exp(-x^2),$$

$$w_2(x, y, z, t) = zy^3 (\frac{(2t - \frac{5}{2}t^2 + \frac{t^3}{3})^2}{2})\exp(-x^2),$$

$$\vdots$$

Thus from (30), we have in series from:

$$w(x, y, z, t) = (1 + (2t - \frac{5}{2}t^2 + \frac{t^3}{3}) + \frac{(2t - \frac{5}{2}t^2 + \frac{t^3}{3})^2}{2} + \cdots)zy^3 \exp(-x^2) = zy^3 \exp(-x^2 + 2t - \frac{5}{2}t^2 + \frac{t^3}{3}).$$

Now from equations (31) and (32) the exact values of $T(x, y, z, t)$ and $p(t)$ can be obtained. This problem bas been solved by the variational iteration method in [36].

4. Conclusion

In this paper, the HPM has been successfully employed to obtain an unknown parameter in a semi-linear partial differential equation with given initial and boundary conditions. This method constructs the solution of the problem as a rapid convergent series solution. Implementation of this method is easy and calculation of successive approximations is straightforward. Comparing with some numerical methods [33, 40, 41,42], the HPM solves the problem without any discretization of the variables, therefore is free from rounding off errors in the computational process. Also, it does not require large computer memory or time. The HPM provide the solution in a closed form while the mesh point techniques [33, 40, 41, 42] provide the approximation at mesh points only. Using Adomian decomposition method and the variational iteration method, the same results will be obtained. The advantage of the proposed method over Adomian decomposition method is that homotopy perturbation technique obtain the solution of the problem without calculating of Adomian's polynomials. Also computing the successive terms in HPM is much easier than the variational iteration method. The results show that the HPM is a powerful mathematical tool for finding the analytical solution of inverse problem.

5. References

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