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A single-input linear controller for complete synchronization of a delay financial hyperchaotic system

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Abstract. This paper is involved with the complete synchronization problems for two identical delay financial hyperchaotic system with different initial conditions, and a simple complete synchronization scheme only with a single linear input is proposed. Based on the Lyapunov stability theory, both linear feedback control and adaptive control approaches is derived to complete synchronization between two nearly identical delay financial hyperchaotic systems with unknown parameters is also studied. Numerical simulation results are showing the effectiveness of the proposed hyperchaotic synchronization method.

Keywords: delay financial hyperchaotic system, complete synchronization, linear feedback control, adaptive control.

1. Introduction

In recent years, chaos study has increasingly become an important topic in nonlinear areas. And chaotic synchronization has been developed extensively in the last few years. Since Pecora and Carrol proposed a successful method to synchronize two identical chaotic systems with different initial conditions [1]. Chaos synchronization has been widely explored and studied because of its potential applications in secure communication, chemical reactions, biological systems, information science, plasma technologies, etc. Meanwhile, many synchronization schemes have been proposed [2-7] such as complete synchronization, phase synchronization, anti-synchronization, lag synchronization, generalized synchronization and projective synchronization and so on.

Economic dynamics has recently become more prominent in mainstream economics [8, 9]. However, with the development of economy, the old financial chaotic system cannot meet the needs of the market. Therefore, more and more scholars improve it by adding an additional state variable [10-13]. Recently, a novel financial hyperchaotic system was brought up [14]. The dynamical behaviors of the new system are more complex, and effective controls are implemented. But there are barely any studies on its synchronization which is the main job we did in this paper. And in a practical way, a smaller number of controllers and simpler form of controllers are greatly practical. The linear feedback control technique was used to synchronize chaotic system[15,16] in various research works.

In this paper, we will adapt the single-input linear feedback controller to investigate the synchronization between these two identical time-delay financial hyperchaotic systems. Base on the Lyapunov stability theory and the adaptive control theory, the single-input adaptive controller associates with estimated update laws to synchronize two nearly identical delay hyperchaotic systems with unknown parameters.

We organize our paper as follows. In Section 2, the financial hyperchaotic system with time delay is present. In Section 3, Synchronization between two identical new delay hyperchaotic systems via single-input linear feedback control and adaptive feedback control laws are proposed and show the synchronization with unknown parameters in the response system. The numerical simulations are also presented. Finally, the conclusions are drawn in Section 4.

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2. The financial hyperchaotic system with time delay

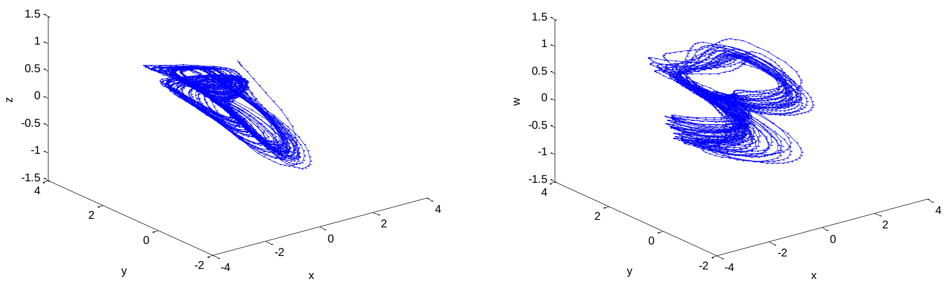
The novel financial hyperchaotic system is a non-delay financial hyperchaotic system in paper [14]. Based on the novel financial hyperchaotic system in this paper, we put forward a delay financial hyperchaotic system by plus a time delay on the average profit margin *w* in the first equation.

The model describes the time variations of four state variables: the interest rate *x*, the investment demand *y*, the price exponent *z*, and the average profit margin *w*.

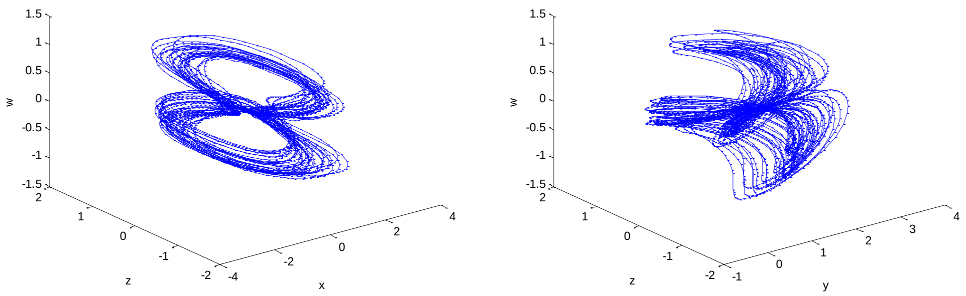
The financial hyperchaotic system with time delay is described as

$$\begin{cases} \dot{x}_1 = z_1 + (y_1 - a)x_1 + w_1(t-\tau), \\ \dot{y}_1 = 1 - by_1 - x_1^2, \\ \dot{z}_1 = -x_1 - cz_1, \\ \dot{w}_1 = -dx_1y_1 - kw_1, \end{cases} \tag{1}$$

where $\tau > 0$ is the time delay, a, b, c, d, k are the parameters of the system (1), and they are positive constants. When $\tau = 0$, system (1) is the financial hyperchaotic system [14]. For convenience, we call it delay financial hyperchaotic system. When the parameters are $a = 0.9$, $b = 0.2$, $c = 1.5$, $d = 0.2$, and $k = 0.17$, the four Lyapunov exponents of the system (4) calculated with Wolf algorithm are $L_1 = 0.034432$, $L_2 = 0.018041$, $L_3 = 0$, and $L_4 = -1.1499$. Figure 1(*a*)–(*d*) show the 3-dimensional phase portraits of financial hyperchaotic system (4).



(*a*) 3*D* view in the *x*-*y*-*z* space. (*b*) 3*D* view in the *x*-*y*-*w* space.



(*c*) 3*D* view in the *x*-*z*-*w* space. (*d*) 3*D* view in the *y*-*z*-*w* space.

Fig. 1**:** Phase portraits of hyperchaotic finance system (1).

3. Complete synchronization of delay financial hyperchaotic system

In this subsection, we will investigate the synchronization of two identical delay financial hyperchaotic system via a single feedback control only with one variable. For this purpose, the drive hyperchaotic time-delay system is chosen as (1), and the response system is given as follows:

$$\begin{cases} \dot{x}_2 = z_2 + (y_2 - a)x_2 + w_2(t-\tau) + u_1, \\ \dot{y}_2 = 1 - by_2 - x_2^2, \\ \dot{z}_2 = -x_2 - cz_2, \\ \dot{w}_2 = -dx_2y_2 - kw_2. \end{cases} \tag{2}$$

In which u_1 is the control law to be designed. Subtracting the drive system (1) from the response system (2), we have the following error dynamics:

$$\begin{cases} \dot{e}_1 = e_3 + (y_2 - a)e_1 + e_2x_1 + e_2(t-\tau) + u_1, \\ \dot{e}_2 = -be_2 - e_1(x_1 + x_2), \\ \dot{e}_3 = -e_1 - ce_3, \\ \dot{e}_4 = -dx_2e_2 - dy_1e_1 - ke_4, \end{cases} \tag{3}$$

where $e_1=x_2-x_1$, $e_2=y_2-y_1$, $e_3=z_2-z_1$, $e_4=w_2-w_1$, $u_1=-k_1e_1$ and k_1 is the positive feedback gain.

3.1. A single linear input for complete synchronization of a delay financial hyperchaotic system

Theorem 1 For enough large feedback gain k_1, the hyperchaotic systems (1) and (2) can be completely synchronized under the following linear control law:

$$u_1 = -k_1e_1. \tag{4}$$

Proof Construct a positive definite Lyapunov function as follows:

$$V = \frac{1}{2}\left[e_1^2 + e_2^2 + e_3^2 + e_4^2\right] + \beta\int_{t-\tau}^{t} e_4^2 \mathrm{d}t, \tag{5}$$

where $\beta>0$. The time derivative of the Lyapunov function along the trajectory is

$$\begin{aligned} \dot{V} &= e_1\dot{e}_1 + e_2\dot{e}_2 + e_3\dot{e}_3 + e_4\dot{e}_4 + \beta e_4^2 - \beta e_4^2(t-\tau) \\ &= e_1\left[e_3 + (y_2 - a)e_1 + e_2(t-\tau)\right] - k_1e_1^2 + e_2\left[-be_2 - e_1(x_1 + x_2)\right] \\ &\quad + e_3\left[-e_1 - ce_3\right] + e_4\left[-dx_2e_2 - dy_1e_1 - ke_4\right] + \beta e_4^2 - \beta e_4^2(t-\tau) \\ &= -(a - y_2 + k_1)e_1^2 - be_2^2 - ce_3^2 - (k-\beta)e_4^2 - x_2e_1e_2 - dy_1e_1e_4 \\ &\quad - dx_2e_2e_4 + e_1e_4(t-\tau) - \beta e_4^2(t-\tau). \end{aligned} \tag{6}$$

Since a chaotic system has bounded trajectories, there exists a positive constant M such that $|x_i|$, $|y_i|$, $|z_i|$, and $|w_i| \le M$ (i= 1, 2), thus,

$$\begin{aligned} \dot{V} &\le -(a - M + k_1 - \frac{1}{2}\lambda^{-1})e_1^2 - be_2^2 - ce_3^2 - (k-\beta)e_4^2 \\ &\quad + M|e_1||e_2| + dM|e_1||e_4| + dM|e_2||e_4| - (\beta - \frac{1}{2}\lambda)e_4^2(t-\tau) \\ &= -\left(|e_1|,|e_2|,|e_3|,|e_4|\right)P\left(|e_1|,|e_2|,|e_3|,|e_4|\right)^T - (\beta - \frac{1}{2}\lambda)e_4^2(t-\tau), \end{aligned} \tag{7}$$

where

$$P = \begin{bmatrix} k_1 + a_{11} & a_{12} & 0 & a_{14} \\ a_{12} & a_{22} & 0 & a_{24} \\ 0 & 0 & a_{33} & 0 \\ a_{14} & a_{24} & 0 & a_{44} \end{bmatrix} \tag{8}$$

$$a_{11} = a - M - \frac{1}{2}\lambda^{-1}, a_{12} = -\frac{1}{2}M, a_{14} = -\frac{1}{2}dM, a_{22} = b,$$

$$a_{24} = -\frac{1}{2}dM, a_{33} = c, a_{44} = k - \beta, \text{ and } \lambda>0.$$

Denote

$$\begin{aligned} &A_1 : k_1 > -a_{11}, \\ &A_2 : k_1 > a_{12}^2 / a_{22} - a_{11}, \\ &A_3 : k_1 > a_{12}^2 / a_{22} - a_{11}, \\ &A_4 : k_1 > \left(a_{12}^2 a_{33} a_{44} + 2a_{12} a_{14} a_{33} a_{24} + a_{14}^2 a_{22} a_{33} \right) \\ &\qquad / \left(a_{22} a_{33} a_{44} - a_{24}^2 a_{33} \right) - a_{11}. \end{aligned} \tag{9}$$

It is obvious that, for suitable values of λ, β, and k_1, the conditions in A_1-A_4 can be satisfied. Then the matrix P is positive definite, and $\dot{V}$ is negative definite. So one obtains $e_i \to 0$ as $t \to \infty$, namely, $\lim_{x\to\infty} \|e\| = 0$. It follows that the states of the response system (2) and the states of the drive system (1) are ultimately completely synchronized asymptotically.

In the numerical simulations, we assume the time delay τ=1, the feedback gain k_1=14.2, the parameters are a = 0.9, b = 0.2, c = 1.5, d = 0.2, and k = 0.17, and the initial values of the drive systems and the response systems chosen as (-1.5, -2.5, -3.5, -4.5) and (-4.5, -3.5, -2.5, -1.5). Figure 2 presents us the time evolution of synchronization error between drive and response systems.

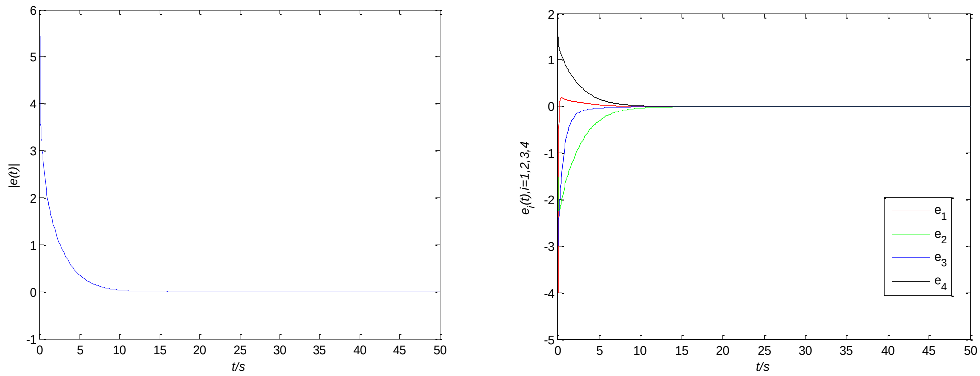


Fig. 2: The time evolution of synchronization error between drive and response systems.

3.2. A single adaptive feedback control for complete synchronization of a delay financial hyperchaotic system

Please acknowledge collaborators or anyone who has helped with the paper at the end of the text.

The linear feedback control laws proposed in Theorem 1 have a fixed feedback gain no matter what the initial error values start. This means that the feedback gain must be maximal to induce a kind of waste in practice. In this section, the adaptive control technology is applied to accomplish the synchronization of two identical delay hyperchaotic systems.

Theorem 2 The response system (2) can completely synchronize the drive system (1) globally and asymptotically, if the adaptive control law is selected as

$$u_1 = -k_1 e_1 , \tag{10}$$

where k_1 is an estimated feedback gain updated according to the following adaptation algorithm

$$\dot{k}_1 = e_1^2 \, . \tag{11}$$

Proof The positive definite Lyapunov function is chosen as follows:

$$V = \frac{1}{2}\left[e_1^2 + e_2^2 + e_3^2 + e_4^2 + (k_1 - l_1)^2 \right] + \beta \int_{t-\tau}^{t} e_4^2 \mathrm{d}t \, , \tag{12}$$

where $\beta, l_1 > 0$. Calculating the time derivative of the Lyapunov function (12) along the trajectory of system(3), we arrive at

$$\begin{aligned} \dot{V} &= e_1\dot{e}_1 + e_2\dot{e}_2 + e_3\dot{e}_3 + e_4\dot{e}_4 + (k_1 - l_1)\dot{k}_1 + \beta e_4^2 - \beta e_4^2(t-\tau) \\ &= e_1\left[e_3 + (y_2 - a)e_1 + e_2(t-\tau) \right] - l_1 e_1^2 + e_2\left[-be_2 - e_1(x_1 + x_2) \right] \\ &\quad + e_3\left[-e_1 - ce_3 \right] + e_4\left[-dx_2 e_2 - dy_1 e_1 - ke_4 \right] + \beta e_4^2 - \beta e_4^2(t-\tau) \\ &= -(a - y_2 + l_1)e_1^2 - be_2^2 - ce_3^2 - (k-\beta)e_4^2 - x_2 e_1 e_2 - dy_1 e_1 e_4 \\ &\quad - dx_2 e_2 e_4 + e_1 e_4(t-\tau) - \beta e_4^2(t-\tau). \end{aligned} \tag{13}$$

Because Eq. (13) is similar to Eq. (6), it is obvious that, for suitable values of λ, β and enough large value of l_1, $\dot{V}$ will be negative definite. So $e_i \to 0$ can be obtained as $t \to \infty$, namely, $\lim\limits_{x\to\infty}\|e\| = 0$. The complete synchronization of systems (1) and (2) with the controller (10) associated with (11) can be observed. This completes the proof.

For the numerical simulation in this example, the initial values of the drive systems and the response systems chosen as (-2, -3, -4, -5) and (-5, -4, -3, -2). Figure 3 show the time evolution of synchronization error between drive and response systems.

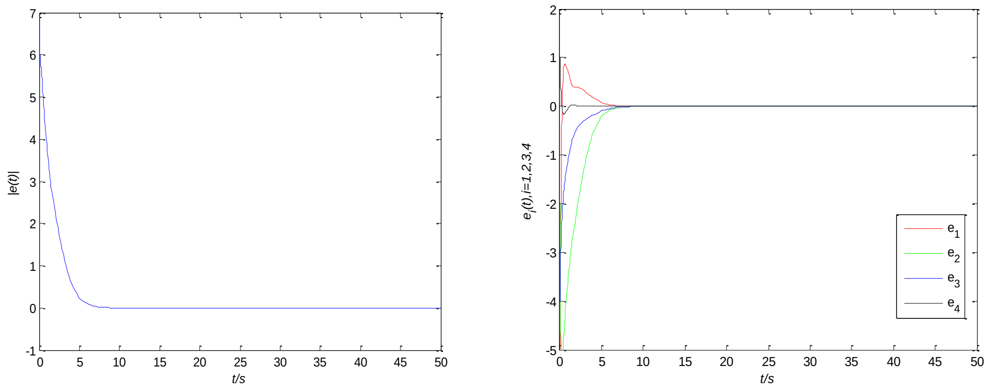


Fig. 3: The time evolution of synchronization error between drive and response systems with the estimation of feedback gain k_1.

3.3. A single adaptive feedback control for complete synchronization of a delay financial hyperchaotic system with uncertain parameters

In practical applications, some or all of the system parameters cannot be exactly known in advance. Therefore, the synchronization problem of chaotic systems in the presence of unknown parameters is essentially to be studied. In the following, the aforementioned adaptive control law is used to synchronize two nearly identical delay hyperchaotic systems with unknown parameters of the response system in spite of the difference in initial conditions.

For this case, the drive system is designed as (1), and the response system is modeled as follows:

$$\begin{cases} \dot{x}_2 = z_2 + (y_2 - a(t))x_2 + w_2(t-\tau) + u_1, \\ \dot{y}_2 = 1 - b(t)y_2 - x_2^2, \\ \dot{z}_2 = -x_2 - c(t)z_2, \\ \dot{w}_2 = -d(t)x_2y_2 - k(t)w_2, \end{cases} \tag{14}$$

where u_1 is a controller to be constructed, and $a(t)$, $b(t)$, $c(t)$, $d(t)$, $k(t)$ are unknown parameters which need to be estimated.

Theorem 3 Systems (1) and (14) will approach complete synchronization for any initial condition if the adaptive controller is taken as

$$u_1 = -k_1 e_1 \tag{15}$$

and the adaptive update laws of the feedback gain k_1 and the unknown parameters are chosen as the following adaptation algorithm:

$$\begin{aligned} &\dot{k} = e_1^2, \\ &\dot{a}(t) = x_2 e_1, \\ &\dot{b}(t) = y_2 e_2, \\ &\dot{c}(t) = z_2 e_3, \\ &\dot{d}(t) = x_2 y_2 e_4, \\ &\dot{k}(t) = w_2 e_4. \end{aligned} \tag{16}$$

Proof Subtracting the drive system (1) from the response system (14) yields the following error dynamical system:

$$\begin{cases} \dot{e}_1 = e_3 + y_2e_1 + e_2x_1 - (a(t)-a)x_2 - ae_1 + e_4(t-\tau) + u_1, \\ \dot{e}_2 = -(b(t)-b)y_2 - be_2 - (x_1+x_2)e_1, \\ \dot{e}_3 = -e_1 - (c(t)-c)z_2 - ce_3, \\ \dot{e}_4 = -(d(t)-d)x_2y_2 - dx_2e_2 - dy_1e_1 - (k(t)-k)w_2 - ke_4. \end{cases} \tag{17}$$

Construct a positive definite Lyapunov function as follows:

$$\begin{aligned} V = \frac{1}{2}\Big[& e_1^2 + e_2^2 + e_3^2 + e_4^2 + (k_1 - l_1)^2 + (a(t)-a)^2 + (b(t)-b)^2 \\ & + (c(t)-c)^2 + (d(t)-d)^2 + (k(t)-k)^2 \Big] \\ & + \beta\int_{t-\tau}^{t} e_4^2 \mathrm{d}t, \end{aligned} \tag{18}$$

where $\beta>0$ and $l_1> 0$.

Calculating the time derivative of the Lyapunov function (18) along the trajectory of system (17) associated with control law (15) and estimated parameters law (16), we have

$$
\begin{aligned}
\dot{V} &= e_1\dot{e}_1 + e_2\dot{e}_2 + e_3\dot{e}_3 + e_4\dot{e}_4 + (k_1 - l_1)\dot{k}_1 + \beta e_4^2 - \beta e_4^2(t-\tau) \\
&\quad + (a(t)-a)\dot{a}(t) + (b(t)-b)\dot{b}(t) + (c(t)-c)\dot{c}(t) \\
&\quad + (d(t)-d)\dot{d}(t) + (k(t)-k)\dot{k}(t) \\
&= e_1\left[e_3 + y_2 e_1 + e_2 x_1 - (a(t)-a)x_2 - a e_1 + e_4(t-\tau) - k_1 e_1\right] \\
&\quad + e_2\left[-(b(t)-b)y_2 - b e_2 - (x_1 + x_2)e_1\right] \\
&\quad + e_3\left[-e_1 - (c(t)-c)z_2 - c e_3\right] \\
&\quad + e_4\left[-(d(t)-d)x_2 y_2 - d x_2 e_2 - d y_1 e_1 - (k(t)-k)w_2 - k e_4\right] \\
&\quad + (k_1 - l_1)e_1^2 + \beta e_4^2 - \beta e_4^2(t-\tau) + (a(t)-a)x_2 e_1 + (b(t)-b)y_2 e_2 \\
&\quad + (c(t)-c)z_2 e_3 + (d(t)-d)x_2 y_2 e_4 + (k(t)-k)w_2 e_4 \\
&= -(a - y_2 + l_1)e_1^2 - b e_2^2 - c e_3^2 - (k-\beta)e_4^2 + x_1 e_1 e_2 \\
&\quad + e_1 e_4(t-\tau) - (x_1 + x_2)e_1 e_2 - d x_2 e_2 e_4 - d y_1 e_1 e_4 - \beta e_4^2(t-\tau).
\end{aligned}
\tag{19}
$$

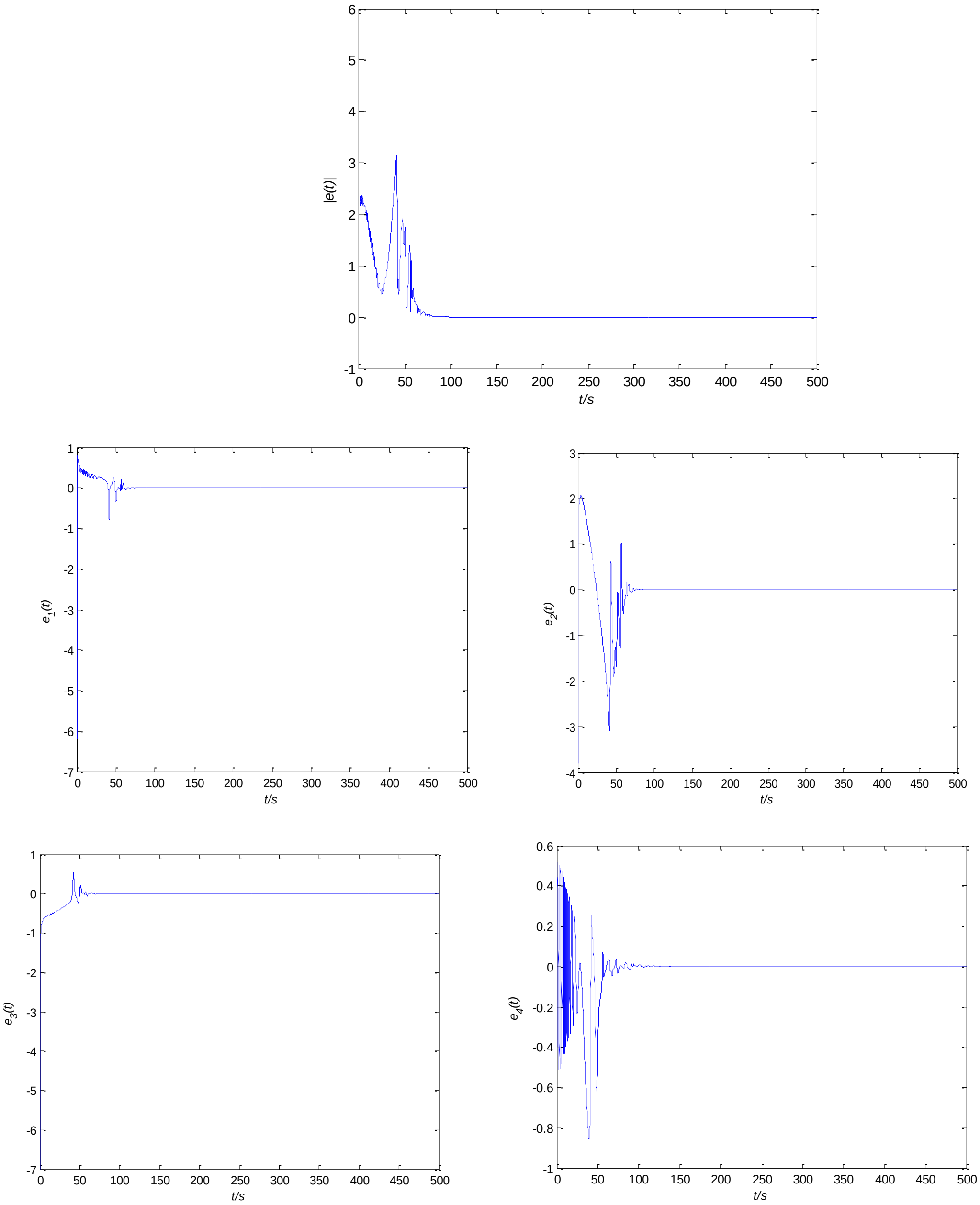


Fig.4: The time evolution of synchronization error between drive and response systems with the estimation of feedback gain k_1 and parameters.

It is obvious that Eq. (19) is similar to Eq. (6). Therefore, with suitable values of λ, β and sufficient large value of l_1, $\dot{V}$ will be negative definite, and the zero solution of the error dynamical system (17) can be globally asymptotically stable. So the complete synchronization of systems (1) and (14) with the controller (15) associated with (16) can be obtained. This completes the proof.

For this numerical simulation, we also assume the time delay τ=1, the initial values of the drive systems and the response systems chosen as (-3, -4, -5, -6) and (-6, -5, -4, -3). Figure 4 gives the time response of states for the drive system(1) and the response system(14). And Figure 5 shows the evolution of the five parameters estimation with time *t*.

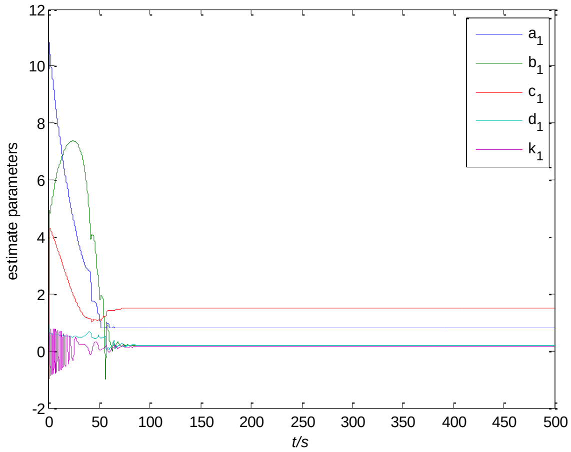


Fig. 5: Evolution of the five parameters estimation with time *t*.

4. Conclusion

This paper is concerned with state feedback control to synchronize two identical delay financial hyperchaotic systems. Based on the Lyapunov stability theory the drive and response systems could be synchronized with only a linear feedback controller, This paper only to control the interest rate of financial hyperchaotic system, not only easy to operate in practice, but also to reduce the financial risks effectively, thus which is important significance on using this method for applications. Numerical simulations show the effectiveness of the analytical results.

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