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An Application of Integer Linear Programming Problem in Tea Industry of Barak Valley of Assam, India under Crisp and Fuzzy Environments

Nabendu Sen[1], Laxminayaran Sahoo[2] and Asoke Kumar Bhunia [3]

[1] Department of Mathematics, Assam University, Silchar, India
E-mail: nsen08@yahoo.com

[2] Department of Mathematics, Raniganj Girls'College, Raniganj, India
E-Mail: lxsahoo@gmail.com

[3] Department of Mathematics, The University of Burdwan, Burdwan, India
E-Mail: bhuniaak@rediffmail.com

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Abstract. The current paper is concerned with a decision making problem of a tea industry of Barak Valley of Assam, India. The goal of this problem is to maximize the overall profit of the industry subject to the given resource constraints. Firstly, the problem has been formulated as an integer linear programming problem in crisp as well as in fuzzy environment. For fuzzification of the problem, some of the parameters are assumed to be different types of fuzzy numbers viz., triangular fuzzy number, parabolic fuzzy number and trapezoidal fuzzy number. Thereafter, the problem has been defuzzified by graded mean integration method to convert the crisp problem. The converted problem has been solved by the software LINGO13.0. Finally, to illustrate the problem and its results, a numerical example has been solved.

Keywords: Linear programming, Fuzzy Set, Industrial problem

1. Introduction

Due to competitive market situation as well as global economy of the market, the manager of an industry is always searching for the global solution for their problems to take appropriate (optimum) decision. Also due to uncertain situation, some of the parameters of the industry problem are not precise. These parameters may be imprecise. To tackle the problem with such imprecise numbers, generally stochastic, fuzzy and fuzzy stochastic approaches are applied and the corresponding problems are converted to crisp problems for solving those. In solving the converted problem, mathematical programming plays an important role.

In tea industry, there arise different types of decision making problems. To the best of our knowledge, no much work has been done on the applications of mathematical programming in tea industry. Deb (1999) first studied the transportation problem of tea industries of Barak Valley of Assam. Motivating from this work, Sinha and Sen (2011) developed a goal programming model for same industries. Recently, Sen (2012) extended the work of Sinha and Sen (2011) by considering several goals based on profit, production, demand, use of processing machines. In all these studies, the values of the system parameters were precise i.e. the values of the parameters were considered in crisp environment. However, in reality, these parameters may not be precise due to uncertainty (more precisely due to human error, improper storage facilities and other unexpected factors relating to environment).Therefore, in modeling of the problems, these parameters are considered as imprecise. To tackle the problem with such imprecise numbers, generally stochastic, fuzzy and fuzzy stochastic approaches are applied and the corresponding problems are converted to deterministic problems for solving those.

2. Representation of Fuzzy Number

The concept of 'fuzzy' was first introduced by Zadeh (1965) in his famous research paper "Fuzzy Sets" to represent the impreciseness/fuzziness or vagueness of a parameter mathematically. The approach of fuzzy set is an extension of classical set theory. In classical set theory, the membership of each element in relation to a set is assessed according to a crisp condition; an element either belongs to or does not belong to the set.

In contrast, a fuzzy set theory permits the gradual assessment of the membership of each element in relation to a set; this can be shown with the help of a membership function. In classical set theory, a membership function may act as an indicator function, mapping all elements to either 1 or 0. In a fuzzy set, a membership function is defined for each element of the referential set. After Zadeh (1965), the subject was enriched by Zimmermann (1976) and Bellman and Zadeh (1970).To tackle the problem with fuzzy parameters, first of all the problem is to be defuzzified. For this defuzzification, there are several techniques available in literature. In this connection, one may refer to the works of Ming et al. (2000), Yager et al. (1993) and Chen and Hsieh (1999). Among them, Chen and Hsieh (1999) proposed graded mean integration representation method on integral value of graded mean α-level of generalized fuzzy number for defuzzification of generalized fuzzy number.

2.1. Fuzzy Set

A fuzzy set $\tilde{A}$ in a universe of discourse X is defined as the set of pairs $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) : x \in X\}$, where $\mu_{\tilde{A}} : X \to [0,1]$ is a mapping and $\mu_{\tilde{A}}(x)$ is called the membership function of $\tilde{A}$ or grade of membership of x in $\tilde{A}$.

2.1.1. Convex Fuzzy Set

A fuzzy set $\tilde{A}$ is called convex if and only if for all $x_1, x_2 \in X$, $\mu_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \ge \min\{\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)\}$, where $\lambda \in [0,1]$.

2.2.2. Support of a fuzzy set

The support of a fuzzy set $\tilde{A}$ denoted by $S(\tilde{A})$ is the crisp set of all $x \in X$ such that $\mu_{\tilde{A}}(x) > 0$.

2.2.3. α -level Set

The set of elements that belong to the fuzzy set $\tilde{A}$ at least to the degree α is called the α -level set or α -cut and is given by $\tilde{A}_\alpha = \{x \in X : \mu_{\tilde{A}}(x) \ge \alpha\}$. If $\tilde{A}_\alpha = \{x \in X : \mu_{\tilde{A}}(x) > \alpha\}$, it is called strong α -level set or strong α -cut.

2.2.4. Normal Fuzzy Set

A fuzzy set $\tilde{A}$ is called a normal fuzzy set if there exists at least one $x \in X$ such that $\mu_{\tilde{A}}(x) = 1$.

2.3. Fuzzy Number

A fuzzy number is a special case of a fuzzy set. In this connection, different definitions and properties of fuzzy numbers are encountered in the literature. However, in all these definitions, the main theme is that a fuzzy number represents the notion of a set of real numbers 'closer to a' where 'a' is the number being fuzzified. A fuzzy number is a fuzzy set which is both convex and normal.

Here we shall discuss three different types of fuzzy numbers, viz. Triangular, and Parabolic and Trapezoidal fuzzy numbers.

2.3.1. Triangular Fuzzy Number (TFN)

A triangular fuzzy number $\tilde{A}$ is represented by the triplet (a_1, a_2, a_3) and is defined by its continuous membership function $\mu_{\tilde{A}}(x) : X \to [0,1]$ given by

$$\mu_{\tilde{A}}(x)=\begin{cases}\dfrac{x-a_1}{a_2-a_1} & \text{if } a_1\le x\le a_2\\ 1 & \text{if } x=a_2\\ \dfrac{a_3-x}{a_3-a_2} & \text{if } a_2\le x\le a_3\\ 0 & \text{otherwise}\end{cases}$$

2.3.2. Parabolic Fuzzy Number (PFN)

A parabolic fuzzy number $\tilde{A}$ is represented by the triplet (a_1,a_2,a_3) and is defined by its continuous membership function $\mu_{\tilde{A}}(x):X\to[0,1]$ given by

$$\mu_{\tilde{A}}(x)=\begin{cases}1-\left(\dfrac{a_2-x}{a_2-a_1}\right)^2 & \text{if } a_1\le x\le a_2\\ 1 & \text{if } x=a_2\\ 1-\left(\dfrac{x-a_2}{a_3-a_2}\right)^2 & \text{if } a_2\le x\le a_3\\ 0 & \text{otherwise}\end{cases}$$

2.3.3. Trapezoidal Fuzzy Number (TrFN)

A trapezoidal fuzzy number $\tilde{A}$ is represented by the quadruplet (a_1,a_2,a_3,a_4) and is defined by its continuous membership function $\mu_{\tilde{A}}(x):X\to[0,1]$ given by

$$\mu_{\tilde{A}}(x)=\begin{cases}\dfrac{x-a_1}{a_2-a_1} & \text{if } a_1\le x\le a_2\\ 1 & \text{if } a_2\le x\le a_3\\ \dfrac{a_4-x}{a_4-a_3} & \text{if } a_3\le x\le a_4\\ 0 & \text{otherwise}\end{cases}$$

In this paper, we have considered a problem of tea industry of Barak Valley of Assam. This problem has been formulated in crisp as well as in fuzzy environments. In case of fuzzy problem, the imprecise parameters have been represented by three different fuzzy numbers (viz. triangular fuzzy number, parabolic fuzzy numbers and trapezoidal fuzzy number).Then the corresponding problems have been defuzzified by graded mean integration method (Chen and Hsieh, 1999). Both the problems (crisp problem and defuzzified problem) are integer linear programming problem. These problems have been solved by LINGO13.0. Finally; these problems have been illustrated with the numerical example.

2.4. Generalized Fuzzy Number

The generalized fuzzy number $\tilde{A}$ with membership function $\mu_{\tilde{A}}(x)$ (Figure 1) exhibits a fuzzy subset of the real line $\mathbb{R}$, where

$$\mu_{\tilde{A}}(x)=\begin{cases}L(x) & a_1\le x\le a_2\\ 1 & a_2\le x\le a_3\\ R(x) & a_3\le x\le a_4\\ 0 & \text{otherwise}\end{cases}$$

Where $L(x)$ and $R(x)$ are continuous functions of x. Moreover, $L(x)$ is strictly monotonic increasing and $R(x)$ strictly monotonic decreasing function of x in $a_1\le x\le a_2$ and $a_3\le x\le a_4$ respectively.

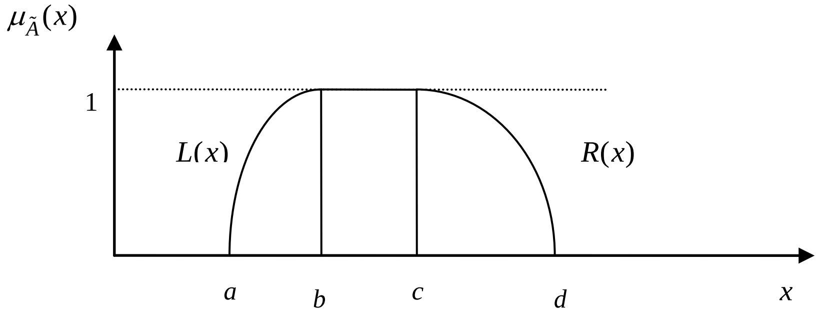

Fig. 1: Generalized fuzzy number

2.4.1. Graded Mean Integration Value of Fuzzy Number

For the generalized fuzzy number $\tilde{A}$ with membership function $\mu_{\tilde{A}}(x)$, according to Chen and Hsieh (1999), the Graded Mean Integral Value $P_{dGw}(\tilde{A})$ of $\tilde{A}$ is given by

$$P_{dGw}(\tilde{A}) = \frac{\int_0^1 x\left\{(1-w)L^{-1}(x) + wR^{-1}(x)\right\}dx}{\int_0^1 x dx}$$

$$= 2\int_0^1 x\left\{(1-w)L^{-1}(x) + wR^{-1}(x)\right\}dx,$$

Where, the pre-assigned parameter $w \in [0,1]$ refers the degree of optimism. $w = 1$,represents an optimistic decision makers' point of view, $w = 0$ represents a pessimistic optimistic decision makers' point of view and $w = 0.5$ indicates a moderately optimistic decision makers' point of view.

For Triangular Fuzzy Number (TFN) $\tilde{A} = (a_1, a_2, a_3)$, $P_{dG0.5}(\tilde{A}) = \frac{1}{6}(a_1 + 4a_2 + a_3)$

For Parabolic Fuzzy Number (PFN) $\tilde{A} = (a_1, a_2, a_3)$, $P_{dG0.5}(\tilde{A}) = \frac{1}{15}(4a_1 + 7a_2 + 4a_3)$

For Trapezoidal Fuzzy Number (TrFN) $\tilde{A} = (a_1, a_2, a_3, a_4)$, $P_{dG0.5}(\tilde{A}) = \frac{1}{6}(a_1 + 2a_2 + 2a_3 + a_4)$

3. Assumptions and Notations:

The following assumptions and notations have been used throughout the paper.

3.1. Assumptions:

(i) The Company produces more than one number of varieties of tea.

(ii) The demand of each variety of tea is deterministic and its value is a certain fraction of production of that variety.

(iii) The different varieties of tea are processed in different machines (say *m* machines).

(iv)There is a particular amount of budget allocation for total production.

(v) The produced different varieties of tea will be stored in a warehouse.

3.2. Notations:

n The number of varieties of tea grade

M	Number of machines in which different varieties of tea are processed.
x_i	Produced quantity of i-th grade of tea, $i = 1, 2, ..., n$
α_i	Unit profit of i-th grade of tea.
γ_i	Cost of production per unit of i-th grade of tea
t_{ji}	Processing time for unit production of i-th grade of tea in j-th machine,j=1,2,…n
χ_i	Budget per unit for i-th grade of tea
A_i	Storage space of X_i ($X_i = 25x_i$)
l_i	Fraction of production of i-th grade of tea
χ	Total budget allocation
β_j	Available time for j-th machine
P	Expected total production
E	The total expenditure (factory expenditure)
D	Expected total demand
A	Total space of ware house of the tea estate
$\mu_{\tilde{A}}(x)$	membership function of x of fuzzy set $\tilde{A}$
$L(x)$	left shape function of x of fuzzy set $\tilde{A}$
$R(x)$	right shape function of x of fuzzy set $\tilde{A}$
$P_{dGw}(\tilde{A})$	graded mean integral value of $\tilde{A}$ with degree of optimism w

4. Mathematical formulation of the problem

Let us consider a tea estate, viz. XYZ tea estate located in Barak valley of Assam. The company produces *n* varieties of tea according to the size. The production of these grades of tea undergoes different process in *m* machines. It is found from the observation that these grades of tea take different time in respective machines. The total production of all varieties of tea depends upon the expenditure (factory expenditure) and the demand on different markets around the locality of the said tea industry and outside also. The objective of the problem is to maximize the overall profit subject to the given resource constraints.

According to the assumptions and notations, the mathematical form of the optimization problem in crisp environment is as follows:

$$\text{Maximize } z = \sum_i \alpha_i x_i \tag{1}$$

Subject to the constraints

$$\sum_i x_i \geq P,\ \sum_i l_i x_i \geq D,\ \sum_i \gamma_i x_i \leq E,\ \sum_i \chi_i x_i \leq \chi,\ \sum_i t_{1i} x_i \leq \beta_1,\ \sum_i t_{2i} x_i \leq \beta_2,\ \sum_i A_i x_i \leq A$$

$x_i \geq 0$ and are integers, $i = 1, 2, ...n$.

If some of the parameters of the problems are imprecise and these imprecisenesses are represented by fuzzy valued numbers, then the problem (1) reduces to

$$\text{Maximize } \tilde{z} = \sum_i \tilde{\alpha}_i x_i \tag{2}$$

subject to $\sum_i x_i \ge P$, $\sum_i \tilde{l}_i x_i \ge \tilde{D}$, $\sum_i \tilde{\gamma}_i x_i \le \tilde{E}$, $\sum_i \chi_i x_i \le \chi$, $\sum_i \tilde{t}_{1i} x_i \le \tilde{\beta}_1$, $\sum_i \tilde{t}_{2i} x_i \le \tilde{\beta}_2$, $\sum_i A_i x_i \le A$

$x_i \ge 0$ and are integers, $i = 1, 2, ...n$

Here $\tilde{z}, \tilde{D}, \tilde{E}, \tilde{\beta}_j$ be the fuzzy valued total profit, total demand, total expenditure, available time for *j*-th machine respectively. Again $\tilde{\gamma}_i, \tilde{t}_{ij}, \tilde{l}_i$ represent fuzzy valued cost of production per unit, time taken for unit production in *j*-th machine and fraction of fraction of *i*-th grade of tea respectively.
Now to convert the problem in (2) in crisp form, we use graded mean defuzzification method. By graded mean defuzzification method, problem (2) reduces to the following form:

$$P_{dGw}(\tilde{z}) = \sum_i P_{dGw}(\tilde{\alpha}_i) x_i \qquad (3)$$

Subject to $\sum_i x_i \ge P$, $\sum_i P_{dGw}(\tilde{l}_i) x_i \ge P_{dGw}(\tilde{D})$, $\sum_i P_{dGw}(\tilde{\gamma}_i) x_i \le P_{dGw}(\tilde{E})$, $\sum_i \chi_i x_i \le \chi$,

$\sum_i P_{dGw}(\tilde{t}_{ji}) x_i \le P_{dGw}(\tilde{\beta}_j)$, $\sum_i A_i x_i \le A$.

Here $P_{dGw}(\tilde{z})$ is the defuzzified /GMIV (with degree of optimism w) valued objective function (total profit).

Now our objective is to determine the optimal values $(x_i, i = 1, 2, ..., n)$ of production of different grades of tea along with the optimal profit by solving the problems (1) and (3).Both the problems (1) and (3) are all integer linear programming problems(LPPS).These problems can be solved by different ways. Here we have used LINGO 13.0 for solving both the problems.

5. Numerical Illustration

As an illustration of our proposed approaches, we have solved all integer LPPs with fuzzy valued as well as precise/fixed valued parameters. In case of fuzzy valued parameters, we have used three different numbers (TFN), parabolic fuzzy number (PFN) and Trapezoidal fuzzy number (TrFN) for each imprecise parameter. The crisp as well as fuzzy valued data are given in Tables 1-6. Considering all these data, we have solved the crisp problem, fuzzy problem with TFN, PFN and TrFN by LINGO13.0. and corresponding results have been displayed in Table 7. From Table 7, it is seen that the optimal profit is highest in crisp environment than those in fuzzy environment.

Table-1: Crisp values of different parameters

Parameters	i = 1	i =2	i =3	i =4	i =5
α_i ()	28	15.4	12	19	24
l_i	0.78	0.68	0.85	0.82	0.94
γ_i	94	89	76	68	64
χ_i ($)	118	112	106	100	98
t_{1i}	0.00095	0.00085	0.00080	0.00076	0.00072
t_{2i}	0.00060	0.00055	0.00050	0.00048	0.00045
A_i (sq. meter)	0.45	0.67	0.63	0.74	0.86
P= 2168, D= 1890, A=1500 sq. meter, E= $1730000, χ =$230000					

Table-2: Numerical data for fuzzy valued profit parameters $\tilde{\alpha}_i$

	$\tilde{\alpha}_1$	$\tilde{\alpha}_2$	$\tilde{\alpha}_3$	$\tilde{\alpha}_4$	$\tilde{\alpha}_5$
TFN	(25,28,30)	(13, 15.4,18)	(10,12,15)	(15,19,22)	(20,24,27)
PFN	(25,28,30)	(13, 15.4,18)	(10,12,15)	(15,19,22)	(20,24,27)
TrFN	(24,26,27,28)	(13,14,15.4,16)	(10,11,12,14)	(16,17,19,20)	(21,23,24,26)

Table-3: Numerical data for fuzzy valued parameters $\tilde{l}_i$ and $\tilde{D}$

	$\tilde{l}_1$	$\tilde{l}_2$	$\tilde{l}_3$	$\tilde{l}_4$	$\tilde{l}_5$	$\tilde{D}$
TFN	(0.70,0.78,0.80)	(0.60,0.68,0.75)	(0.70,0.85,0.95)	(0.75,0.82,0.90)	(0.90,0.94,0.99)	(1870,1890,1900)
PFN	(0.70,0.78,0.80)	(0.60,0.68,0.75)	(0.70,0.85,0.95)	(0.75,0.82,0.90)	(0.90,0.94,0.99)	(1870,1890,1900)
TrFN	(0.72, 0.74,0.78,0.80)	(0.64,0.65,0.68, 0.74)	(0.80,0.83,0.85, 0.88)	(0.79,0.82,0.83, 0.85)	(0.90,0.93,0.94, 0.97)	(1885,1888,1890, 1895)

Table-4: Numerical data for fuzzy valued parameters and $\tilde{E}$

	$\tilde{\gamma}_1$	$\tilde{\gamma}_2$	$\tilde{\gamma}_3$	$\tilde{\gamma}_4$	$\tilde{\gamma}_5$	$\tilde{E}$
TFN	(90,94,100)	(85,89,95)	(70,76,80)	(60,68,75)	(60,64,70)	(172000,173000,173500)
PFN	(90,94,100)	(85,89,95)	(70,76,80)	(60,68,75)	(60,64,70)	(172000,173000,173500)
TrFN	(90,93,94,95)	(86,88,89,93)	(74,75,76,78)	(64,67,68,70)	(62,63,64,66)	(172700,172900,173000,173300)

Table-5: Numerical data for fuzzy valued parameters and $\tilde{\beta}_1$

	$\tilde{t}_{11}$	$\tilde{t}_{12}$	$\tilde{t}_{13}$	$\tilde{t}_{14}$	$\tilde{t}_{15}$	$\tilde{\beta}_1$
TFN	(0.00090, 0.00095, 0.00098)	(0.00080, 0.00085, 0.00090)	(0.00075, 0.00080, 0.00084)	(0.00070, 0.00076, 0.00080)	(0.00070, 0.00072, 0.00076)	(120, 130, 139)
PFN	(0.00090, 0.00095, 0.00098)	(0.00080, 0.00085, 0.00090)	(0.00075, 0.00080, 0.00084)	(0.00070, 0.00076, 0.00080)	(0.00070, 0.00072, 0.00076)	(120, 130, 139)
TrFN	(0.00088, 0.00090, 0.00095, 0.00098)	(0.00078, 0.00080, 0.00085 ,0.00090)	(0.00073, 0.00075, 0.00080, 0.00084)	(0.00066, 0.00070, 0.00076, 0.00081)	(0.00068, 0.00070, 0.00072, 0.00078)	(125, 130, 135, 142)

Table-6: Numerical data for fuzzy valued parameters and $\tilde{\beta}_2$

	$\tilde{t}_{21}$	$\tilde{t}_{22}$	$\tilde{t}_{23}$	$\tilde{t}_{24}$	$\tilde{t}_{25}$	$\tilde{\beta}_2$
TFN	(0.00055, 0.00060, 0.00064)	(0.00050, 0.00055, 0.00059)	(0.00045, 0.00050, 0.00060)	(0.00045, 0.00050, 0.00054)	(0.00040, 0.00045, 0.00051)	(118,125,130)
PFN	(0.00055, 0.00060, 0.00064)	(0.00050, 0.00055, 0.00059)	(0.00045, 0.00050, 0.00060)	(0.00045, 0.00050, 0.00054)	(0.00040, 0.00045, 0.00051)	(118,125,130)
TrFN	(0.00053, 0.00055, 0.00060, 0.00064)	(0.00048, 0.00055, 0.00060, 0.00062)	(0.00043, 0.00050, 0.00052, 0.00060)	(0.00043, 0.00045, 0.00048, 0.00053)	(0.00038, 0.00042, 0.00045, 0.000500)	(115,120,125,132)

Table-7: Computational results for different types of data

	Precise/Fixed	**TFN**	**PFN**	**TrFN**
Optimal Profit	55187	51278.89	44953.58	41900.77
x_1	870	734	490	422
x_2	0	0	0	0
x_3	0	295	769	903
x_4	65	0	4	1
x_5	1233	1144	921	861
$\sum_{i=1}^{5} x_i$	2168	2173.000	2184.000	2187.000
$\sum_{i=1}^{5} l_i x_i$	1890.92	1888.340	1887.380	1889.400
$\sum_{i=1}^{5} \gamma_i x_i$	165112	165154.4	164059.2	162535.0
$\sum_{i=1}^{5} \chi_i x_i$	229994	229994.0	229992.0	229992.0
$\sum_{i=1}^{5} t_{1i} x_i$	1.763660	1.757620	1.746688	1.711797
$\sum_{i=1}^{5} t_{2i} x_i$	1.108050	1.105880	1.106192	1.082979
$\sum_{i=1}^{5} A_i x_i$	1499.980	1499.990	1499.990	1499.990

6. Conclusions

In our real life situation, uncertainty plays an important role in formulation a decision making problem of any industry. Due to this uncertainty, several parameters of any decision making problem may be imprecise. The impreciseness of any parameter is represented different ways. Among these, one way is to represent the imprecise parameter by fuzzy number. In this paper, for the first time, we have formulated and solved a decision making problem of tea estate of Barak Valley of Assam, India, in crisp and fuzzy environments. In fuzzy environment, the imprecise parameters are represented by three different types of fuzzy numbers, viz. triangular, parabolic and trapezoidal fuzzy numbers. From computational result, it is observed that the optimal profit obtained for triangular fuzzy number representation is highest in comparison to that of parabolic and trapezoidal number representations.

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