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Modified Projective Synchronization of New Hyper-chaotic Systems

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Abstract. In this paper, we discuss modified projective synchronization of two identical new hyper-chaotic systems by active non linear control. And the stability results derived in this paper for modified projective synchronization of two identical new hyper-chaotic systems are established using Lyapunov stability. Since the Lyapunov exponents are not required for these calculations, the active nonlinear control method very effective and convenient two achieve modified projective synchronization of two identical new hyper-chaotic systems. Numerical simulations are shown to validate and demonstrate the effectiveness of the Modified projective synchronization schemes derived in this paper.

Keywords: Active nonlinear control modified projective synchronization, new hyper-chaotic systems, Lyapunov Stability.

1. Introduction

Chaos synchronization is a phenomenon that occurs when two or more chaotic oscillators are coupled or when a chaotic oscillator derives another chaotic oscillator and it has wide application in several fields such as nonlinear science, biological science, ecological science and secure communication etc.

In 1990 Pecora and Carroll [1] induced a method to synchronize two identical chaotic systems with different initial conditions; the research of chaos synchronization has been intensively studied in last two decades. There are different method have been proposed for achieving the chaos synchronization such as PC method [1], OGY method, active control, parameter adaptive method, coupling control, sliding mode control, etc.

However, In most of the chaos synchronization have been proposed the master (drive) and slave (response) has used, some of the aforementioned methods are primarily concerned with the synchronization of the chaotic systems with low dimensional attractor, characterized by one positive Lyapunov exponent. Because of chaotic systems with higher dimensional attractor generating more complex dynamics, it is believed that hyper-chaotic systems have much wider applications. Recently hyper-chaotic systems were also considered with quickly increasing interest. Hyper-chaotic system is usually classified as a chaotic system with more than one positive Lyapunov exponent. Synchronization strategies of hyper-chaotic systems have been investigated in [2].

So far, many types of chaos synchronization method have been presented such as complete synchronization, Phase synchronization, lag synchronization, generalised synchronization and projective synchronization [3], etc. Recently generalized projective synchronization which has a more general form of synchronization has been studied [4]. And complete synchronization is characterized by the convergence of the two chaotic trajectories and has been observed in mutually coupled, unidirectionally coupled and even noise induced chaotic oscillator. Projective synchronization is characterized by the fact that the master and slave systems synchronize up to a scaling factor, α .Complete synchronization and anti-synchronization are the special cases of generalized projective synchronization where the scaling factor $\alpha = 1, \alpha = -1$, respectively. Very

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recently, Li [5] introduced a modified projective synchronization (MPS), where the responses of the synchronized dynamical states synchronize up to a constant scaling matrix.

This paper addresses the modified projective synchronization of two identical new Hyperchaotic systems by active control.

2. Methodology

Consider the chaotic system described by

$$\dot{x} = Ax + f(x) \tag{2.1}$$

where $x \in R^n$ are the states of the system, A is the $n \times n$ matrix of the system parameters and $f : R^n \to R^n$ is the nonlinear part of the system. We consider the system (2.1) as the master or drive system.

As the slave or response system, we consider the following chaotic system described by the dynamics.

$$\dot{y} = By + g(y) + u \tag{2.2}$$

where $y \in R^n$ are the states of the system, B is the $n \times n$ matrix of the system parameters, $g : R^n \to R^n$ nonlinear part of the system and $u \in R^n$ is the controller of the slave system.

If $A = B$ and $f = g$ then x and y are the states of the two identical chaotic systems. If $A \neq B$ or $f \neq g$ then x and y are the states of two different chaotic systems.

In the nonlinear feedback control approach, we design the feedback controllers u, which modified projective synchronized the states of the master (2.1) and slave system (2.2) for all initial conditions $x(0), y(0) \in R^n$.

We define the modified projective synchronization error as

$$e = x - \alpha y \tag{2.3}$$

Then the modified projective error dynamics is obtained as

$$\dot{e} = Ax + f(x) - \alpha[By + g(y) + u] \tag{2.4}$$

To find a controller u to stabilize the error dynamics (2.4) for all initial conditions $e(0) \in R^n$

$\lim_{t\to\infty} \|e(t)\| = 0$ for all initial conditions

$$e(0) \in R^n \tag{2.5}$$

We take the Lyapunov function as

$$V(e) = e^T Pe \tag{2.6}$$

Where P is positive definite matrix and $V : R^n \to R^n$ is a positive definite function.

We find a controller u such that

$$\dot{V}(e) = -e^T Qe \tag{2.7}$$

Where Q is positive definite matrix, and $\dot{V} : R^n \to R^n$ is a negative definite function.

Thus, by Lyapunov stability theory [6], the error dynamics (2.4) is globally exponentially stable and hence the condition (2.5) will be satisfied for all initial conditions $e(0) \in R^n$. Then the states of the master and slave systems will be modified projective synchronized.

3. Error estimation

The master and slave systems are given below respectively.

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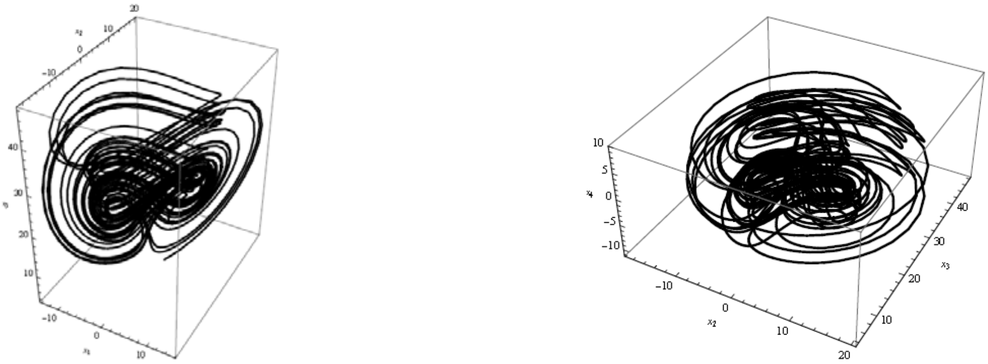
$$\begin{aligned}
\dot{x}_1 &= a(x_2 - x_1) + x_4 \\
\dot{x}_2 &= bx_1 + cx_2 - x_1x_3 \\
\dot{x}_3 &= x_1^{\,2} - hx_3 + x_1x_2 \\
\dot{x}_4 &= -dx_1 + x_2
\end{aligned} \tag{3.1}$$

$$\begin{aligned}
\dot{y}_1 &= a(y_2 - y_1) + y_4 + u_1 \\
\dot{y}_2 &= by_1 + cy_2 - y_1y_3 + u_2 \\
\dot{y}_3 &= y_1^{\,2} - hy_3 + y_1y_2 + u_3 \\
\dot{y}_4 &= -dy_1 + y_2 + u_4
\end{aligned} \tag{3.2}$$



(a) (b)

Figure.1. Hyperchaotic attractor (a) $x_1 - x_2 - x_3$ space (b) $x_2 - x_3 - x_4$ space.

where u_i ($i = 1,2,3,4$) are the nonlinear control such that two chaotic systems can be synchronized with a scaling factor α. Define the error signals as $e_i = x_i - \alpha y_i (i = 1,2,3,4)$.

Then the error dynamics is given as

$$\begin{aligned}
\dot{e}_1 &= a(e_2 - e_1) + e_4 - \alpha u_1 \\
\dot{e}_2 &= be_1 + ce_2 - x_1x_3 + \alpha y_1y_3 - \alpha u_2 \\
\dot{e}_3 &= (x_1^{\,2} - \alpha y_1^{\,2}) - he_3 + x_1x_2 - \alpha y_1y_2 - \alpha u_3 \\
\dot{e}_4 &= -de_1 + e_2 - \alpha u_4
\end{aligned} \tag{3.3}$$

For two identical chaotic systems without control $(u_i = 0)$, if the initial condition of two systems is different, the trajectories of the two identical systems will quickly separate from each other and become independent. However, for the two controlled chaotic systems, the two systems will approach synchronization for any initial condition by appropriate control.

The active control is defined as.

$$\begin{aligned}
u_1 &= \frac{1}{\alpha}(-v_1) \\
u_2 &= \frac{1}{\alpha}(-x_1x_3 + \alpha y_1y_3 - v_2) \\
u_3 &= \frac{1}{\alpha}(x_1^{\,2} - \alpha y_1^{\,2} + x_1x_2 - \alpha y_1y_2 - v_3) \\
u_4 &= \frac{1}{\alpha}(-v_4)
\end{aligned} \tag{3.4}$$

Then the error dynamics becomes

$$\begin{aligned} \dot{e}_1 &= a(e_2 - e_1) + e_4 + v_1 \\ \dot{e}_2 &= be_1 + ce_2 + v_2 \\ \dot{e}_3 &= -he_3 + v_3 \\ \dot{e}_4 &= -de_1 + e_2 + v_4 \end{aligned} \tag{3.5}$$

Where v_1, v_2, v_3, v_4 are function of e_1, e_2, e_3, e_4 and the system e_1, e_2, e_3 and e_4 converge to zero as t tends to infinity. This implies that the synchronization is achieved and chose

$$\begin{aligned} v_1 &= -ae_2 - e_4 \\ v_2 &= -be_1 - ce_2 - e_2 \\ v_3 &= 0 \\ v_4 &= de_1 - e_2 - e_4 \end{aligned} \tag{3.6}$$

The error dynamics becomes

$$\dot{e}_1 = -ae_1, \dot{e}_2 = -e_2, \dot{e}_3 = -he_3, \dot{e}_4 = e_4 \tag{3.7}$$

Consider the Lyapunov function as

$$V = \frac{1}{2}(e_1^2 + e_2^2 + e_3^2 + e_4^2)$$

The time derivative of the Lyapunov function along the trajectories is

$$\dot{V} = e_1\dot{e}_1 + e_2\dot{e}_2 + e_3\dot{e}_3 + e_4\dot{e}_4 = -[ae_1^2 + e_2^2 + he_3^2 + e_4^2]$$

Thus by Lyapunov stability theory [6], the error dynamical system converge to zero and hence the synchronization is achieved.

4. Numerical Simulation

The initial condition of the master and slave system are taken as $x_1(0) = 2, x_2(0) = 2.5, x_3(0) = 4, x_4(0) = 3.5$ and $y_1(0) = 1, y_2(0) = 1, y_3(0) = -3, y_4(0) = -3$ respectively and $\alpha = 3$ then the initial condition of error dynamics is $e_1(0) = -1, e_2(0) = -0.5, e_3(0) = 13, e_4(0) = 12.5$. MPS the systems (3.1) and (3.2) via active control (3.4) are shown in figure-2. Figure-2 shows the time response of the modified projective errors $e_1, e_2, e_3, e_4 \to 0$ as $t \to \infty$ implying that all the states tend to synchronized in a proportional.

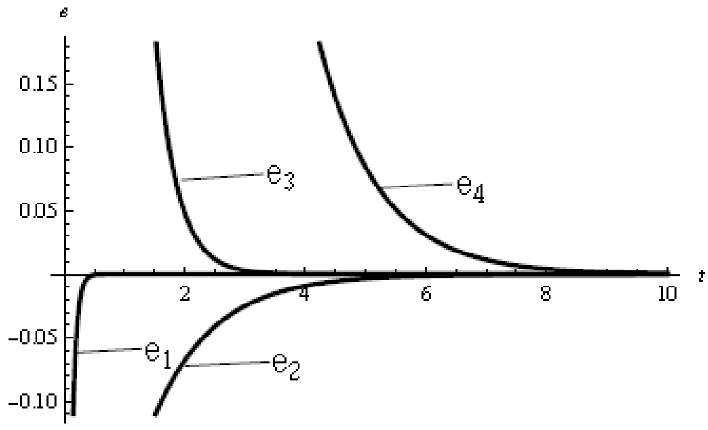


Fig.2. Error signals between master and slave systems.

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