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Application of an Ant Colony System – Node (ACS – N) algorithm in the Vehicle Routing Problem (VRP)

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Abstract. Ant colony Optimization (ACO) is a relatively new class of metaheuristic search techniques for hard optimization problems. In this paper we focus on the definition and minimization of the objective function of the VPR using an Ant Colony System – Node (ACS – N) algorithm. The (ACS – N) algorithm is implemented for an eight node graph with respective demands. Moreover, in this paper we study the effect of the number of the ants to the value of the objective function.

Keywords: Ant Colony Optimization (ACO), Vehicle Routing Problem (VRP), Ant System (AS).

1. Introduction

The Vehicle Routing Problem (VRP), which was introduced by Dantzig and Ramser [1], is an important combinatorial optimization problem in the field of operations management and logistics.

The VRP is an NP-hard problem, and can be described as follows: items are to be delivered to a set of customers by a fleet of vehicles from a common depot. The locations of the customers and the depot are given. The aim is to determine a set of vehicle routes:

i) of minimum total cost,
ii) starting and ending at the common depot,

and

iii) each customer is served exactly once by exactly one vehicle,
iv) the total duration of each route must not exceed the constraint
v) the total demand of any route does not exceed the capacity of the vehicle

A number of researchers developed heuristics techniques to solve the VRP such as tabu search (TS) [2], simulated annealing (SA) [3] and genetic algorithms (GA) [4].

More recently Ant Colony Optimization (ACO) algorithm has been evolved for solving difficult Optimization problems.

ACO is a new Metaheuristic that is based on the foraging behaviour of the real ants. One of the main ideas behind this approach is that the ants can communicate with one another through indirect means by making modification to the concentration of highly volatile chemical called pheromones in their immediate environment [5].

In this paper we present the Ant Colony System – Node (ACS-N) algorithm through which we control the pheromone. In most cases, the structure of the problem described with the use of a graph. The use of the (ACS-N) algorithm demands the pheromone to be placed at the nodes of the graph saving memory of $O\left(n^2\right)$ comparing with the classic structure of the pheromone that it is used at the basic algorithm ACO (AS, ACS, Max – Min Ant System).

The paper has the following structure: in the section 2 it is presented a sort introduction in the Ant Colony optimization and especially we insert a stochastic model, concerning the pheromone distribution through difference equations.

Section 3 we give the problem formulation of the VRP problem. Section 4 we present the mathematical model of (ACS-N) algorithm. Section 6 we present the case study. Finally in section 7 we draw some conclusions.

2. ANT COLONY OPTIMIZATION

The Ant Colony Optimization (ACO) was proposed by Dorigo and co-authors [6] use artificial ants, called agents, to solving various Combinatorial Optimization problems. The artificial ants to mimic the cooperative of real ants behaviour, plus additional capabilities that make them more effective, such as a memory of past actions.

Generally, an ant walking from the nest to food sources and vice versa deposits on the ground a chemical substance called pheromone, forming a pheromone trail.

We suppose that there are two possible paths to reach a food source, as shown in Fig. 1. Also, supposing M ants cover per second the paths, to every direction with constant velocity $u\ cm/s$, and deposit a unit of pheromone in each path. In this paper the evaporation of pheromone is neglect.

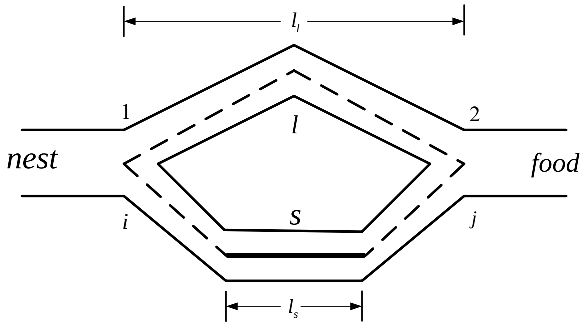

Figure 1. The pheromone deposition of ants

If l_s and l_l are the lengths in cm, of the sorter (s) and longer (l) path, and t_s, t_l respectively.

Is the time that an ant choose to cover the sorter (s) and longer (l) path, then the probability $P_{ia}(t)$ for an ant to reach the decision point $i \in \{1.2\}$ and choose the path $a \in \{s, l\}$ is [7]:

$$P_{ia} = \frac{(t_s + \Phi_{is}(t))^a}{(t_s + \Phi_{is}(t))^a + (t_s + \Phi_{il}(t))^a} \quad (1)$$

Where $\Phi_{i\alpha}(t)$ is the total amount of pheromone on the paths, which is proportional to the number of ants use the paths in time t. experimentally to accept α=2.

The amount of pheromone $\Phi_{is}(t)$ (respectively $\Phi_{il}(t)$) which is deposited on the S path (respectively l) is given by solution of the following difference equation system [8]:

$$\Phi_{is}(t+1) - \Phi_{is}(t) = M\left[P_{js}(t - t_s) + P_{is}(t)\right] \quad (2)$$

$$\Phi_{il}(t+1) - \Phi_{il}(t) = M\left[P_{jl}(t - t_s) + P_{il}(t)\right] \quad (3)$$

In fact, the difference equations (2), (3) are of the first degree, non homogenous.

To point $\Phi_{is}(t-1) \equiv y_{t+1}, \Phi_{is}(t) \equiv y_t$ and $M\left[P_{js}(t - t_s) + P_{is}(t)\right] \equiv g_t$. The difference equation (2) to transform:

$$y_{t+1} - y_t \equiv g_t \quad (4)$$

The general solution is: $y_t = k + \sum_{\lambda=1}^{t-1} g_\lambda$

$$\text{or } y_t = k + \sum_{\lambda=1}^{t-1} M\left[P_{js}(t - t_s) + P_{is}(t)\right]$$

$$\text{or } \Phi_{is}(t) = k + \sum_{\lambda=1}^{t-1} M\left[P_{js}(t - t_s) + P_{is}(t)\right] \quad (5)$$

Similarly the solution of the difference equation (3) is:

$$\Phi_{il}(t) = k + \sum_{\lambda=1}^{t-1} M\left[P_{jl}(t-t_s) + P_{il}(t)\right] \tag{6}$$

The equation (5) expresses the deposit of the pheromone in time t of the path s at the decision point i. It is given by the constant number, of the number M of ants multiplied with the probability of ants to choose the sorter path at the decision point j in time t-t_s and with the probability of ants to choose the shorter path at the decision point i at time t. Finally the constant k is added. The equation (3) expresses the deposit of pheromone on the longer path.

3. PROBLEM FORMULATION

Suppose a set of n customers, each with a known location. Vehicles with a known capacity Q must deliver ordered quantities q_i (i=1, …, n) of items from a single depot (i=0) to n customers.

Knowing the distance between customers the aim is to define and minimize the objective function of the VRP, using an Ant Colony System – Node (ACS-N) algorithm.

The VRP from mathematical view can be described using a complete directed graph G=(V,A), where V={0,1, …, n} is the set of nodes and $A = \{(i, j) \mid i, j \in V\}$ is a set of arcs. Nodes i, …, n represent customers.

To each customer i are associated a nonnegative demand q_i and a nonnegative service duration t_i.

Node 0 is the depot at which is based fleet of m homogenous vehicles of capacity Q. To each arc(i,j) is associated a travelling cost C_{ij} calculated with the following equation:

$$C_{ij} = \sqrt{(i_x - j_x)^2 + (i_y - j_y)^2}$$

Where i_x is the coordinate x for the customer i and i_y is the coordinate y for customer i.

The VRP consist be design of m vehicle routes on G such as:

- each customer is visited only once,
- the total demand of the route, does not exceed the vehicle capacity q,
- the length of any route does not exceed a pre-set maximal route length L,
- in some version m is fixed a priori in other is a decision variable,
- the total cost of all vehicle routes is minimized.

The VRP is more complex that the travelling salesman problem (TSP) as every route in VRP is as TSP. Generally, if we have M vehicles in a VRP, then in order to get a best set of routes for the M vehicles the M number of TSP have to be solved.

The value of the objective function of the VRP is calculated using the following equation [9]:

$$F = \sum \lambda_{ij} C_{ij}^{-1} \tag{7}$$

Where: λ_{ij} : shows the number of traverses that took place for the arc joining nodes i and j

C_{ij}^{-1} : is the cost of traverse for the arc.

The value of this objective function finally, arise from adding the cost of every arc used, multiplied with the number of times that is traversed. Building a path network with the lowest cost arise from minimizing equation (7).

4. MATHEMATICAL MODEL OF (ACS-N) ALGORITHM

In (ACS-N) algorithm an ant represent a vehicle. We concentrate on the way of choosing the next node by every ant. This selection is related with the pheromone quantity that has been displayed on every arc also with the traverse cost of this arc. Equations (8) and (9) are used to select the next node j for ant k currently at node i. Equation (8) is a greedy selection technique favoring nodes which possess the best combination of short path and large pheromone quantity. Equation (9), (is called random proportional transition rule) balances this by allowing a probabilistic selection of the next node.

$$j = \begin{cases} \arg\max\left\{[\tau_{ij}(t)] * [n_{ij}]^{\beta}\right\}, if \quad q \le q_0 \text{ (exploitation)} \\ Equation \quad (3) \quad , otherwise \quad \text{(exploitation)} \end{cases} \tag{8}$$

$$P_{ij}^{k}(t) = \begin{cases} \dfrac{\tau_{ij}(t)[n_{ij}]^{\beta}}{\sum_{r \in J_i^k} [\tau_{ir}(t)] * [n_{ir}]^{\beta}}, if \quad j \in J_i^k \\ 0 \quad , otherwise \end{cases} \tag{9}$$

Where:

$\tau_{ij}(t)$: is the amount of pheromone on arc (i,j) at time t,

n_{ij}: is the inverse of the distance between nodes i and j, called visibility,

β: is a parameter which controls the relative weight of the visibility,

q: is a uniform random number $q \in [0.1]$

q_0: is a parameter,

J_i^k: is the set of the nodes to which ant k can move when being located in node i.

In this paper the pheromone placed at the nodes of the graph G and not at the arcs, so the local update rule is [10]:

$$\tau(j) \leftarrow (1-\rho)\tau(j) + \rho\tau_0\left(1 - \frac{d(u_k(i-1), j) - d_{\min}}{d_{\max} - d_{\min}}\right) \tag{10}$$

Where:

$u_k(i-k)$ is the node from which ant k just arrived

$d_{\min}$ and $d_{\max}$ are the lengths of the shortest and longest links in the problem respectively.

The global update rule is:

$$\tau(j) \leftarrow (1-\rho)\tau(j) + \rho\frac{Q}{L}\left(1 - \frac{d(i, j) - d_{\min}}{d_{\max} - d_{\min}}\right) \tag{11}$$

Where:

i: is the node that precedes node j in the global best solution.

L: is the length of the best (shortest) tour to date,

Q: is a problem dependent parameter

ρ : is a parameter governing pheromone decay, 0< ρ <1.

5. CASE STUDY

A computer program has been constructed which shows how the concepts described above can be put into practice and tests the corrections of the proposed algorithm.

The results taken from this program are illustrated using the input data of the table1.

Table 1The example of VRP (Input data form)

node	need
1	0
2	500
3	900
4	200
5	90
6	800
7	1000

8	50

The run of the program has been done for the following examples:

Example 1: Using the algorithm (ACS-N) the objective function of VRP should be minimized with the following characteristics:

- Initial value of pheromone trails: 500
- Evaporation parameter (ρ): 0.1
- Cost parameter (β): 2
- Maximum load of an ant: 50
- Number of ants: 10

The program run for 75 circles and the results are given at the Figure 2.

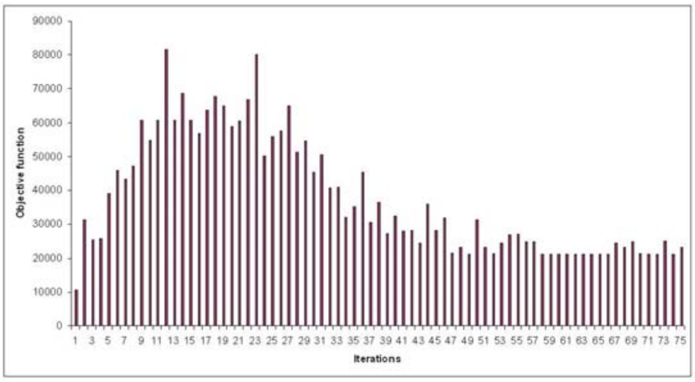

Figure 2. Objective function optimization in connection with time

From Figure 2 we have the following results:

A. Until the 22nd iteration the objective function shows some fluctuates and specifically some incremental tension. This paradox explained as follows: Because of the operation of the local search, the ants scan all the graph trying to find the best solution, avoiding the phenomenon of the converge of the semi-optimum solutions. Trying many routes it is logical to be created and some solutions that are far from the best.

B. After the 22nd iteration and after the progressive scan of the graph, the ants are starting to stabilize to more and more better solutions and at last (after the 45th iteration) to the best one that has value near to 20.000.

Example 2: Using the algorithm (ACS-N) the objective function of VRP should be minimized with the following characteristics:

- Initial value of pheromone trails: 500
- Evaporation parameter (ρ): 0.1
- Cost parameter (β): 2
- Maximum load of an ant: 50
- Number of ants: 20

The minimizing of the objective function of the VRP is given at the Figure 3

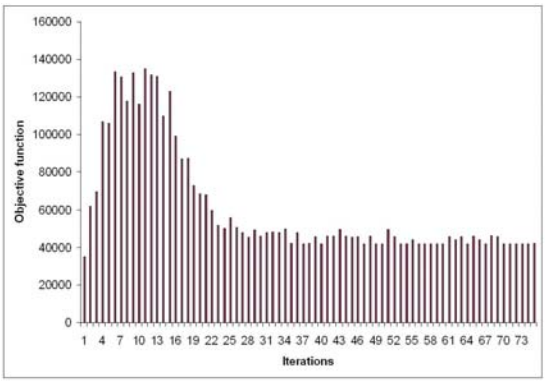

Figure 3. Objective function optimization in connection with time

From Figure 3 we have the following results:

Initially we have some incremental tensions of the objective function caused by the local search. This time after the 17th iteration the ants converge with rapid speed to the best solution which they found and keep from the 28th iteration. It is obvious that the number of the ants that we have chosen is the critical factor for the course of the algorithm and the best solution speed up. The best value of the objective function is close to the 40.000 that it is normal, because we used double number of ants and consequently has been done the double volume of crossing to the best route.

6. CONCLUSION

In the ACO meta-heuristic a set of agents, called ants, build solutions on the given problem cooperating through pheromone mediated indirect communications. In this paper we use an Ant Colony System-Node (ACS-N) algorithm of family algorithms ACO for solving the VRP problem.

The basic characteristic of the (ACS-N) algorithm is that it put pheromone at the nodes of the graph, that describes the given problem, and not at the arcs.

Many researchers suggest that the number of the ants should be equal to the number of the nodes of the graph that describe the problem. At the *Example 1* we saw that the 10 ants we used were proven too few, in order to find the best solution in short time. In reverse, when we used 20 ants, *Example 2,* the results were much better. The number of the ants can be chosen like the rational function of the overall demand of the graph in cargo divided by the maximum load that an ant can carry.

Except from the number of the ants, important role at the implementation of the (ACS-N) algorithm are:

- the starting quantity of the pheromone
- evaporation parameter (ρ)
- cost parameter (β) and
- the maximum load of an ant.

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